

Thm 7C5: Let $p(t)$ be any Laurent polynomial satisfying

$$(1) \quad p(1) = \pm 1$$

$$(2) \quad p(t) = p(t^{-1})$$

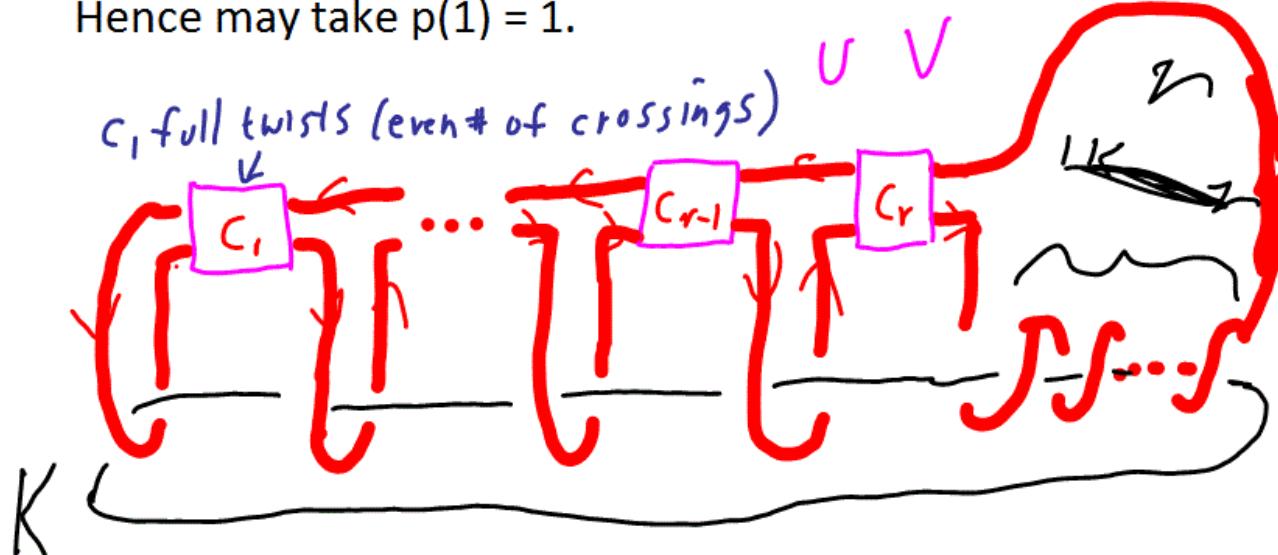
i.e., $p(t) = c_r t^{-r} + \dots + c_1 t^{-1} + c_0 + c_1 t + \dots + c_r t^r$

and $c_r + \dots + c_1 + c_0 + c_1 + \dots + c_r = \pm 1$.

Then \exists Knot K w/ $Alex\ poly p(t)$

Proof: If the Alexander polynomial of a knot K is $q(t)$, then $\pm t^k q(t)$ is also an Alexander polynomial for K .

Hence may take $p(1) = 1$.



