Final Exam PART A, Dec., 2005 SHOW ALL WORK
Math 25 Calculus I Either circle your answers or place on answer line.

All problems required on this part of the exam.

1.) Suppose $f(x) = \sqrt{4x-3}$ and $g(x) = x$.

$\sqrt{4x-3} = x$

$(4x-3) = x^2$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$. Hence $x = 1, 3$

Take $2 \in (1, 3)$

$x = 2 : \sqrt{4x-3} = \sqrt{5}$

$x = 2 : x = 2$

$\sqrt{5} > 2$. Hence $\sqrt{4x-3} > x$ on $(1,\ 3)$

[3] 1a.) Set up, **but do NOT evaluate**, an integral for the area of the region enclosed by f and g.

height $= \sqrt{4x-3} - x$, width $= dx$

Area $= \int_1^3 (\sqrt{4x-3} - x)dx$

[4] 1b.) Set up, **but do NOT evaluate**, an integral for the volumne of the solid obtained by rotating the region bounded by the curves f and g about the line $x = 8$ (hint: use cylindrical shells).

Area $= 2\pi rh$, $h = \sqrt{4x-3} - x$, $r = 8 - x$,

Area $= 2\pi(8-x)(\sqrt{4x-3} - x)$

width or thickness of cylindrical shell $= dx$.

Volume $= 2\pi \int_1^3 (8-x)(\sqrt{4x-3} - x)dx$

[4] 1c.) Set up, **but do NOT evaluate**, an integral for the volumne of the solid obtained by rotating the region bounded by the curves f and g about the line $y = -4$ (hint: use washers).

Area $= \pi(R^2 - r^2)$, $R = \sqrt{4x-3} - (-4) = \sqrt{4x-3} + 4$, $r = x - (-4) = x + 4$

width or thickness of washer $= dx$.

Volume $= \pi \int_1^3 [(\sqrt{4x-3} + 4)^2 - (x+4)^2]dx$

[1] 2.) If $h(x) = 3x^2$, then the slope of the tangent line at the point (1, 3) is $\underline{6}$

$h'(x) = 6x$, $h'(1) = 6(1) = 6$

[10] 3.) Find the derivative of $g(x) = ln(\frac{4e^{x^3}-2}{5x+2})$

$$g'(x) = \frac{1}{\frac{4e^{x^3}-2}{5x+2}}[\frac{4e^{x^3}(3x^2)(5x+2)-(4e^{x^3}-2)5}{(5x+2)^2}] = \frac{5x+2}{4e^{x^3}-2}[\frac{4e^{x^3}(3x^2)(5x+2)-(4e^{x^3}-2)5}{(5x+2)^2}]$$

$$= \frac{4e^{x^3}(3x^2)(5x+2)-(4e^{x^3}-2)5}{(4e^{x^3}-2)(5x+2)}$$

Answer 3.) $\underline{\frac{4e^{x^3}(3x^2)(5x+2)-(4e^{x^3}-2)5}{(4e^{x^3}-2)(5x+2)}}$

4.) Find the following integrals:

[10] a.) $\int_1^e \frac{ln(x^2)}{x}dx = \underline{1}$ [SIMPLIFY your answer]

$\int_1^e \frac{ln(x^2)}{x}dx = \int_1^e \frac{2ln(x)}{x}dx$

Let $u = ln(x)$

$du = \frac{1}{x}dx$

$x = 1$: $u = ln(1) = 0$

$x = e$: $u = ln(e) = 1$

$\int_0^1 2udu = u^2|_0^1 = (1)^2 - (0)^2 = 1$

[3] b.) $\int \frac{3}{\sqrt{1-x^2}}dx = \underline{3sin^{-1}(x) + C}$

[10] 5.) Find the following limit (SHOW ALL STEPS): $lim_{x\to\infty}x^4e^{-3x^2} = \underline{0}$

$$lim_{x\to\infty}x^4e^{-3x^2} = lim_{x\to\infty}\frac{x^4}{e^{3x^2}} \quad (\text{"}\frac{\infty}{\infty}\text{"})$$

$$= lim_{x\to\infty}\frac{4x^3}{6xe^{3x^2}}$$

$$= lim_{x\to\infty}\frac{2x^2}{3e^{3x^2}} \quad (\text{"}\frac{\infty}{\infty}\text{"})$$

$$= lim_{x\to\infty}\frac{4x}{18xe^{3x^2}}$$

$$= lim_{x\to\infty}\frac{4}{18e^{3x^2}} = 0$$

6.) Find the following for $f(x) = x^{\frac{3}{2}} - 2x^{\frac{1}{2}} = x^{\frac{1}{2}}(x-2)$ (if they exist; if they don't exist, state so). Use this information to graph f.

