Exam 2 Nov. 10, 2005 SHOW ALL WORK
Math 25 Calculus I Either circle your answers or place on answer line.

[16] 1.) If $f'(x) = 3x^4 + 2 + 4x^{-1}$ and $f(1) = 8$, find f

$f(x) = \frac{3}{5}x^5 + 2x + 4ln(x) + C$ for some C.

$f(1) = 8$, $f(1) = 8 = \frac{3}{5}(1)^5 + 2(1) + 4ln(1) + C$

$8 = \frac{3}{5} + 2 + 0 + C$

$6 - \frac{3}{5} = \frac{30-3}{5} = \frac{27}{5} = C$

Answer 1.) $f(x) = \frac{3}{5}x^5 + 2x + 4ln(x) + \frac{27}{5}$

[16] 2.) $lim_{x \to 0^+}[x^{x^3}] =$ 1

$lim_{x \to 0^+}[x^{x^3}] = lim_{x \to 0^+}[e^{ln(x^{x^3})}] = lim_{x \to 0^+}[e^{x^3 ln(x)}]$

$lim_{x \to 0^+}[x^3 ln(x)] = lim_{x \to 0^+}[\frac{ln(x)}{x^{-3}}] = lim_{x \to 0^+}[\frac{\frac{1}{x}}{-3x^{-4}}] = lim_{x \to 0^+}[\frac{x^3}{-3}] = 0$

Hence $lim_{x \to 0^+}[e^{x^3 ln(x)}] = e^{lim_{x \to 0^+}[x^3 ln(x)]} = e^0 = 1$

[10] 3a.) Given $x^2 + 2xy + y^3 - x - 3 = 0$, then $y' = \frac{1-2x-2y}{2x+3y^2}$

$\frac{d}{dx}(x^2 + 2xy + y^3 - x - 3) = \frac{d}{dx}(0)$,

$2x + 2(y + xy') + 3y^2y' - 1 = 0$,

$2x + 2y + 2xy' + 3y^2y' - 1 = 0$,

$(2x + 3y^2)y' = 1 - 2x - 2y$,

$y' = \frac{1-2x-2y}{2x+3y^2}$,

[6] 3b.) Find the equation of the tangent line to the curve $x^2 + 2xy + y^3 - x - 3 = 0$, at the point (-2, 1).

$y' = \frac{1-2x-2y}{2x+3y^2}$,

Hence at (-2, 1), $y' = \frac{1-2(-2)-2(1)}{2(-2)+3(1)^2} = \frac{1+4-2}{-4+3} = -3$.

$\frac{y-1}{x-(-2)} = -3$, $y = -3(x+2) + 1 = -3x - 6 + 1 = -3x - 5$

Answer 3b.) $y = -3x - 5$

[16] 4.) If $g(3) = 4$ and $g'(x) \leq 2$, how large can $g(8)$ be? $\underline{\ 14\ }$

Since g' exists, g is differentiable and hence continuous.

By the MVT since g continuous on [3, 8] and g differentiable on (3, 8), then there exists $c \in [3, 8]$ such that

$$\frac{g(8) - g(3)}{8 - 3} = g'(c)$$

Hence $\frac{g(8)-4}{5} = g'(c) \leq 2$.

Thus $g(8) \leq 2(5) + 4 = 14$.

[16] 5.) A tank is in the form of an inverted cone having a height of 16m and a diameter at the top of 8 m. Water is flowing into the tank at the rate of 2 m^3/min. How fast is the water level rising when the water is 5m deep? (Volume of cone $= \frac{1}{3}\pi r^2 h$)

$\frac{dV}{dt} = 2$, $\frac{dh}{dt} =$? when $h = 5$.

NOTE: V, h, r change with respect to time. None of them are constant.

$V = \frac{1}{3}\pi r^2 h$, $\quad \frac{r}{h} = \frac{4}{16}$. Hence $r = \frac{1}{4}h$.

$V = \frac{1}{3}\pi(\frac{1}{4}h)^2 h = \frac{1}{3}\pi(\frac{h^3}{16})$

$\frac{d}{dt}[V] = \frac{d}{dt}[\frac{1}{3}\pi(\frac{h^3}{16})]$

$\frac{dV}{dt} = \frac{1}{3}\pi(\frac{3h^2}{16}\frac{dh}{dt})$

$\frac{dV}{dt} = \pi(\frac{h^2}{16}\frac{dh}{dt})$

$2 = \pi(\frac{5^2}{16}\frac{dh}{dt})$. Hence $\frac{32}{25\pi} = \frac{dh}{dt}$

Alternate answer:

$V = \frac{1}{3}\pi r^2 h$. NOTE: V, h, r change with respect to time.

$\frac{d}{dt}[V] = \frac{d}{dt}[\frac{1}{3}\pi r^2 h]$

$\frac{dV}{dt} = \frac{1}{3}\pi[2r\frac{dr}{dt}h + r^2\frac{dh}{dt}]$

When $h = 5$: $2 = \frac{1}{3}\pi[2r\frac{dr}{dt}(5) + r^2\frac{dh}{dt}]$

$\frac{r}{h} = \frac{4}{16}$. Hence $r = \frac{1}{4}h$. Thus $\frac{dr}{dt} = \frac{1}{4}\frac{dh}{dt}$ and when $h = 5$, $r = \frac{5}{4}$.

Thus, when $h = 5$: $2 = \frac{1}{3}\pi[2(\frac{5}{4})(\frac{1}{4})\frac{dh}{dt}(5) + (\frac{5}{4})^2\frac{dh}{dt}] = \frac{25\pi}{16}\frac{dh}{dt}$. Hence $\frac{32}{25\pi} = \frac{dh}{dt}$

Answer 5.) $\underline{\ \frac{dh}{dt} = \frac{32}{25\pi}\ }$

6.) Find the following for $f(x) = \frac{x}{(x-1)^2}$ (if they exist; if they don't exist, state so). Use this information to graph f.

Note $f'(x) = \frac{-x-1}{(x-1)^3}$ and $f''(x) = \frac{2(x+2)}{(x-1)^4}$

[1] 6a.) critical numbers: $\underline{\ -1\ }$

[1.5] 6b.) local maximum(s) occur at $x = \underline{\ none\ }$

[1.5] 6c.) local minimum(s) occur at $x = \underline{\ -1\ }$

[1.5] 6d.) The global maximum of f on the interval [0, 5] is $\underline{\ none\ }$ and occurs at $x = \underline{\ none\ }$

[1.5] 6e.) The global minimum of f on the interval [0, 5] is $\underline{\ 0\ }$ and occurs at $x = \underline{\ 0\ }$

[1.5] 6f.) Inflection point(s) occur at $x = \underline{\ -2\ }$

[1] 6g.) f increasing on the intervals $\underline{\ (-1, 1)\ }$

[1] 6h.) f decreasing on the intervals $\underline{\ (-\infty, -1) \cup (1, \infty)\ }$

[1.5] 6i.) f is concave up on the intervals $\underline{\ (-2, 1) \cup (1, \infty)\ }$

[1.5] 6j.) f is concave down on the intervals $\underline{(-\infty, -2)}$

[1.5] 6k.) Equation(s) of vertical asymptote(s) $\underline{\ x = 1\ }$

[4] 6l.) Equation(s) of horizontal and/or slant asymptote(s) $\underline{\ y = 0\ }$

[4] 6m.) Graph f