Calc 1 review: Chain rule.

Suppose $z = f(y)$ and $y = g(x)$. Then $z = (f \circ g)(x)$

$\frac{dz}{dy} = f'(y)$ $\qquad$ $\frac{dy}{dx} = g'(x)$ $\qquad$ $\frac{dz}{dx} = (f \circ g)'(x)$

Chain rule $(f \circ g)'(x) = f'(g(x))g'(x)$

$\frac{dz}{dx} = (f \circ g)'(x) = f'(g(x))g'(x) = \frac{dz}{dy}\frac{dy}{dx}$

Thus $\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$

Example: The area of a rectangle is given by the function $A : \mathbf{R}^2 \to \mathbf{R}$, $A(l, w) = lw$

Suppose the length of a rectangle is changing with time according to the function $l(t) = t^2$.

Suppose the width of a rectangle is changing with time according to the function $w(t) = sin(t)$.

We can combine these two functions for l and w into a function $F : \mathbf{R} \to \mathbf{R}^2$ where $F(t) =$

Thus the area of a rectangle is changing with time according to the function:

At time $t = 3$,

The length of the rectangle is

The width of the rectangle is

The area of the rectangle is

The rate of change of the length of the rectangle at $t = 3$ is

The rate of change of the width of the rectangle at $t = 3$ is

The rate of change of the area of the rectangle at $t = 3$ is

Example: The area of a rectangle is given by the function
$A : \mathbf{R}^2 \to \mathbf{R}$, $A(l, w) = lw$

Suppose the length of a rectangle is changing with time according to some unknown function $l : \mathbf{R} \to \mathbf{R}$

Suppose the width of a rectangle is changing with time according to some unknown function $w : \mathbf{R} \to \mathbf{R}$

We can combine these two functions for l and w into a function $F : \mathbf{R} \to \mathbf{R}^2$ where $F(t) =$

Suppose at time $t = 3$, the length of the rectangle is 10m and the width is 8 m. Suppose also that at time $t = 3$, the length is decreasing at a rate of 4m/sec while the width is increasing at a rate of 5m/sec. Find the rate of change of the area of the rectangle at $t = 3$.