Note $f'(x) = \frac{3}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}} = x^{-\frac{1}{2}}(\frac{3}{2}x - 1)$ and $f''(x) = \frac{3}{4}x^{-\frac{1}{2}} - \frac{-1}{2}x^{-\frac{3}{2}} = x^{-\frac{3}{2}}(\frac{3}{4}x + \frac{1}{2})$

[1] 6a.) critical numbers: $\underline{x = 0, \frac{2}{3}}$

[1] 6b.) local maximum(s) occur at $x =$ <u>*none*</u>

[1] 6c.) local minimum(s) occur at $x = \underline{\frac{2}{3}}$

[1] 6d.) The global maximum of f on the interval [0, 5] is $\underline{3\sqrt{5}}$ and occurs at $x = \underline{5}$

[1] 6e.) The global minimum of f on the interval [0, 5] is $\underline{-\frac{4}{3}\sqrt{\frac{2}{3}}}$ and occurs at $x = \underline{\frac{2}{3}}$

[1] 6f.) Inflection point(s) occur at $x =$ <u>*none*</u>

[1] 6g.) f increasing on the intervals $\underline{(\frac{2}{3}, \infty)}$

[1] 6h.) f decreasing on the intervals $\underline{(0, \frac{2}{3})}$

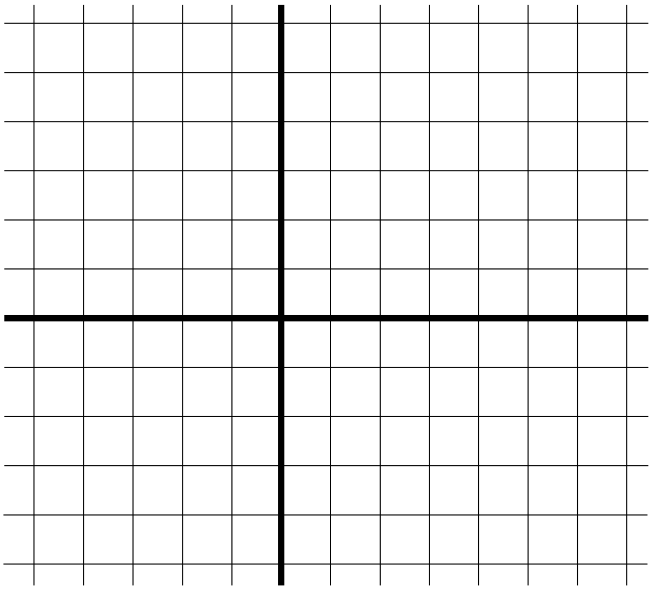
[1] 6i.) f is concave up on the intervals $\underline{[0, \infty)}$

[1] 6j.) f is concave down on the intervals <u>*none*</u>

[1] 6k.) What is the domain of f? $\underline{[0, \infty)}$

[1] 6l.) What is the range of f? $\underline{[-\frac{4}{3}\sqrt{\frac{2}{3}}, \infty)}$

[4] 6m.) Graph f



Final Exam PART B, Dec., 2005 SHOW ALL WORK
Math 25 Calculus I Either circle your answers or place on answer line.

Choose 4 out of the following 5 problems: **Clearly indicate which 4 problems you choose.** Each problem is worth 10 points You may do all the problems for up to five points extra credit.

I have chosen the following 4 problems: ______________________________

A.) Find the derivative of $f(x) = (x+1)^{x^3}$

$(x+1)^{x^3} = e^{ln[(x+1)^{x^3}]} = e^{x^3 ln(x+1)}$

Thus $f'(x) = e^{x^3 ln(x+1)}[3x^2 ln(x+1) + \frac{x^3}{x+1}]$

$= e^{ln[(x+1)^{x^3}]}[3x^2 ln(x+1) + \frac{x^3}{x+1}]$

$= (x+1)^{x^3}[3x^2 ln(x+1) + \frac{x^3}{x+1}]$

Answer A.) $\underline{(x+1)^{x^3}[3x^2 ln(x+1) + \frac{x^3}{x+1}]}$

B.) Express the following integral as a limit of Riemann sums. Do not evaluate the limit: $\int_0^5 x^2 ln(4x+1)dx$.

$\Delta x = \frac{5-0}{n} = \frac{5}{n}$ = width

$x_i = 0 + \frac{5i}{n} = \frac{5i}{n}$. Thus $f(x_i) = (\frac{5i}{n})^2 ln(4(\frac{5i}{n}) + 1) = (\frac{5i}{n})^2 ln(\frac{20i}{n} + 1)$ = height

Answer B.) $\underline{lim_{n\to\infty} \Sigma_{i=1}^{n} [(\frac{5i}{n})^2 ln(\frac{20i}{n} + 1)]\frac{5}{n}}$

