Exam 1 Oct. 6, 2005 SHOW ALL WORK

Math 25 Calculus I Either circle your answers or place on answer line.

[15] 1.) Calculate the following limit: $lim_{x\to+\infty}\frac{\sqrt{4x^2+9x+8}}{5x+4}$

$lim_{x\to+\infty}\frac{\sqrt{4x^2+9x+8}}{5x+4}\cdot\frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{\sqrt{x^2}}} = lim_{x\to+\infty}\frac{\sqrt{4x^2+9x+8}}{5x+4}\cdot\frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{|x|}}$

$= lim_{x\to+\infty}\frac{\sqrt{4x^2+9x+8}}{5x+4}\cdot\frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{x}}$, since $x\to+\infty$, we can assume x is positive.

What if $x\to-\infty$?

$= lim_{x\to+\infty}\frac{\sqrt{4+\frac{9}{x}+\frac{8}{x^2}}}{5+\frac{4}{x}} = \frac{\sqrt{4}}{5} = \frac{2}{5}$

Alternate method: Factor out highest power in denominator:

$lim_{x\to+\infty}\frac{\sqrt{4x^2+9x+8}}{5x+4} = lim_{x\to+\infty}\frac{\sqrt{x^2(4+\frac{9}{x}+\frac{8}{x^2})}}{x(5+\frac{4}{x})} = lim_{x\to+\infty}\frac{\sqrt{x^2}\sqrt{4+\frac{9}{x}+\frac{8}{x^2}}}{x(5+\frac{4}{x})}$

$= lim_{x\to+\infty}\frac{|x|\sqrt{4+\frac{9}{x}+\frac{8}{x^2}}}{x(5+\frac{4}{x})} = lim_{x\to+\infty}\frac{x\sqrt{4+\frac{9}{x}+\frac{8}{x^2}}}{x(5+\frac{4}{x})}$, since $x\to+\infty$, we can assume x is positive.

What if $x\to-\infty$?

$= lim_{x\to+\infty}\frac{\sqrt{4+\frac{9}{x}+\frac{8}{x^2}}}{5+\frac{4}{x}} = \frac{\sqrt{4}}{5} = \frac{2}{5}$

Answer 1.) $= \frac{2}{5}$

[15] 2.) Find the derivative of $f(x)=\sqrt{x}$ by using the definition of derivative.

$f'(x) = lim_{h\to0}\frac{f(x+h)-f(x)}{h} = lim_{h\to0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$

$= lim_{h\to0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$= lim_{h\to0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$

$= lim_{h\to0}\frac{h}{h(\sqrt{x+h}+\sqrt{x})}$

$= lim_{h\to0}\frac{1}{(\sqrt{x+h}+\sqrt{x})} = \frac{1}{(\sqrt{x}+\sqrt{x})} = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{\frac{-1}{2}}$ IF $x>0$.

Extra credit if you noticed that x must be >0.

Answer 2.) $\frac{1}{2}x^{\frac{-1}{2}},\ x>0$

Find the following derivatives

[15] 3.) $\frac{d}{dx}[\frac{e^x(x^2-x+3)}{cos(2x)}]$

$\frac{[e^x(x^2-x+3)]'cos(2x)-e^x(x^2-x+3)[cos(2x)]'}{cos^2(2x)}$

$= \frac{[(e^x)'(x^2-x+3)+e^x(x^2-x+3)']cos(2x)-e^x(x^2-x+3)[-sin(2x)](2x)'}{cos^2(2x)}$

$= \frac{[(e^x)(x^2-x+3)+e^x(2x-1)]cos(2x)-e^x(x^2-x+3)[-sin(2x)](2)}{cos^2(2x)}$

$= \frac{(e^x)(x^2+x+2)cos(2x)+2e^x(x^2-x+3)sin(2x)}{cos^2(2x)}$

Note it is better if you don't show all the intermediate steps.

Answer 3.) $\underline{\frac{(e^x)(x^2+x+2)cos(2x)+2e^x(x^2-x+3)sin(2x)}{cos^2(2x)}}$

[15] 4.) $\frac{d}{dx}[2sin(e^{x^3}+4)]$

$2[sin(e^{x^3}+4)]' = 2cos(e^{x^3}+4)[e^{x^3}+4]'$

$= 2[cos(e^{x^3}+4)][e^{x^3}(x^3)'+0]$

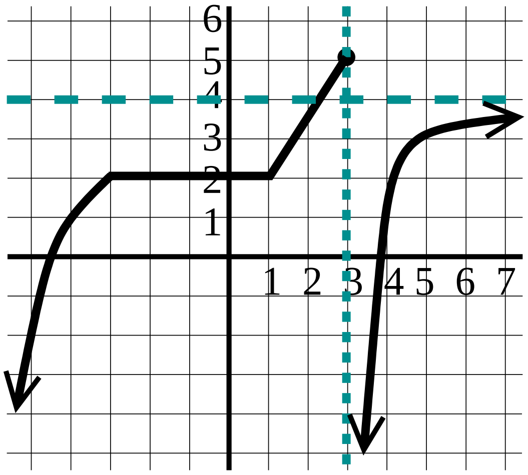
$= 2[cos(e^{x^3}+4)][e^{x^3}(3x^2)]$

$= 6x^2e^{x^3}cos(e^{x^3}+4)$

Note it is better if you don't show all the intermediate steps.

