Exam 1 Feb. 26, 2009 SHOW ALL WORK

Math 28 Calculus III Either circle your answers or place on answer line.

[14] 1.) Use the chain rule to calculate $D(f \circ g)(s,t)$ where $f : \mathbf{R}^2 \to \mathbf{R}^3$, $f(x,y) = (x, y, e^{xy})$ and $g : \mathbf{R}^2 \to \mathbf{R}^2$, $g(s,t) = (t^2, sin(st))$.

Answer: $D(f \circ g)(s,t) =$ ____________________________

[14] 2a.) Suppose $f(x, y) = e^{xy}$. Approximate $f(1.9, 0.1)$ by finding a best linear approximation to f at an appropriate $\mathbf{x} = \mathbf{a}$.

Answer 2a: $f(1.9, 0.1) \sim$ ________________

[6] 2b.) $D_{(3,4)}f(10, 2) =$ ________________________ where $f(x, y) = e^{xy}$.

[5] 3a.) $proj_{(1,2)}(8,6) =$ ______________

[4] 3b.) Suppose that a force $\mathbf{F} = (8,6)$ is acting on an object moving parallel to the vector (1, 2). Decompose the vector (8, 6) into a sum of vectors $\mathbf{F_1}$ and $\mathbf{F_2}$ where $\mathbf{F_1}$ points along the direction of motion and $\mathbf{F_2}$ is perpendicular to the direction of motion.

Answer 3b: $\mathbf{F_1} =$ ______________, $\mathbf{F_2} =$ ______________

[1] 3c.) Verify that $\mathbf{F} = \mathbf{F_1} + \mathbf{F_2}$

[4] 3d.) Use the dot product to verify that $\mathbf{F_1}$ and $\mathbf{F_2}$ are perpendicular to each other. Explain how the dot product can be used to verify that two vectors are perpendicular.

[12] 4.) Find the following limit if it exists. If it doesn't exist, state why you know it doesn't exist.

$$lim_{(x,y)\to(0,0)} \frac{2x^2 - y^2}{x^2 + y^2}$$

[5] 5.) State the limit definition of differentiable:

$f : \mathbf{R}^n \to \mathbf{R}$ is differentiable at $\mathbf{x} = \mathbf{a}$ if

[12] 6a.) Let $f : \mathbf{R}^2 \to \mathbf{R}$, $f(x, y) = x^2 + 4y^2$. Draw several level curves of f (make sure to indicate the height c of each curve). Draw vectors in the direction of the gradient of f at $(\sqrt{12}, -1)$ and at (0, 2). The length of your vectors should denote their relative magnitudes.

[2] 6b.) Identify the quadric surface in 6a: ________________

[12] 7.) State the equation for the line of intersection of the planes $2x - y + 3z = 10$ and $4x + 5y - 10z = 20$

Answer ____________________

8.) Circle T for True and F for False:

[3] a.) Suppose $f : \mathbf{R}^n \to \mathbf{R}$. If f is differentiable, then $\frac{\partial f}{\partial x_i}(\mathbf{a})$ exists and is continuous for $i = 1, ..., n$. T F

[3] b.) Suppose $f : \mathbf{R}^n \to \mathbf{R}$. If $\frac{\partial f}{\partial x_i}(\mathbf{a})$ exists and is continuous for $i = 1, ..., n$, then f is differentiable at $\mathbf{a}$. T F

[3] c.) Suppose $f : \mathbf{R}^n \to \mathbf{R}$. If $D_{\mathbf{v}}(f)(\mathbf{a})$ exists for all $\mathbf{v}$, then f is differentiable at $\mathbf{a}$. T F

[3] d.) If f is continuous, then f is differentiable. T F