C.) Find the horizontal asymptotes of $f(x) = \frac{\sqrt{x^2-3}}{2x-1}$

$$lim_{x\to+\infty} \frac{\sqrt{x^2-3}}{2x-1} = lim_{x\to+\infty} \frac{\sqrt{x^2(1-\frac{3}{x^2})}}{x(2-\frac{1}{x})} = lim_{x\to+\infty} \frac{|x|\sqrt{1-\frac{3}{x^2}}}{x(2-\frac{1}{x})} = lim_{x\to+\infty} \frac{\sqrt{1-\frac{3}{x^2}}}{2-\frac{1}{x}} = \frac{1}{2}$$

$$lim_{x\to-\infty} \frac{\sqrt{x^2-3}}{2x-1} = lim_{x\to-\infty} \frac{|x|\sqrt{1-\frac{3}{x^2}}}{x(2-\frac{1}{x})} = lim_{x\to-\infty} \frac{-x\sqrt{1-\frac{3}{x^2}}}{x(2-\frac{1}{x})} = lim_{x\to-\infty} \frac{-\sqrt{1-\frac{3}{x^2}}}{2-\frac{1}{x}} = -\frac{1}{2}$$

Answer C.) $\underline{y = \frac{1}{2}, \quad y = -\frac{1}{2}}$

D.) A plane flying horizontally at an altitude 400km and at a speed of 2000 km/hr. passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 500km from the station.

$y = 400$, $\frac{dx}{dt} = 2000$ Find $\frac{dz}{dt}$ when $z = 500$

$x^2 + y^2 = z^2$

$x^2 + 400^2 = z^2$

$2x\frac{dx}{dt} + 0 = 2z\frac{dz}{dt}$

$x\frac{dx}{dt} = z\frac{dz}{dt}$.

When $z = 500$: $x^2 + 400^2 = 500^2$. Hence $x^2 = 250000 - 160000 = 90000$. Thus $x = 300$

$(300)(2000) = (500)\frac{dz}{dt}$.

$\frac{dz}{dt} = 300(2000)/500 = 300(20)/5 = 300(4) = 1200$

Answer D.) $\underline{1200km/hr}$

E.) A cylindrical can without a top is made to contain 100 cm^3 of water. Find the dimensions that will minimize the cost of the metal to make the can. Explain why your answer is optimal.

Minimize surface area of the can, $S = 2\pi rh + \pi r^2$

Volume $= \pi r^2 h = 100$. Hence $h = \frac{100}{\pi r^2}$

Minimize $S(r) = 2\pi r(\frac{100}{\pi r^2}) + \pi r^2 = \frac{200}{r} + \pi r^2 = 200r^{-1} + \pi r^2$

$S'(r) = -200r^{-2} + 2\pi r = \frac{-200+2\pi r^3}{r^2}$

$S'(r)$ DNE if $r = 0$

$S'(r) = 0$ implies $\frac{-200+2\pi r^3}{r^2} = 0$

$-200 + 2\pi r^3 = 0$, $r^3 = \frac{100}{\pi}$, $r = (\frac{100}{\pi})^{\frac{1}{3}}$.

$S'(r) > 0$ if $r > (\frac{100}{\pi})^{\frac{1}{3}}$ $\qquad$ $S'(r) < 0$ if $r < (\frac{100}{\pi})^{\frac{1}{3}}$

Hence $S(r)$ is decreasing on $(-\infty, (\frac{100}{\pi})^{\frac{1}{3}})$ and increasing on $((\frac{100}{\pi})^{\frac{1}{3}}, \infty)$

Hence the global minimum occurs at $r = (\frac{100}{\pi})^{\frac{1}{3}}$.

$h = \frac{100}{\pi r^2} = \frac{100}{\pi(\frac{100}{\pi})^{\frac{2}{3}}} = (\frac{100}{\pi})^{\frac{1}{3}}$

Answer E.) $\underline{radius = (\frac{100}{\pi})^{\frac{1}{3}} cm, height = (\frac{100}{\pi})^{\frac{1}{3}}.}$

Final Exam Part C, Dec., 2005 SHOW ALL WORK
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Note the proof problems on this page are completely optional. You may choose to prove one (and only one) of the following statements. If you choose to do one of the following problems, it can replace your lowest point problem in the optional section OR 80% of your lowest point problem in the required section. If you chose to do one of the following problems, clearly indicate your choice.

I have chosen the following problem: ____

I.) Prove f differentiable implies f continuous.

Show: $lim_{x \to a} f(x) = f(a)$

Know: f differentiable at a implies $f'(a)$ exists and $lim_{x \to a} \frac{f(x)-f(a)}{x-a} = f'(a)$

Show $lim_{x \to a}[f(x) - f(a)] = 0$

$$lim_{x \to a}[f(x) - f(a)] = lim_{x \to a} \frac{f(x)-f(a)}{x-a}(x - a) = lim_{x \to a} \frac{f(x)-f(a)}{x-a} lim_{x \to a}(x - a)$$

$$= f'(a)(0) = 0.$$

$lim_{x \to a} f(x) = lim_{x \to a}[f(x)-f(a)+f(a)] = lim_{x \to a}[f(x)-f(a)]+lim_{x \to a} f(a) = 0+f(a) = f(a)$