Thm 5.3: Suppose $T : \mathbf{R}^2 \to \mathbf{R}^2 \in C^1$

$T(u, v) = (T_1(u, v), T_2(u, v)) = (x(u, v), y(u, v))$

If $D^* \subset \mathbf{R}^2$ and if $f : T(D^*) \to \mathbf{R}$ is integrable, then

$\int\int_{T(D^*)} f(x, y)dxdy =$

$$\int\int_{D^*} f((x(u, v), y(u, v)))abs(det(DT(u, v)))dudv$$

Ex: Evaluate $\int\int_{T(D^*)} xydxdy$

where $T(D^*) =$ the parallelogram where the vectors (4, 1) and (2, 3) form two of its sides.

Find D^* and $T : \mathbf{R}^2 \to \mathbf{R}^2$ such that $T(D^*)$ is the parallelogram described above.

Note T is a linear transformation.

$T : \mathbf{R}^2 \to \mathbf{R}^2$, $T(u, v) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$

$$T(1,0) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$T(0,1) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Hence $T : \mathbf{R}^2 \to \mathbf{R}^2$,

$$T(u,v) = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 4u + 2v \\ u + 3v \end{pmatrix} = \begin{pmatrix} x(u,v) \\ y(u,v) \end{pmatrix}$$

Let $x = 4u + 2v$ and $y = u + 3v$

$$DT = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} \text{ and } abs(det \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}) = 12 - 2 = 10$$

$$\int \int_{T(D^*)} (xy) dxdy$$
$$= \int \int_{D^*} f((x(u,v), y(u,v))) abs(det(DT(u,v))) dudv$$

$$= \int \int_{D^*} (4u + 2v)(u + 3v)(10) dudv$$

$$= 10 \int_0^1 \int_0^1 (4u^2 + 14uv + 6v^2) dudv$$

$$= 10 \int_0^1 (\tfrac{4}{3}u^3 + 7u^2v + 6uv^2|_0^1 dv$$

$$= 10 \int_0^1 (\tfrac{4}{3} + 7v + 6v^2) dv$$

$$= 10(\tfrac{4}{3}v + \tfrac{7}{2}v^2 + 2v^3)|_0^1$$

$$= 10(\tfrac{4}{3} + \tfrac{7}{2} + 2)$$