$$c_i = \underbrace{XX \dots XX}_{2c_i \text{ crossings}}$$

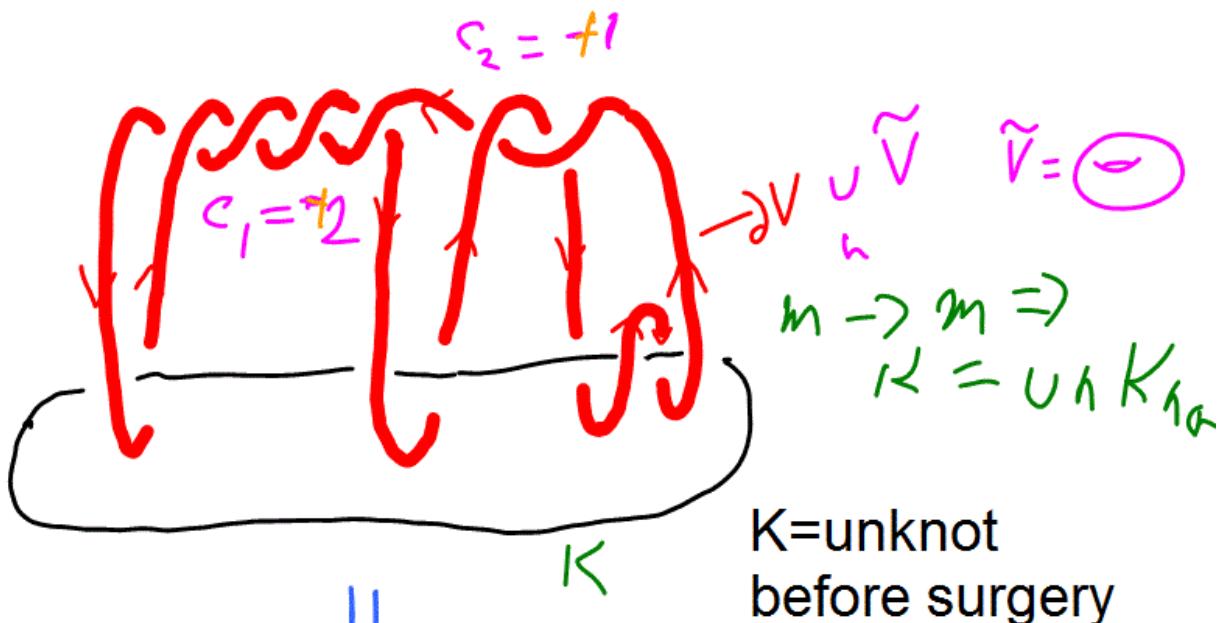
$2c_i$ crossings

Right-handed if $c_i > 0$

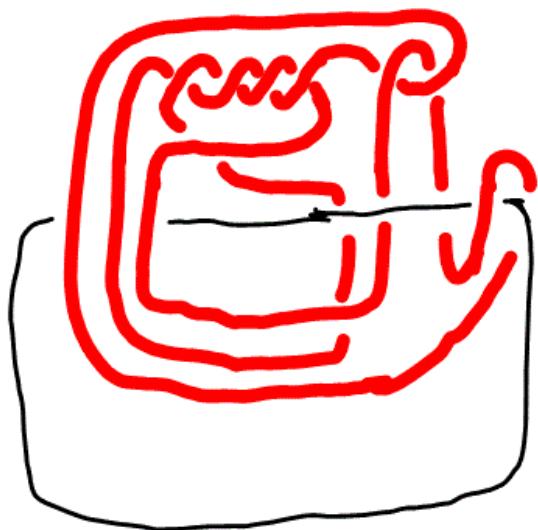
$$\text{or } \underbrace{XX \dots XX}_{2c_i \text{ crossings}}$$

