Final Exam PART A, May 13, 2008 SHOW ALL WORK
Math 28 Calculus III Either circle your answers or place on answer line.

All problems required on this part of the exam.

[13] 1.) Let $f(x) = (2x, e^{3x-3})$. Let $g(x, y) = \sqrt{x^2y - 4}$. Use the chain rule to calculate $D(f \circ g)(1, 4)$ and $D(g \circ f)(1)$

$D(f \circ g)(1, 4) =$ ______________ $D(g \circ f)(1) =$ ______________

[13] 2.) **Evaluate** the following integral by transforming this integral in Cartesian coordinates to one in polar coordinates. **Sketch** the region of integration for the integral in Cartesian coordinates and the region of integration for the integral in polar coordinates.

$\int_{-2}^{2} \int_{0}^{\sqrt{4-x^2}} e^{-(x^2+y^2)} \, dydx =$ ______________________________________.

[13] 3.) Let $S$ denote the surface of the cylinder $x^2 + y^2 = 9$, $-1 \leq z \leq 1$.

A parametrization of $S$ is ______________________________

Use this parametrization to calculate $\int\int_S 1 dS =$ ______________.

The surface area of $S$ is ______________.

[13] 4.) Use a Lagrange multiplier to find the largest sphere centered at the origin that can be inscribed in the ellipsoid $3x^2 + 2y^2 + z^2 = 6$.

Final Exam PART B, May 13, 2008 SHOW ALL WORK
Math 28 Calculus III Either circle your answers or place on answer line.

Choose 4 out of the following 7 problems: **Clearly indicate which 4 problems you choose.** Each problem is worth 12 points You may do more than 4 problems for up to five points extra credit.

I have chosen the following 4 problems: ______________________________

A.) Find the following limit if it exists. If it doesn't exist, state why you know it doesn't exist.

$$lim_{(x,y)\to(0,0)} = \frac{xy - 2x^2}{x^2 + y^2}$$

B.) Let $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbf{R}^n$.

Is the scalar product associative (i.e., does $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c}) = (\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$)?

Is the cross product associative (i.e., does $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$)?

Prove that $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{b}$.

C.) Show that the vector field $\mathbf{F}(x, y) = (y^2 + 2x + 4)\mathbf{i} + (2xy + 4y - 5)\mathbf{j}$ is conservative.

Find a scalar potential function for $\mathbf{F}$

Evaluate $\int_X \mathbf{F} \cdot d\mathbf{s}$ along the path $\mathbf{x} : [2, 5] \to \mathbf{R}^2$, $\mathbf{x}(t) = (t\sqrt{t^2 + 1}, 2t^2 + 3)$

D.) Find the arclength parameter $s = s(t)$ for the path $\mathbf{x}(t) = (t^3, t^2)$, $0 \leq t \leq 10$

The length of this path is ________________.

Express the original parameter $t$ in terms of $s$: ________________________________.

Reparametrize $\mathbf{x}$ in terms of $s$:

E.) Let $f(x, y, z) = x^2 sin(yz)$. Calculate the directional derivative of $f$ at $\mathbf{a} = (3, 0, 2)$ in the direction parallel to the vector (3, 4, 0).

F.) Let $\mathbf{x}(t) = (ln(t), 2t, e^{3t})$.

The velocity of this path when $t = 1$ is _______________

The speed of this path when $t = 1$ is _______________

The acceleration of this path when $t = 1$ is _______________

The tangential component of acceleration of this path when $t = 1$ is _______________

The normal component of acceleration of this path when $t = 1$ is _______________

G.)