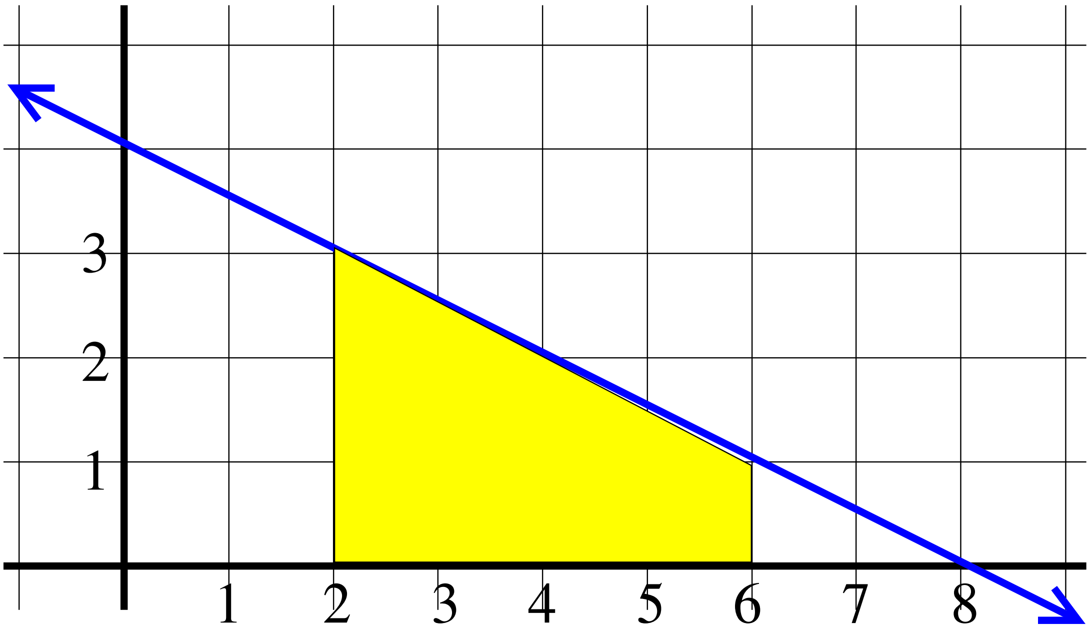
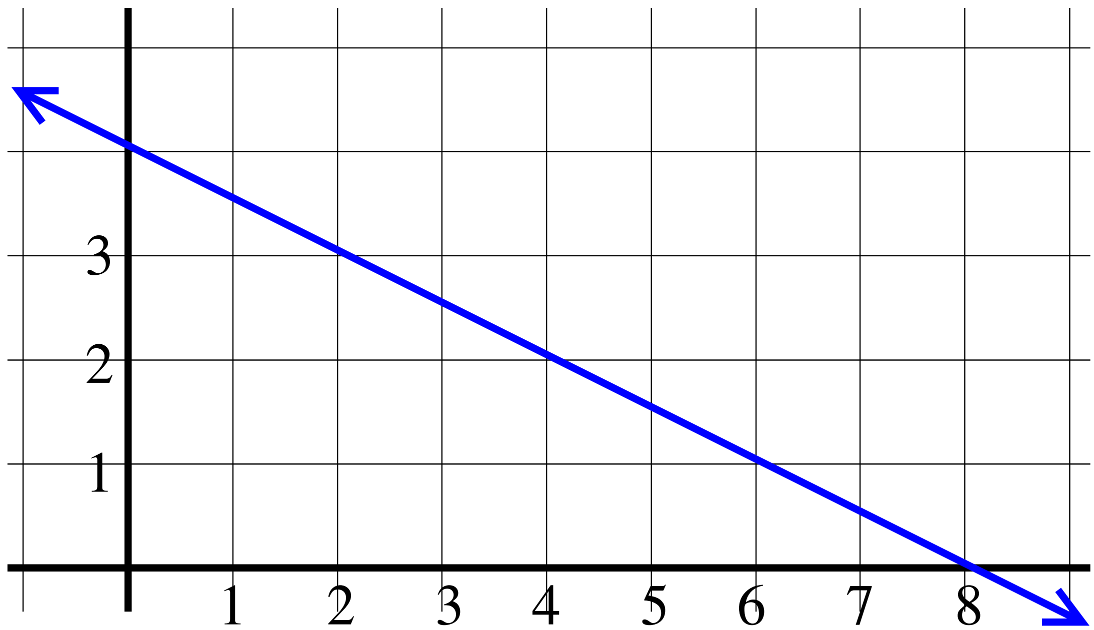
Find the area under the curve $f(x) = -\frac{1}{2}x + 4$, above the $x-$axis and between $x = 2$ and $x = 6$.



Method 1: In this case our function is very simple, so we can determine the area without calculus:

Method 2: Estimate using rectangles.