$2c_i$ crossings

Left-handed if $c_i < 0$



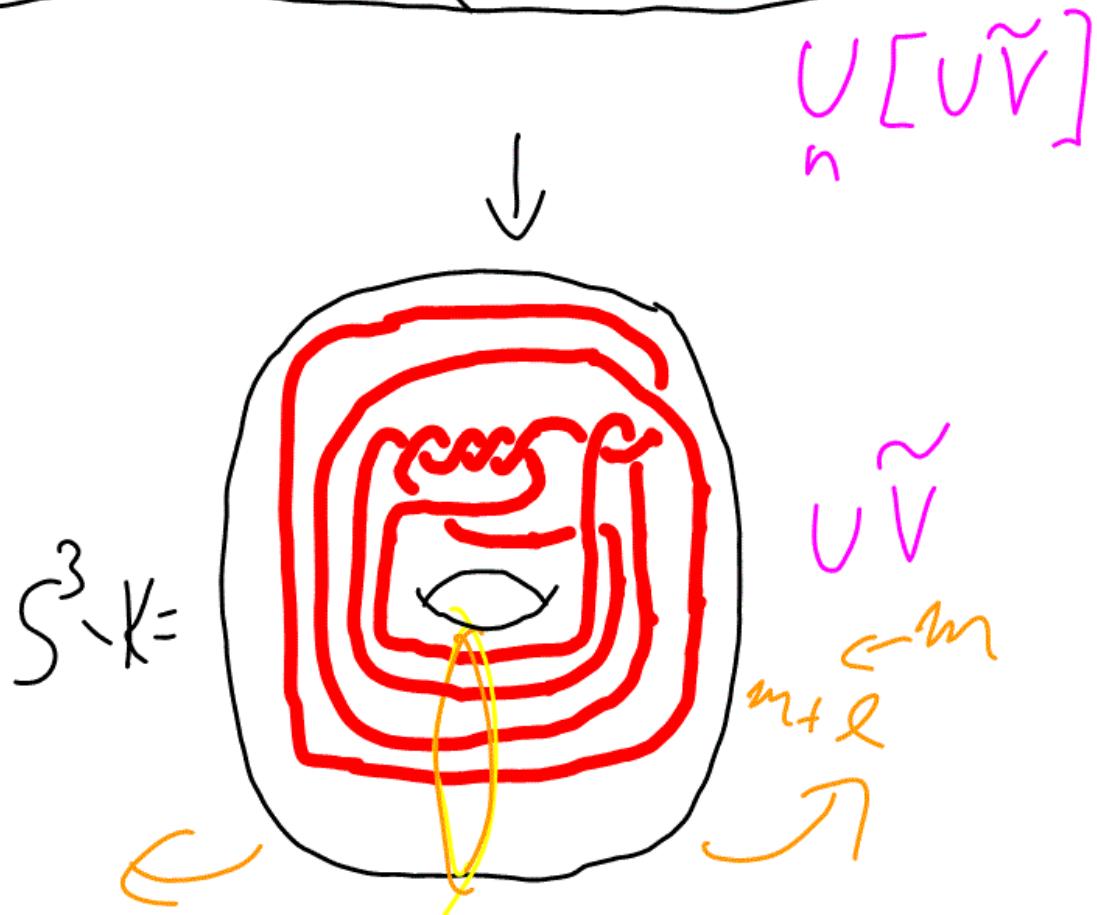
