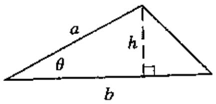
Geometry

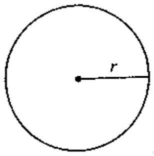
Geometric Formulas

Formulas for area A, circumference C, and volume V:

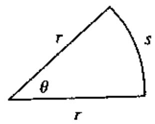
Triangle
$A = \frac{1}{2}bh$
$= \frac{1}{2}ab \sin \theta$



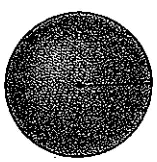
Circle
$A = \pi r^2$
$C = 2\pi r$



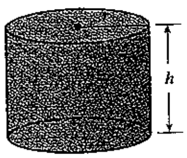
Sector of Circle
$A = \frac{1}{2}r^2\theta$
$s = r\theta$ (θ in radians)



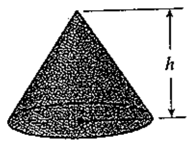
Sphere
$V = \frac{4}{3}\pi r^3$
$A = 4\pi r^2$



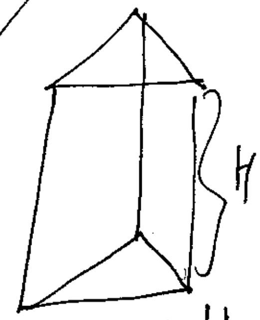
Cylinder
$V = \pi r^2 h$



Cone
$V = \frac{1}{3}\pi r^2 h$
$A = \pi r\sqrt{r^2 + h^2}$



Perimeter, Area, Volume of square, rectangle, cube, etc.



$V = BH$
$V = \frac{1}{2}bh\,H$

Distance and Midpoint Formulas

Distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of $\overline{P_1P_2}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

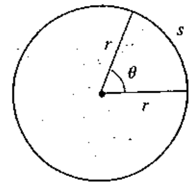
Angle Measurement

π radians $= 180°$

$1° = \frac{\pi}{180}$ rad $\qquad$ 1 rad $= \frac{180°}{\pi}$

$s = r\theta$

(θ in radians)



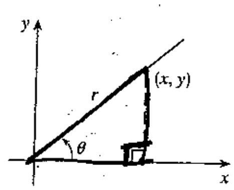
Trigonometric Functions

right Δ

$\sin \theta = \frac{y}{r}$ $\qquad$ $\csc \theta = \frac{r}{y}$

$\cos \theta = \frac{x}{r}$ $\qquad$ $\sec \theta = \frac{r}{x}$

$\tan \theta = \frac{y}{x}$ $\qquad$ $\cot \theta = \frac{x}{y}$



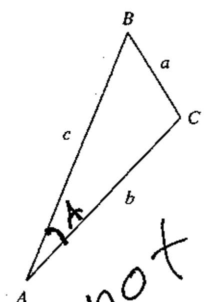
$x^2 + y^2 = r^2$

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

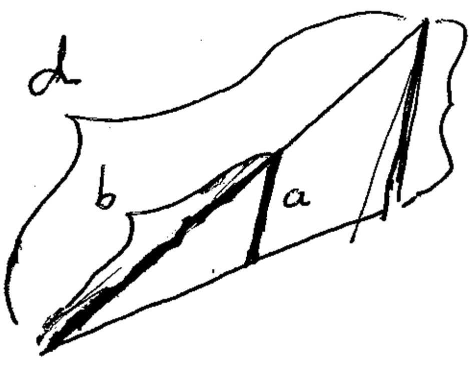
The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$



not a right Δ

Similar triangles:



$\frac{a}{b} = \frac{c}{d}$ for right and not right Δ

Find change in Volume wrt to time of a cylinder as height changes, but radius remains fixed

