[10] 1.) Give the formula involving iterated integral(s) for calculating the average value of $f(x, y) = xe^y$ over the region $x^2 + 4y^2 \leq 1$ (you do not need to evaluate the integrals).

[5] 2.) For the following integral, sketch the region of integration in $\mathbf{R}^3$. Give a geometric argument as to why the value of this integral is zero.

$$\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \int_{-\sqrt{y-x^2}}^{\sqrt{y-x^2}} z \ dzdxdy$$

[10] 3.) For the following integral, sketch the region of integration and reverse the order of integration (do not evaluate the integral).

$$\int_0^4 \int_0^{\sqrt{16-x^2}} \sqrt{x^2 + y^2} \; dydx = \underline{\hspace{6cm}}.$$

[10] 4.) Transform the given integral in Cartesian coordinates to one in polar coordinates (do not evaluate the integral).

$$\int_0^4 \int_0^{\sqrt{16-x^2}} \sqrt{x^2 + y^2} \; dydx = \underline{\hspace{6cm}}.$$

5.) Let $f(x, y) = x^2 - xy$

[5] 5a.) Calculate the Hessian matrix of f at $(x, y) = (0, 2)$

[5] 5b.) Find the first order Taylor polynomial for f at $(x, y) = (0, 2)$

[5] 5c.) Find the second order Taylor polynomial for f at $(x, y) = (0, 2)$

[5] 5d.) Use the fact that the total differential df approximates the incremental change Δf to estimate $f(0.98, 2.1)$.

[3] 5e.) Is the value of the function near $(x, y) = (0, 2)$ more sensitive to changes in x or y?

Problem 5 continued.)

[3] 5f.) Give an equation for the tangent plane to f at $(x, y) = (0, 2)$

[4] 5g.) The critical points of f are _____

[3] 5h.) Use the sequence of principal minors of the Hessian of f to determine the nature of the critical points (i.e, local max/min/saddle or no info).

6.) Let $F(x, y, z) = (0, xy, z)$

[5] 6a.) The divergence of $F =$ _____

[5] 6b.) The curl of $F =$ _____

7.) Let $x(t) = (4cos(t), 4sin(t), 3t)$, $1 \leq t \leq 5$, be a path in $\mathbf{R}^3$.

[5] 7a.) Find the arclength parameter $s = s(t)$ for this path.

[3] 7b.) The length of this path is _____ .

[5] 7c.) Express the original parameter t in terms of s and reparametrize the path $\mathbf{x}$ in terms of s.

[5] 7d.) Determine the moving frame $[\mathbf{T}, \mathbf{N}, \mathbf{B}]$

$\mathbf{T} = $ _____ , $\mathbf{N} = $ _____ , $\mathbf{B} = $ _____

[5] 7e.) The curvature of this path is _____

[5] 7f.) The torsion of this path is _____