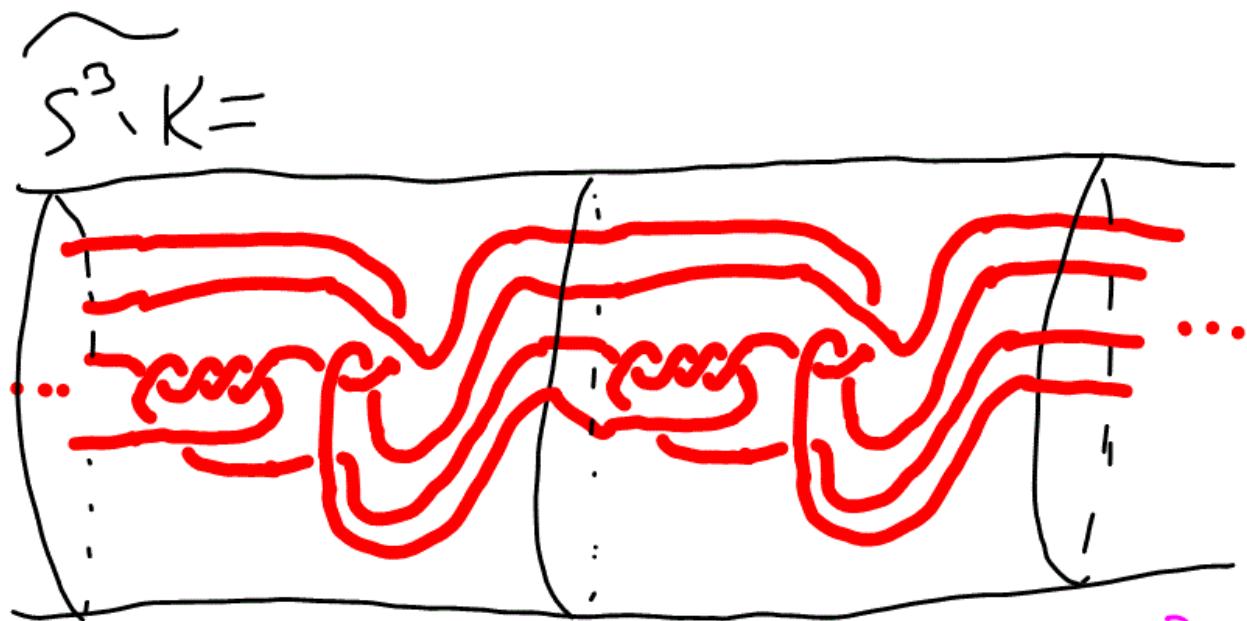
$K = \text{unknot}$
before surgery
on red torus