Answer 4.) $\underline{6x^2e^{x^3}cos(e^{x^3}+4)}$

5.) Answer the following questions based on the graph of f given below.



[2] 5a.) domain of $f =$ $\underline{\ R\ }$

[2] 5b.) range of $f =$ $\underline{\ (-\infty, 5]\ }$

[1] 5c.) Is f one-to-one? $\underline{\ No\ }$

[2] 5d.) Does f^{-1} exist? $\underline{\ No\ }$

[1] 5e.) $f(1) =$ $\underline{\ 2\ }$

[2] 5g.) $f'(-1) =$ $\underline{\ 0\ }$

[2] 5f.) Solve $f(x) = 1$: $\underline{\ -4, 4\ }$

[2] 5i.) $lim_{x \to +\infty} f(x) =$ $\underline{\ 4\ }$

[2] 5j.) $lim_{x \to -\infty} f(x) =$ $\underline{\ -\infty\ }$

[2] 5k.) $lim_{x \to 3^+} f(x) =$ $\underline{\ -\infty\ }$

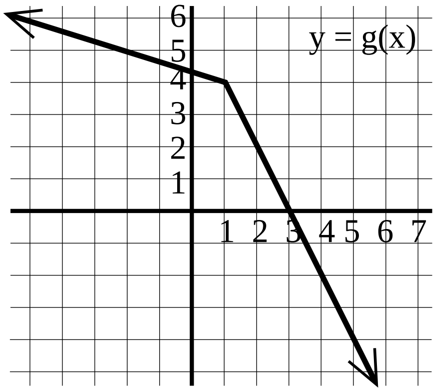
[2] 5l.) $lim_{x \to 3^-} f(x) =$ $\underline{\ 5\ }$

[2] 5m.) State all points where f is not continuous: $\underline{\ x = 3(or(3, 5))\ }$

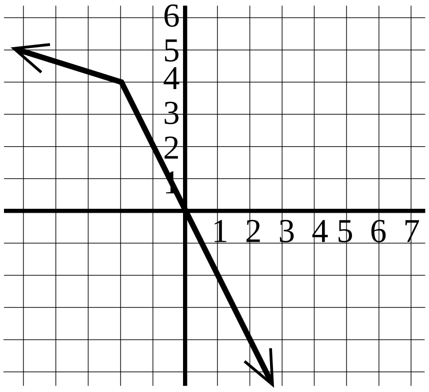
[2] 5n.) State all points where f is not differentiable: $\underline{\ x = 3, -3, 1, (or(3, 5), (-3, 2), (1, 2))\ }$

Note there is a corner at $x = -3$. The transition where f is not constant for $x < -3$ to where f is constant between -3 and 1 is not a smooth transition. However, if you thought it was a smooth transition due to my lack of drawing skills, you will not be docked if you missed this point.

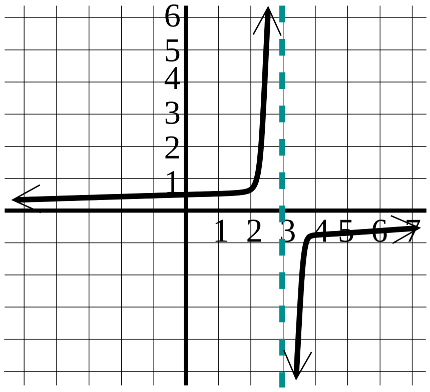
6.) Given the graph of y = g(x) below, draw the following graphs:



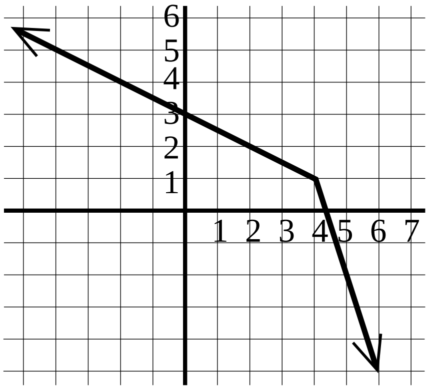
[4] 6a.) y = g(x +3)



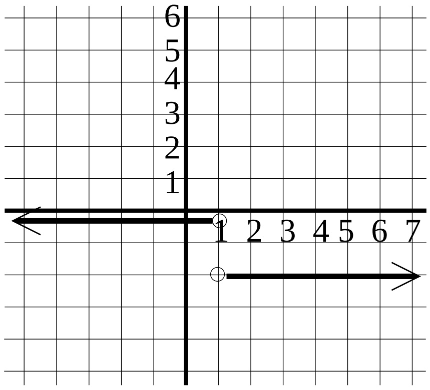
[4] 6b.) $y = \dfrac{1}{g(x)}$



[4] 6c.) $y = g^{-1}(x)$



[4] 6d.) y = g'(x)



[2] 6e.) Where is g differentiable?

Everywhere except at x = 1.