constant

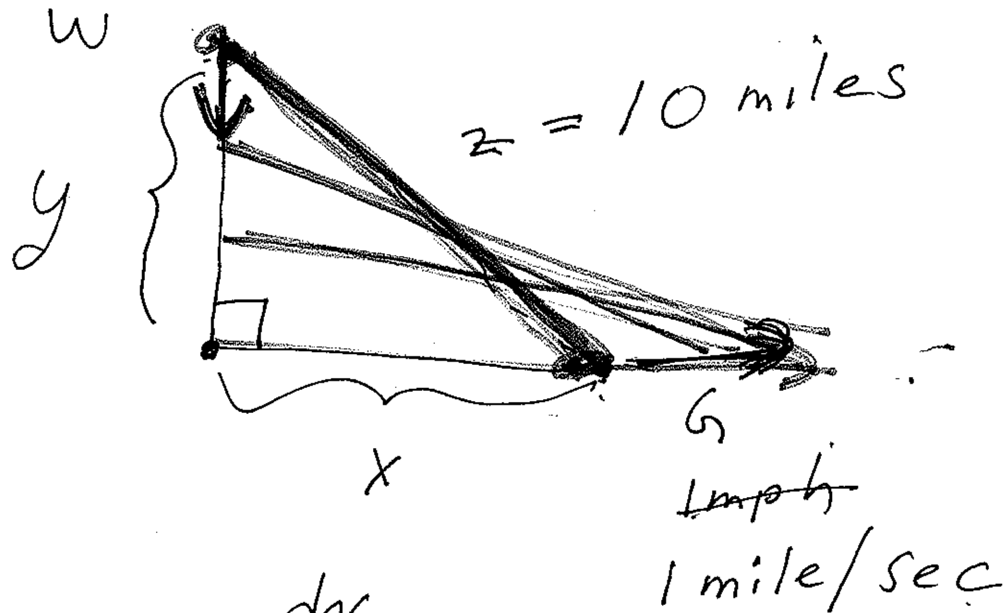
$$\frac{d(V)}{dt} = \frac{d(\pi r^2 h)}{dt}$$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

Pay attention to what remains constant and what is changing ladders Example 2

2.) Suppose the distance between two planes must be maintained at 10 miles. Suppose plane W is north of a radio tower and moving south while plane G is east of the same radio tower. If plane G is moving east at 1 mile/second, how fast should plane W be moving when plane G is 6 miles from the radio tower?

$\frac{dy}{dt} = ?$ when plane G is 6 miles from radio tower



$$\frac{dx}{dt} = 1$$

$$x^2 + y^2 = z^2$$

$$x^2 + y^2 = 10^2$$

$$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(100)$$

$$\cancel{2}x \cdot \frac{dx}{dt} + \cancel{2}y \cdot \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

pay attention to what is constant vs what is changing

$$\frac{dy}{dt} = ?$$

$$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

$$= \frac{-6}{8}(1)$$

$$= -\frac{3}{4} \text{ miles/sec}$$

when $x = 6$

$$\frac{dx}{dt} = 1$$

$$x^2 + y^2 = 100$$

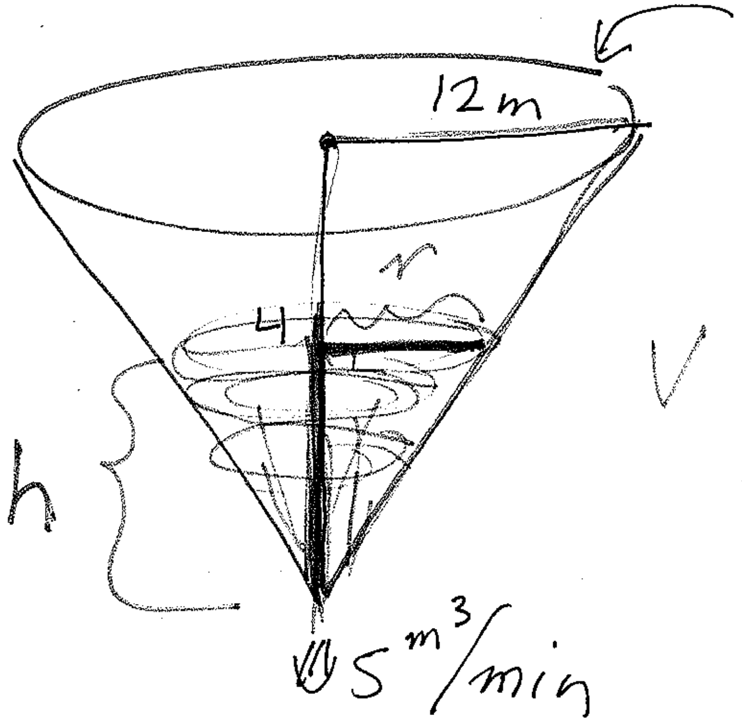
$$36 + y^2 = 100$$

$$y = \sqrt{100 - 36}$$

$$y = \sqrt{64} = 8$$

Plane W <u>approaches</u> radio tower at $\frac{3}{4}$ miles/sec

3.) A water tank has the shape of an inverted circular cone with base radius 12 m and height 4 m. Suppose water is leaking out of the cone at a rate of 5 m^3/min. while water is being pumped into the cone at a rate of 9 m^3/min. Find the rate at which the water level is rising when the water is 1.5 m deep.



