Feb 11: 4D10 Knots in Solid tori and Companionship.

Note Title   2/7/2010

Ex 4D3: Suppose L is a closed subset of a solid torus V, then TFAE:

   (a)   L is geometrically essential (i.e., L intersects every meridinal disk of V).

   (b)   There does not exist a 3-ball B s.t. $L \subset B \subset V$.

   (c)   The inclusion homomorphism $\pi_1(\partial V) \to \pi_1(V - L)$ is 1:1.

Let L be geometrically essential link in $V = S^1 \times D^2$.

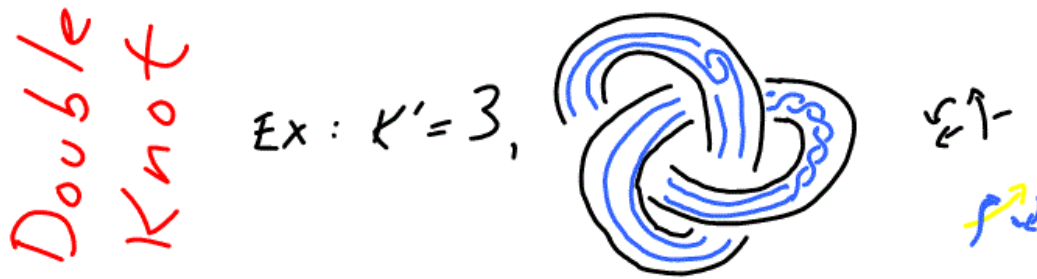Let K' be a knot and let V' be a tubular nbhd of K'. Let h: $V \to V'$, h(L) = J. Then J is a *satellite link* with *companion knot* K'.

Ex: *Double knots*: L =

   If companion K' = the unknot, then satellite J = twist knot.

$$Ex: \quad h(\ell) = \ell + m \quad \to \quad$$

To uniquely define J, require h to be *faithful*: $h(S^1 \times 1)$ = preferred longitude.
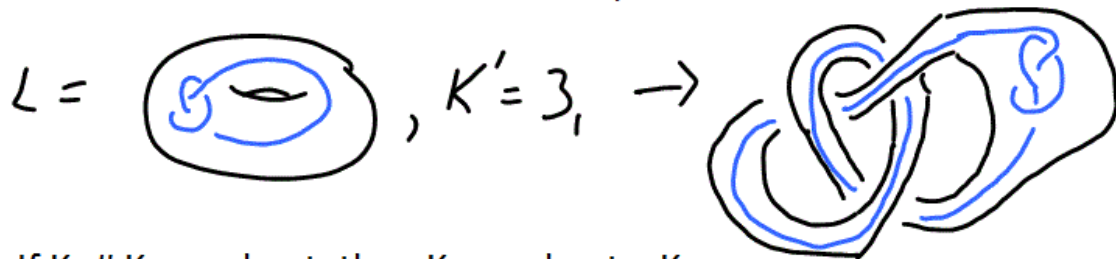
Double Knot

Ex : $K' = 3$,

Ex: *Cable knot*: L = (p, q) torus knot, p ≠ 0, and the companion K' is non-trivial. If h is faithful, then J is called the (p, q) cable of K'.

If p = ±1, then J = K' .

Thm 4D9: If K' is a companion of J, then $\pi_1(J)$ contains a subgroup isomorphic to $\pi_1(K')$

Cor 4D10: Any knot with non-trivial companion is non-trivial.

Note: $K_1 \# K_2$ is a satellite knot with companion K_1.

$L = $ ⬭ , $K' = 3$, →

Cor: If $K_1 \# K_2$ = unknot, then K_1 = unknot = K_2.

Define K' ≤ J if J is a satellite knot with companion K'.
Note ≤ is reflexive, transitive and anti-symmetric for prime knots (composites?) and thus ≤ gives a partial order of (prime?) knot types.

Thm (Thurston 1978): A knot is either a (1) torus knot or (2) satellite knot or (3) a hyperbolic knot.

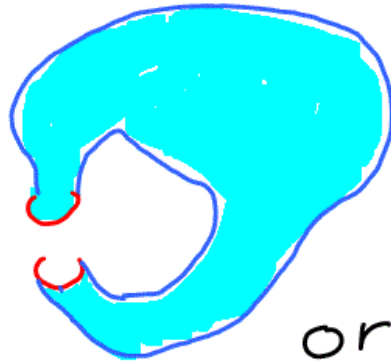Minimal genus Seifert surfaces are not unique:

Satellite
Knot
w/ companion
4_1

← not trivial
Knot

$+$

or

$+$ $=$