

Potential exam questions:

i.) Define: $f : X \rightarrow Y$ is 1:1 iff

ii.) Define: $f : X \rightarrow Y$ is NOT 1:1 iff

iii.) Prove that $f : R \rightarrow R$, $f(x) = \underline{\hspace{2cm}}$ is NOT 1:1.

i.) State the Intermediate Value Theorem:

ii.) See HW problems

Use the limit definition of the derivative to find the derivative of $f(x) = \underline{\hspace{2cm}}$

Prove: If c is a constant and f is differentiable at x , then $(cf)'(x) = c(f'(x))$

I will choose two of the above for the following exam 1 question:

#.) Choose one of the following (clearly indicate your choice).

#A.)

#B.)

Suppose $2x^2y - 3y^2 = 4$. Find y''

1.) First find y' :

Easiest method is to use implicit differentiation. Take derivative (with respect to x) of both sides.

$$\frac{d}{dx}(2x^2y - 3y^2) = \frac{d}{dx}(4)$$

$$4xy + 2x^2y' - 6yy' = 0$$

Solve for y' (note this step is easy as one can factor y' from some terms):

$$y'(2x^2 - 6y) = -4xy$$

$$y' = \frac{-4xy}{2x^2 - 6y} = \frac{-2(2xy)}{-2(3y - x^2)} = \frac{2xy}{3y - x^2}$$

Hence $y' = \frac{2xy}{3y - x^2}$

2.) To find y'' , take derivative of y' :

$$y'' = \left(\frac{2xy}{3y - x^2}\right)' = \frac{(2xy' + 2y)(3y - x^2) - 2xy(3yy' - 2x)}{(3y - x^2)^2}$$

Since $y' = \frac{2xy}{3y - x^2}$,

$$y'' = \frac{(2x\left(\frac{2xy}{3y - x^2}\right) + 2y)(3y - x^2) - 2xy\left(3y\left(\frac{2xy}{3y - x^2}\right) - 2x\right)}{(3y - x^2)^2}$$