2.4 Higher order derivatives:

$\frac{\partial^2 f}{\partial x_{i_1} \partial x_{i_2}} = \frac{\partial}{\partial x_{i_1}}(\frac{\partial}{\partial x_{i_2}}(f))$

Ex: Let $f(x, y, z) = x^2 ln(yz)$

$\frac{\partial f}{\partial x} =$ $\frac{\partial f}{\partial y} =$ $\frac{\partial f}{\partial z} =$

$\frac{\partial^2 f}{\partial x^2} =$ $\frac{\partial^2 f}{\partial x \partial y} =$ $\frac{\partial^2 f}{\partial z \partial y} =$

$\frac{\partial^3 f}{\partial x^3} =$ $\frac{\partial^3 f}{\partial x^2 \partial z} =$ $\frac{\partial^3 f}{\partial x \partial y \partial z} =$

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Defn: Let $V$ be a nonempty open subset of $R^n$, $f : V \to R^m$, $p \in \mathbf{N}$.

i.) $f$ is $C^p$ on $V$ is each partial derivative of order $k \leq p$ exists and is continuous on $V$.

ii.) $f$ is $C^\infty$ on $V$ if $f$ is $C^p$ on $V$ for all $p \in \mathbf{N}$ ($f$ is *smooth*).

Ex: $g(x, y) = (x + y, x)$

Cor 1.7 If $f \in C^r$ on $U$, then $\frac{\partial^k f}{\partial x_{i_1} \partial x_{i_2} ... \partial x_{i_k}} = \frac{\partial^k f}{\partial x_{j_1} \partial x_{j_2} ... \partial x_{j_k}}$ where $(j_1, j_2, ..., j_k)$ is a permutation of $(i_1, i_2, ..., i_k)$