$$\boxed{\frac{dh}{dt} = \,?\, \text{ when } h = 1.5\text{m}}$$

$$\frac{dV}{dt} = 9 - 5 = \frac{4\,m^3}{min}$$

$$V = \frac{1}{3}\pi r^2 h$$

$\cancel{\frac{dr}{dt}}$ need to eliminate r from equation

$$\frac{r}{h} = \frac{12}{4} = 3$$

$$r = 3h$$

Side note: $\frac{dr}{dt} = 3\frac{dh}{dt}$ ← not needed for this problem

$$V = \frac{1}{3}\pi(3h)^2 h$$

$$V = \frac{1}{3}\pi 9h^3$$

$$V = 3\pi h^3$$

$$\frac{d(V)}{dt} = \frac{d(3\pi h^3)}{dt}$$

$$\frac{dV}{dt} = 3\pi\, 3h^2 \cdot \frac{dh}{dt}$$

$$\frac{dV}{dt} = 9\pi h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{dV}{dt}\left(\frac{1}{9\pi h^2}\right) = 4\left(\frac{1}{9\pi(1.5)^2}\right)$$

height increasing at ↗ m/min

3.11

$f(x) = x^{1/2}$

Find the linearization of $\sqrt{x}$ at $x = 4$
i.e, find a linear approximation of $\sqrt{x}$ for x close to 4.
i.e, find equation of tangent line to $\sqrt{x}$ at $x = 4$.

Find slope $\quad f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

at $x = 4$ slope $= f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

point: $(4, \sqrt{4}) = (4, 2)$

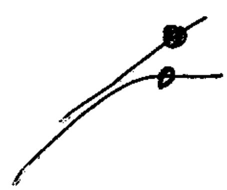
$$\frac{y-2}{x-4} = \frac{1}{4} \Rightarrow y - 2 = \frac{1}{4}(x-4) = \frac{1}{4}x - 1$$

$$y = \frac{1}{4}x + 1$$

Approximate $\sqrt{5}$

Method 1: Use equation of tangent line

$$\sqrt{5} \approx \frac{1}{4}(5) + 1 = \frac{9}{4}$$



overestimate

Method 2 (even easier): Use $\Delta y \sim dy$

Recall: slope of secant line $= \frac{\Delta y}{\Delta x}$

$\Delta x = x + h - x, \quad \Delta y = f(x+h) - f(x) = f(x + \Delta x) - f(x)$

slope of tangent line $= f'(x) = \frac{dy}{dx}$. Thus $dy = f'(x)dx$.

If $\Delta x = dx$, then $\Delta y \sim dy$

$f(x + \Delta x) = f(x) + \Delta y \sim f(x) + dy$

We sometimes refer to the slope of the tangent line to a curve at a point **the curve** at the point. The idea is that if we zoom in far enough toward the looks almost like a straight line. Figure 2 illustrates this procedure for the Example 1. The more we zoom in, the more the parabola looks like a line. the curve becomes almost indistinguishable from its tangent line.

