$$p_2(\mathbf{x}) = f(\mathbf{a}) + Df(\mathbf{a})(\mathbf{x}-\mathbf{a}) + \frac{1}{2}(\mathbf{x}-\mathbf{a})^T Hf(\mathbf{a})(\mathbf{x}-\mathbf{a})$$

If $f : \mathbf{R}^2 \to \mathbf{R}$,

$$p_2(x,y) = f(a_1,a_2) + \begin{pmatrix} \frac{\partial f}{\partial x}(a_1,a_2) & \frac{\partial f}{\partial y}(a_1,a_2) \end{pmatrix} \begin{pmatrix} x-a_1 \\ y-a_2 \end{pmatrix}$$

$$+\frac{1}{2}(x-a_1, y-a_2) \begin{pmatrix} \frac{\partial^2 f}{\partial^2 x}(a_1,a_2) & \frac{\partial^2 f}{\partial y \partial x}(a_1,a_2) \\ \frac{\partial^2 f}{\partial y \partial x}(a_1,a_2) & \frac{\partial^2 f}{\partial^2 y}(a_1,a_2) \end{pmatrix} \begin{pmatrix} x-a_1 \\ y-a_2 \end{pmatrix}$$

$$f(a_1,a_2) + \begin{pmatrix} f_x(a_1,a_2) & f_y(a_1,a_2) \end{pmatrix} \begin{pmatrix} x-a_1 \\ y-a_2 \end{pmatrix}$$

$$+\frac{1}{2}(x-a_1, y-a_2) \begin{pmatrix} f_{xx}(a_1,a_2) & f_{xy}(a_1,a_2) \\ f_{yx}(a_1,a_2) & f_{yy}(a_1,a_2) \end{pmatrix} \begin{pmatrix} x-a_1 \\ y-a_2 \end{pmatrix}$$

$$= f(a_1,a_2) + \begin{pmatrix} f_x & f_y \end{pmatrix} \begin{pmatrix} x-a_1 \\ y-a_2 \end{pmatrix} + \frac{1}{2}(x-a_1, y-a_2) \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} \begin{pmatrix} x-a_1 \\ y-a_2 \end{pmatrix}$$

$$= f(a_1,a_2) + f_x[x-a_1] + f_y[y-a_2] + \frac{1}{2}(x-a_1, y-a_2) \begin{pmatrix} f_{xx}[x-a_1] + f_{xy}[y-a_2] \\ f_{yx}[x-a_1] + f_{yy}[y-a_2] \end{pmatrix}$$

$$= f(a_1,a_2) + f_x[x-a_1] + f_y[y-a_2]$$

$$+\frac{1}{2}(f_{xx}[x-a_1]^2 + f_{xy}[x-a_1][y-a_2] + f_{yx}[x-a_1][y-a_2] + f_{yy}[y-a_2]^2)$$

$$= f(a_1,a_2) + f_x[x-a_1] + f_y[y-a_2] + \frac{1}{2}(f_{xx}[x-a_1]^2 + (f_{xy}+f_{yx})[x-a_1][y-a_2] + f_{yy}[y-a_2]^2)$$

$$p_2(x,y) = f(a_1,a_2) + f_x(a_1,a_2)[x-a_1] + f_y(a_1,a_2)[y-a_2]$$

$$+\frac{1}{2}(f_{xx}(a_1,a_2)[x-a_1]^2 + (f_{xy}+f_{yx})(a_1,a_2)[x-a_1][y-a_2] + f_{yy}(a_1,a_2)[y-a_2]^2)$$