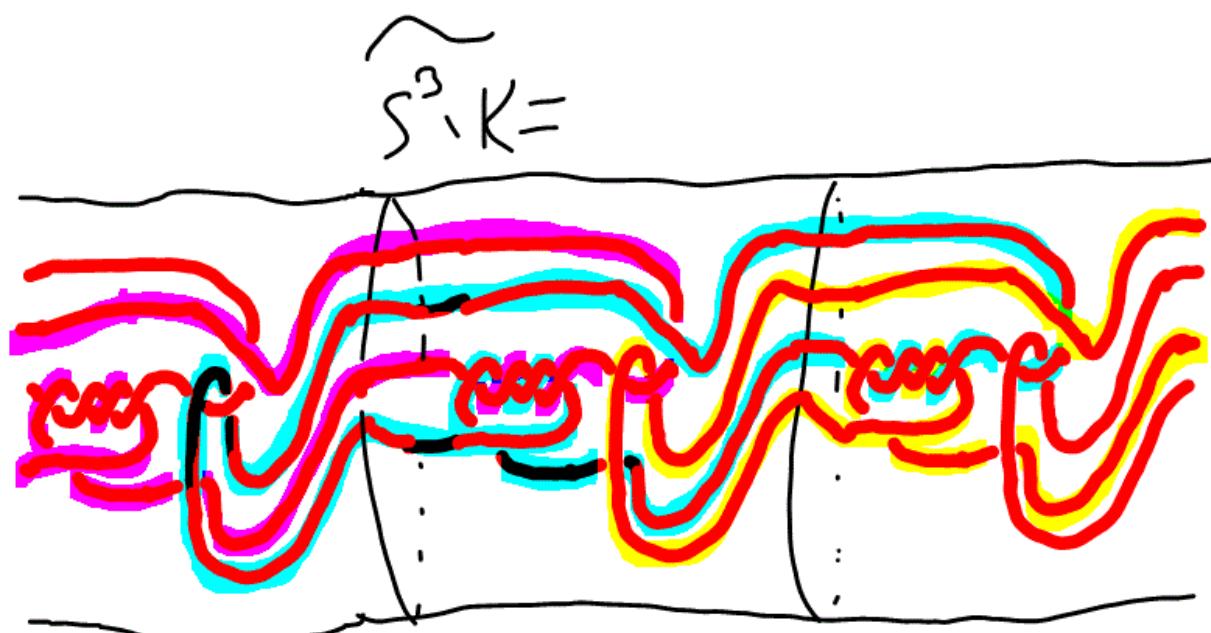
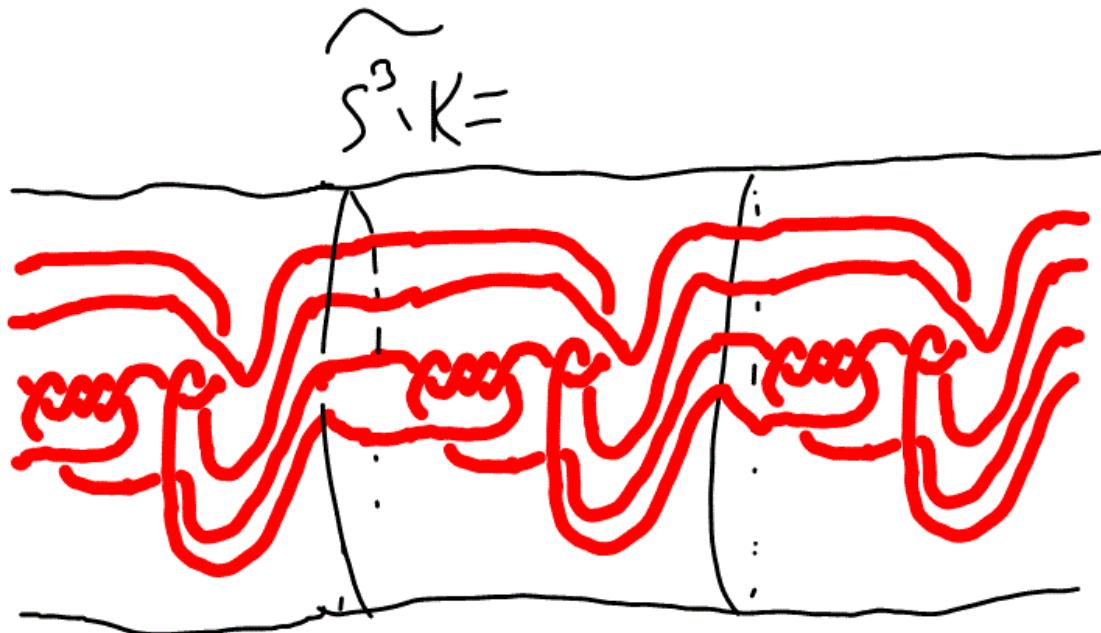


