Section 6.1

Find the area between the curve $y^2 = 2x - 2$ and $y = x - 5$.

Method 1: Using dx

1.) Find points of intersection between the two curves.

$y^2 = 2x - 2$ and $y = x - 5$.

$(x-5)^2 = 2x - 2$

$x^2 - 10x + 25 = 2x - 2$

$x^2 - 12x + 27 = 0$

$(x-3)(x-9) = 0$. Hence $x = 3, 9$.

2.) Determine which is larger.

Between 1 and 3: $\sqrt{2x-2} > -\sqrt{2x-2}$

Between 3 and 9: $\sqrt{2x-2} > x - 5$

3.) Write as integral(s)

Note that between 1 and 3, the height of the rectangles is $\sqrt{2x-2} - (-\sqrt{2x-2})$ and the width is dx.

Note that between 3 and 9, the height of the rectangles is $\sqrt{2x-2} - (x-5)$ and the width is dx.

$\int_1^3 [\sqrt{2x-2} - (-\sqrt{2x-2})]dx + \int_3^9 [\sqrt{2x-2} - (x-5)]dx$

4.) Evaluate the integral

$\int_1^3 [2\sqrt{2x-2}]dx + \int_3^9 [\sqrt{2x-2} - (x-5)]dx$

$= \int_1^3 [2\sqrt{2x-2}]dx + \int_3^9 (\sqrt{2x-2})dx - \int_3^9 (x-5)dx$

Let $u = 2x - 2$, $du = 2dx$,
$x = 1 : u = 2(1) - 2 = 0$;
$x = 3 : u = 2(3) - 2 = 4$;
$x = 9 : u = 2(9) - 2 = 16$

$= \int_0^4 u^{\frac{1}{2}} du + \int_4^{16} \frac{1}{2} u^{\frac{1}{2}} du + \int_3^9 (-x+5)dx$

$= \frac{2}{3} u^{\frac{3}{2}} |_0^4 + \frac{1}{3} u^{\frac{3}{2}} |_4^{16} + (-\frac{1}{2}x^2 + 5x)|_3^9$

$$= \frac{2}{3}(4^{\frac{3}{2}} - 0^{\frac{3}{2}}) + \frac{1}{3}(16^{\frac{3}{2}} - 4^{\frac{3}{2}}) + (-\frac{1}{2}(9)^2 + 5(9)) \\ -(-\frac{1}{2}(3)^2 + 5(3))$$

$= \frac{1}{3}[2(8) + 64 - 8] - \frac{81}{2} + 45 + \frac{9}{2} - 15 = 16$

$= \frac{72}{3} - \frac{72}{2} + 30 = 24 - 36 + 30 = 18$

Find the area between the curve $y^2 = 2x - 2$ and $y = x - 5$.

Method 2: Using dy

1.) Find points of intersection between the two curves.

$x = \frac{y^2+2}{2}$ and $x = y + 5$.

$\frac{y^2+2}{2} = y + 5$

$y^2 + 2 = 2y + 10$

$y^2 - 2y - 8 = 0$. Hence, $(y+2)(y-4) = 0$, $y = -2, 4$.

(or note from method 1, that when $x = 3, y = 3 - 5 = -2$ and when $x = 9, y = 9 - 5 = 4$

2.) Determine which function is larger.

Between $y = -2$ and 4:

when $y = 0$, $x = \frac{y^2+2}{2} = \frac{0^2+2}{2}$ and $x = y + 5 = 0 + 5 = 5$.

Hence $y + 5 > \frac{y^2+2}{2}$ when $y \in (-2, 4)$.

3.) Write as integral(s)

Note that between -2 and 4, the height of the rectangles is $y + 5 - (\frac{y^2+2}{2})$ and the width is dy.

$\int_{-2}^{4} [y + 5 - (\frac{y^2+2}{2})] dy$

4.) Evaluate the integral:

$\int_{-2}^{4} [y + 5 - (\frac{y^2+2}{2})] dy = \int_{-2}^{4} [y + 5 - (\frac{y^2}{2} + 1)] dy$

$= \int_{-2}^{4} [y + 4 - \frac{y^2}{2}] dy$

$= \frac{1}{2}y^2 + 4y - \frac{1}{6}y^3 |_{-2}^{4}$

$= \frac{1}{2}(4)^2 + 4(4) - \frac{1}{6}(4)^3 - [\frac{1}{2}(-2)^2 + 4(-2) - \frac{1}{6}(-2)^3]$

$= \frac{1}{2}(16 - 4) + 4(4 + 2) - \frac{1}{6}[64 + 8]$

$= 6 + 24 - \frac{1}{6}[72] = 30 - 12 = 18$