

2.5 Defn:  $f$  is continuous at  $a$  if  $\lim_{x \rightarrow a} f(x) = f(a)$

(i.e., if  $\lim_{x \rightarrow a} f(x) = f(\lim_{x \rightarrow a} x)$ )

Examples:

Read left and right continuity

If  $f, g$  continuous at  $a$ ,  $c \in \mathcal{R}$ , then  $f + g, fg, cf, f/g$  (if  $g(a) \neq 0$ ) are continuous.

If  $g$  continuous at  $a$  and  $f$  continuous at  $g(a)$ , then  $f \circ g$  continuous at  $a$ .

Ex:  $\lim_{x \rightarrow 0} \frac{x^2 - e^{x^3}}{\cos(x)} =$

Intermediate value theorem: Suppose  $f$  continuous on  $[a, b]$ ,  $f(a) \neq f(b)$  and  $n$  is between  $f(a)$  and  $f(b)$ , then there exists  $c \in (a, b)$  such that  $f(c) = N$ .

Example: Show that  $x^2 - 7x + 1$  has a root between 0 and 1.