After surgery,
 $K = \text{knot with}$
Alexander
polynomial ??

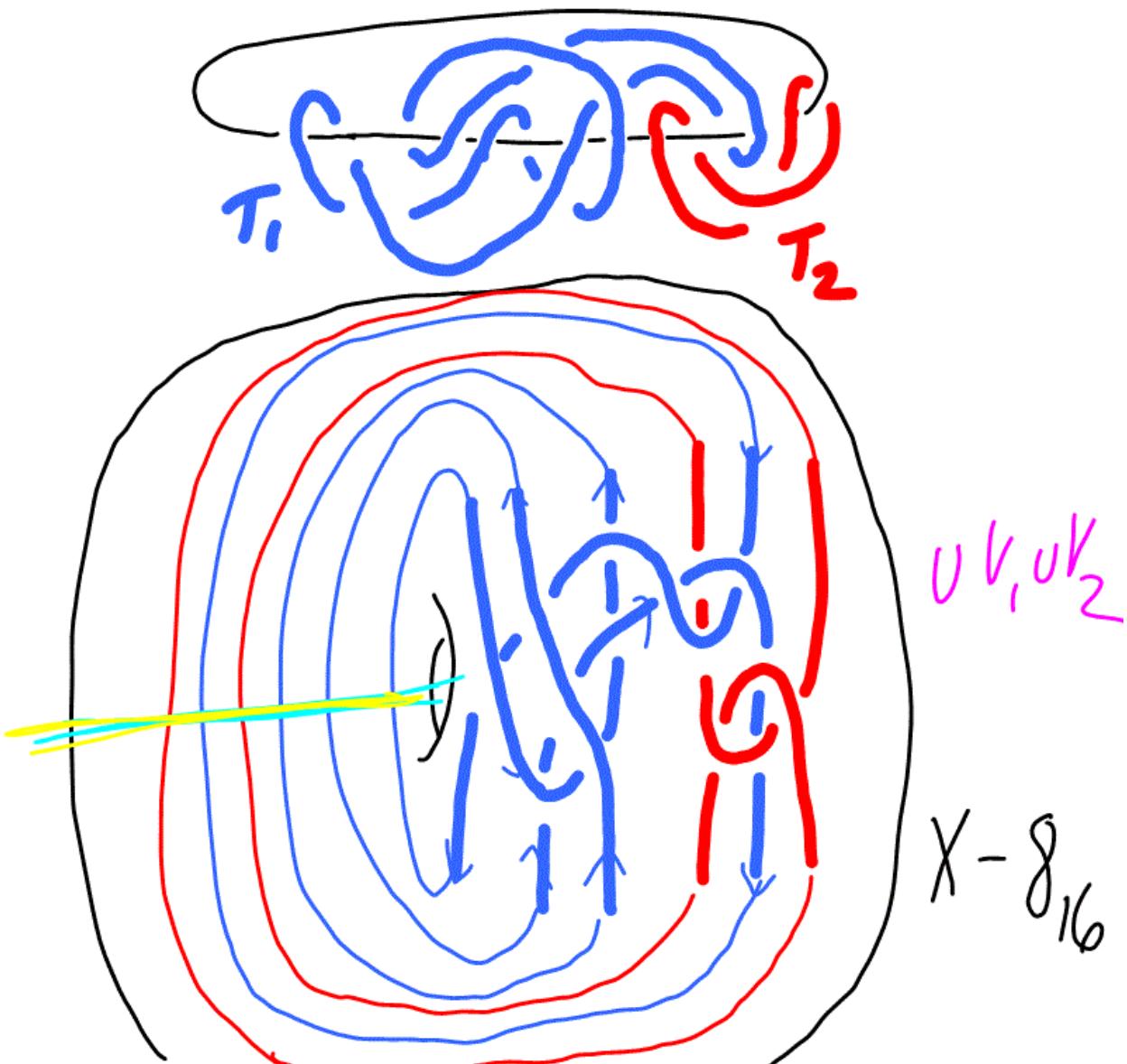
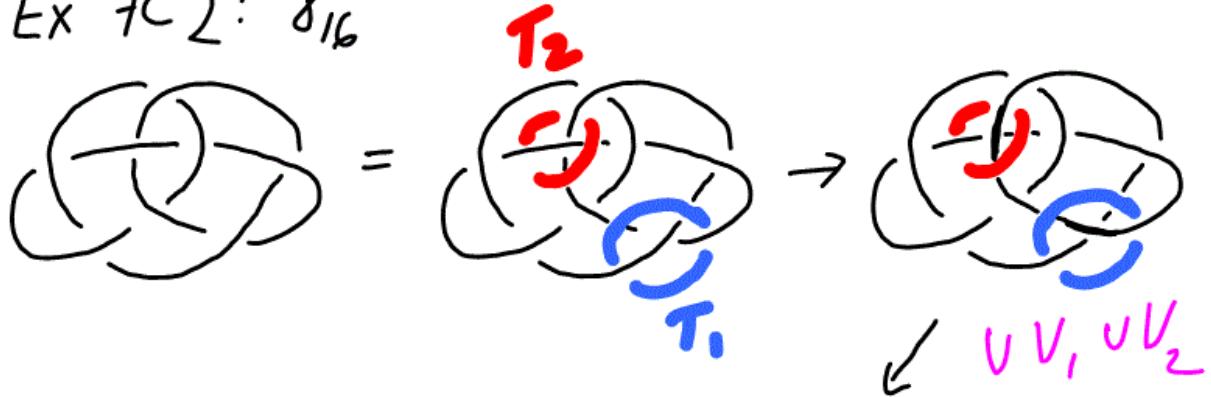
$$\omega r = 5$$