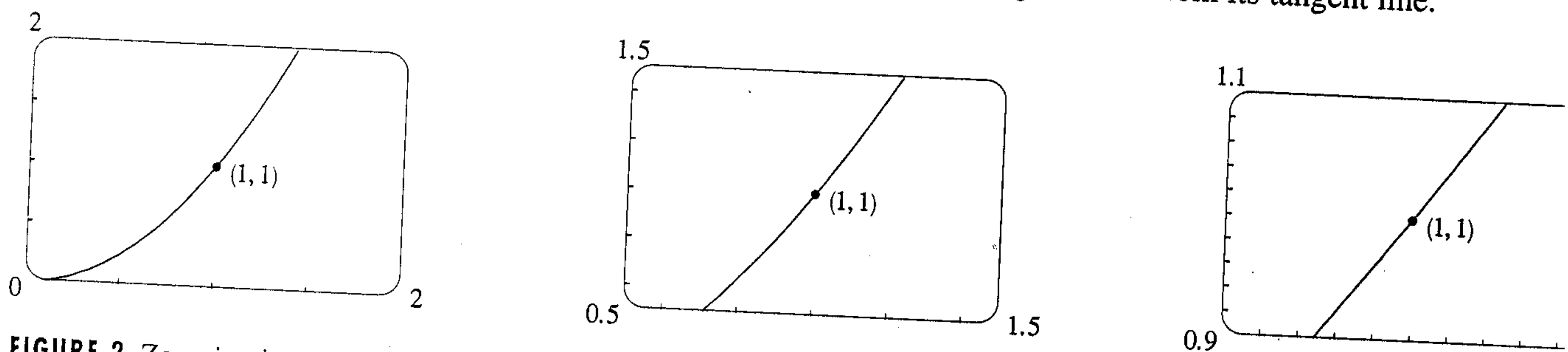
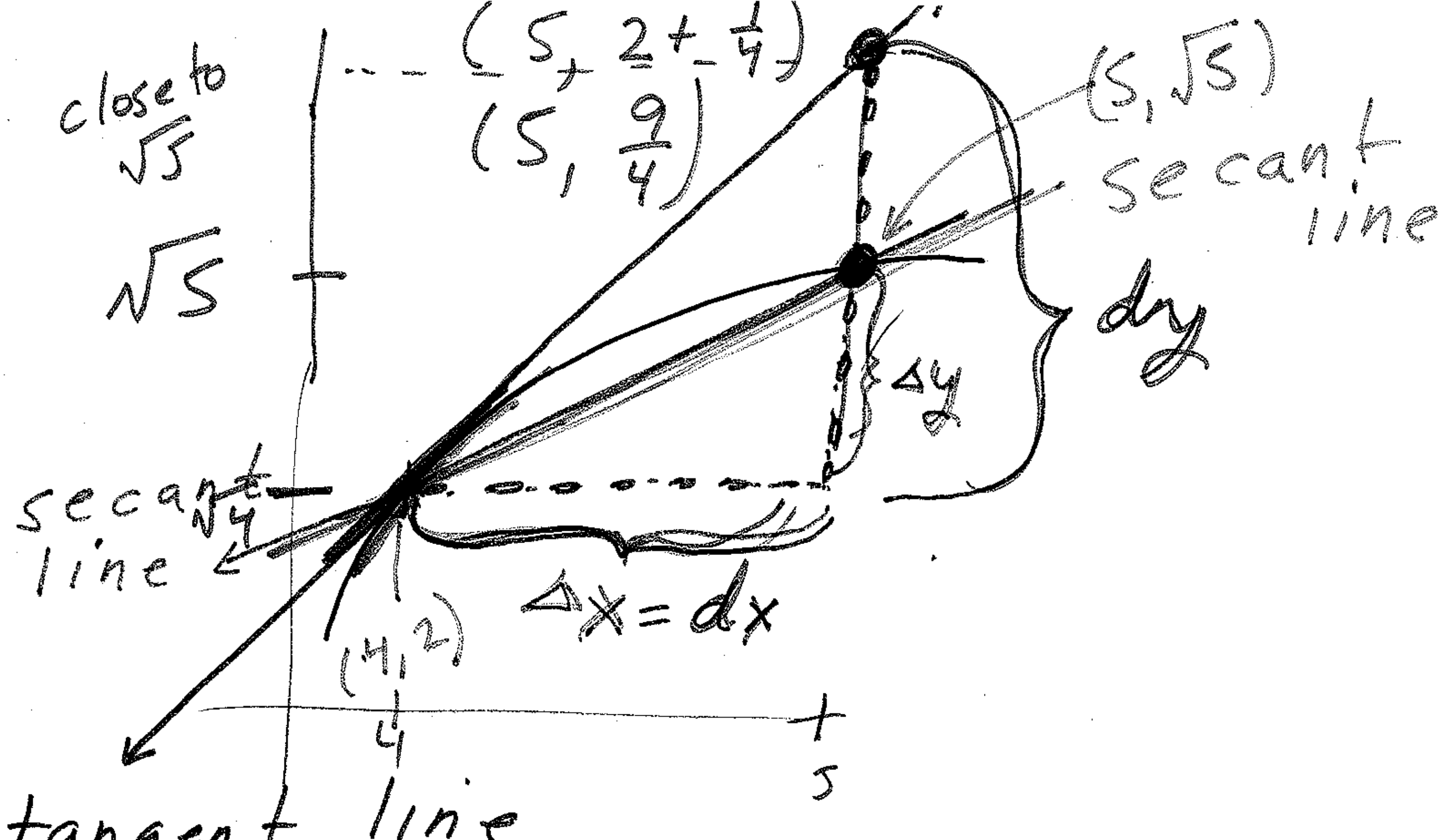


FIGURE 2 Zooming in toward the point (1, 1) on the parabola $y = x^2$



slope of secant line $= \dfrac{\Delta y}{\Delta x}$

Slope of tangent line $= \dfrac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{4}$$

If $\Delta x = dx = 1 = \overset{x_2 - x_1}{(5-4)}$

$$dy = \frac{1}{4}\,dx = \frac{1}{4}(1) = \frac{1}{4}$$