

$$p_2(\mathbf{x}) = f(\mathbf{a}) + Df(\mathbf{a})(\mathbf{x} - \mathbf{a}) + \frac{1}{2}(\mathbf{x} - \mathbf{a})^T Hf(\mathbf{a})(\mathbf{x} - \mathbf{a})$$

If $f : \mathbf{R}^2 \rightarrow \mathbf{R}$,

$$p_2(x, y) = f(a_1, a_2) + \begin{pmatrix} \frac{\partial f}{\partial x}(a_1, a_2) & \frac{\partial f}{\partial y}(a_1, a_2) \end{pmatrix} \begin{pmatrix} x - a_1 \\ y - a_2 \end{pmatrix} \\ + \frac{1}{2}(x - a_1, y - a_2) \begin{pmatrix} \frac{\partial^2 f}{\partial x^2}(a_1, a_2) & \frac{\partial^2 f}{\partial y \partial x}(a_1, a_2) \\ \frac{\partial^2 f}{\partial y \partial x}(a_1, a_2) & \frac{\partial^2 f}{\partial y^2}(a_1, a_2) \end{pmatrix} \begin{pmatrix} x - a_1 \\ y - a_2 \end{pmatrix}$$

$$f(a_1, a_2) + \begin{pmatrix} f_x(a_1, a_2) & f_y(a_1, a_2) \end{pmatrix} \begin{pmatrix} x - a_1 \\ y - a_2 \end{pmatrix} \\ + \frac{1}{2}(x - a_1, y - a_2) \begin{pmatrix} f_{xx}(a_1, a_2) & f_{xy}(a_1, a_2) \\ f_{yx}(a_1, a_2) & f_{yy}(a_1, a_2) \end{pmatrix} \begin{pmatrix} x - a_1 \\ y - a_2 \end{pmatrix}$$

$$= f(a_1, a_2) + \begin{pmatrix} f_x & f_y \end{pmatrix} \begin{pmatrix} x - a_1 \\ y - a_2 \end{pmatrix} + \frac{1}{2}(x - a_1, y - a_2) \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} \begin{pmatrix} x - a_1 \\ y - a_2 \end{pmatrix}$$

$$= f(a_1, a_2) + f_x[x - a_1] + f_y[y - a_2] + \frac{1}{2}(x - a_1, y - a_2) \begin{pmatrix} f_{xx}[x - a_1] + f_{xy}[y - a_2] \\ f_{yx}[x - a_1] + f_{yy}[y - a_2] \end{pmatrix}$$

$$= f(a_1, a_2) + f_x[x - a_1] + f_y[y - a_2] \\ + \frac{1}{2}(f_{xx}[x - a_1]^2 + f_{xy}[x - a_1][y - a_2] + f_{yx}[x - a_1][y - a_2] + f_{yy}[y - a_2]^2)$$

$$= f(a_1, a_2) + f_x[x - a_1] + f_y[y - a_2] + \frac{1}{2}(f_{xx}[x - a_1]^2 + (f_{xy} + f_{yx})[x - a_1][y - a_2] + f_{yy}[y - a_2]^2)$$

$$p_2(x, y) = f(a_1, a_2) + f_x(a_1, a_2)[x - a_1] + f_y(a_1, a_2)[y - a_2] \\ + \frac{1}{2}(f_{xx}(a_1, a_2)[x - a_1]^2 + (f_{xy} + f_{yx})(a_1, a_2)[x - a_1][y - a_2] + f_{yy}(a_1, a_2)[y - a_2]^2)$$