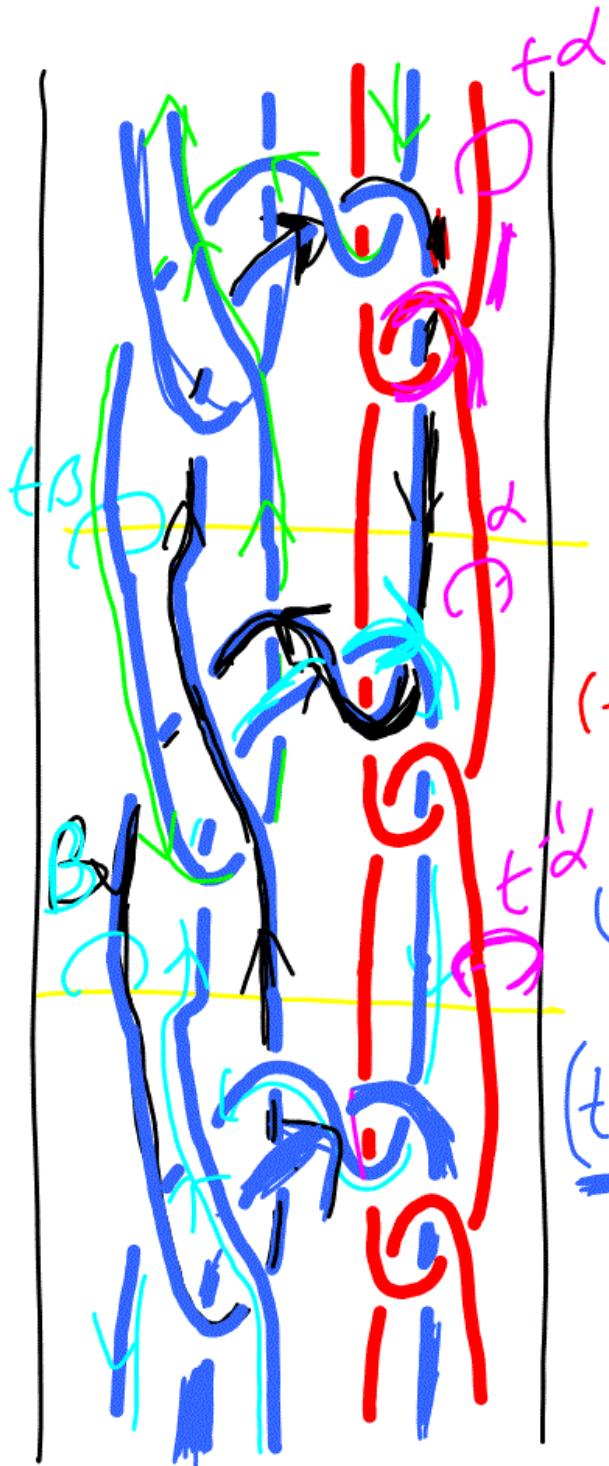






Ex 7C2: 8_{16}

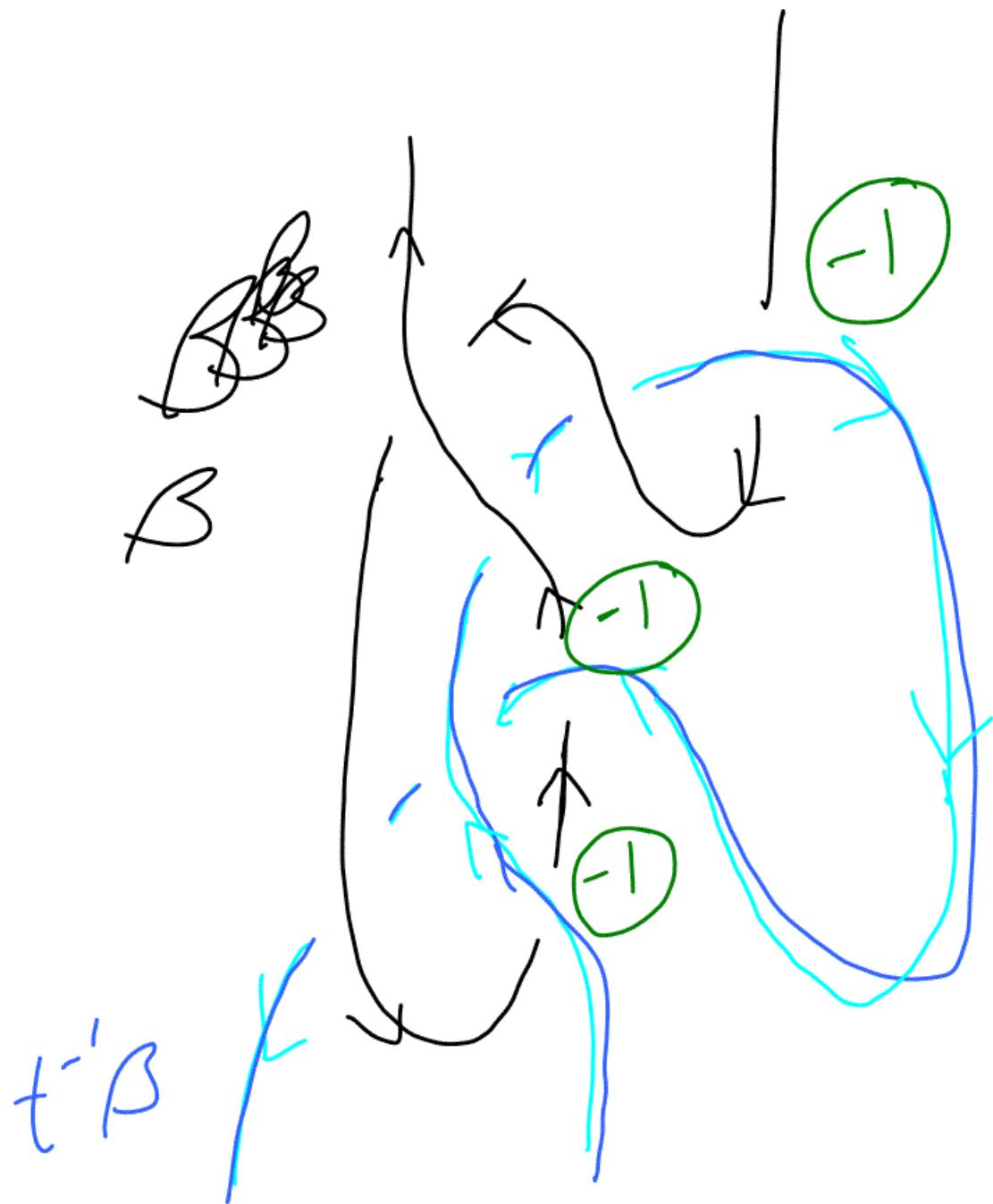




$$H_1(\tilde{X}) = \\ (\alpha, \beta | r_1, r_2)$$

$\cup_{i \in \mathbb{Z}} V_i$ (red circle)
 $(-t^{-1} + 1 - t) \alpha + (t^{-1} - 1) \beta = 0$

$\cup_{i \in \mathbb{Z}} V_i$ (blue oval)
 $\frac{(t^{-2} - 3t^{-1} + 5 - 3t + t^2)}{+ (t-1) \alpha} \beta$
 $+ 1$



$$\begin{aligned}
 & (-t^{-1} + 1 - t) \alpha + (t^{-1} - 1) \beta = 0 \\
 & (t - 1) \alpha + (t^{-2} - 3t^{-1} + 5 - 3t + t^2) \beta = 0 \\
 & \hline
 & -t^{-1} \alpha + (t^{-2} - 2t^{-1} + 5 - 3t + t^2) \beta = 0
 \end{aligned}$$

$$\Rightarrow \alpha = (t^{-1} - 2 + 5t - 3t^2 + t^3) \beta$$

$$\begin{aligned}
 & (-t^{-1} + 1 - t)(t^3 - 3t^2 + 4t - 2 + t^{-1}) \beta + (t^{-1} - 1) \beta \\
 & = (-t^{-2} + 4t^{-1} - 8 + 9t - 8t^2 + 4t^3 - t^4) \beta = 0
 \end{aligned}$$

$$\Rightarrow H_1(\tilde{X}) = \bigwedge_{-t^{-3} + 4t^{-2} - 8t^{-1} + 9 - 8t^{-1} + 4t^2 - t^3}$$

$$\text{Note } p(t) = p(t^{-1}) \nmid -1 + 4 - 8 + 9 - 8 + 4 - 1 = -1$$