Inscribed rectangles with $\Delta x = 1$:

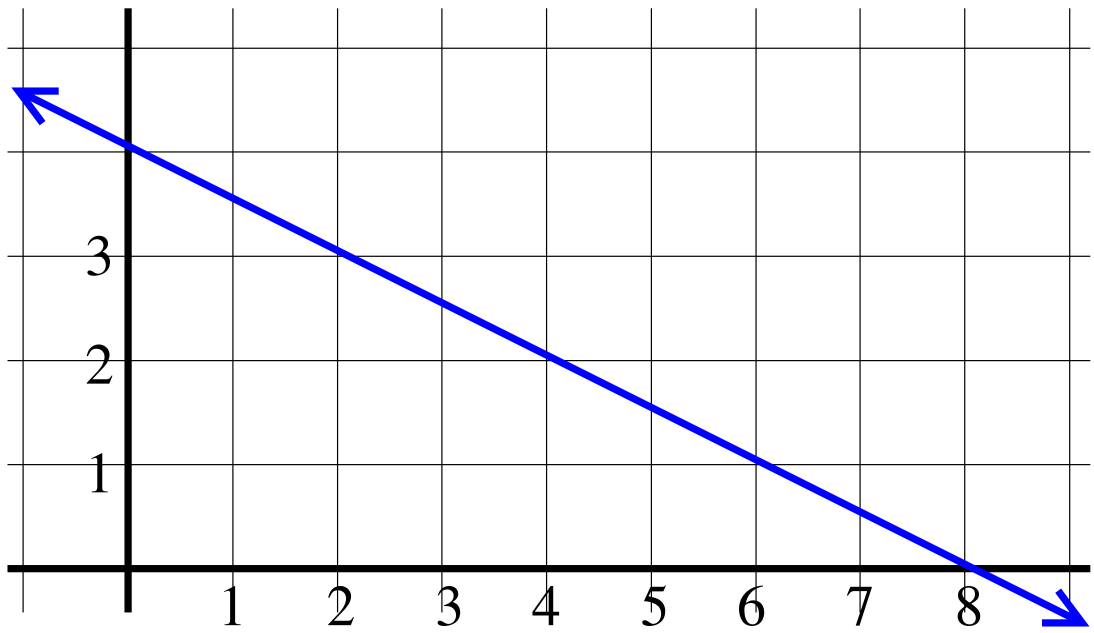


$$f(3)(1) + f(4)(1) + f(5)(1) + f(6)(1) =$$

$$= [-\tfrac{1}{2}(3) + 4](1) + [-\tfrac{1}{2}(4) + 4](1) + [-\tfrac{1}{2}(5) + 4](1) + [-\tfrac{1}{2}(6) + 4](1)$$

$$= \tfrac{5}{2}(1) + 2(1) + \tfrac{3}{2}(1) + 1(1) = 7$$

Inscribed rectangles with $\Delta x = \frac{1}{2}$:



$$f(\tfrac{5}{2})(\tfrac{1}{2}) + f(3)(\tfrac{1}{2}) + f(\tfrac{7}{2})(\tfrac{1}{2}) + f(4)(\tfrac{1}{2})$$
$$+ f(\tfrac{9}{2})(\tfrac{1}{2}) + f(5)(\tfrac{1}{2}) + f(\tfrac{11}{2})(\tfrac{1}{2}) + f(6)(\tfrac{1}{2})$$

$$= [-\tfrac{1}{2}(\tfrac{5}{2}) + 4](\tfrac{1}{2}) + [-\tfrac{1}{2}(3) + 4](\tfrac{1}{2}) + [-\tfrac{1}{2}(\tfrac{7}{2}) + 4](\tfrac{1}{2})$$

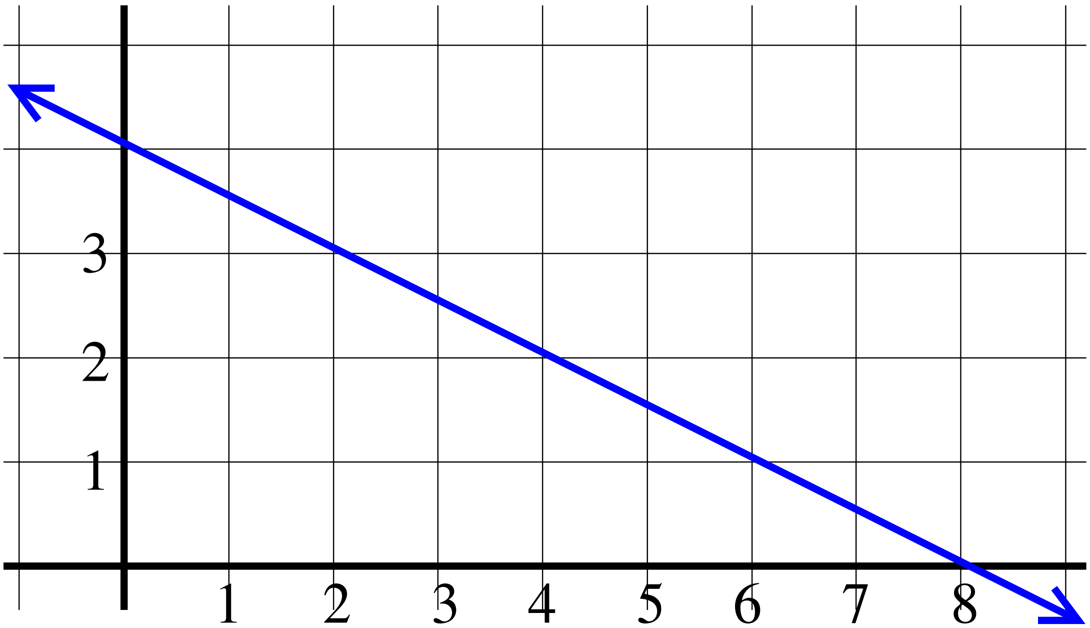
$$+ [-\tfrac{1}{2}(4) + 4](\tfrac{1}{2}) + [-\tfrac{1}{2}(\tfrac{9}{2}) + 4](\tfrac{1}{2}) + [-\tfrac{1}{2}(5) + 4](\tfrac{1}{2})$$

$$+ [-\tfrac{1}{2}(\tfrac{11}{2}) + 4](\tfrac{1}{2}) + [-\tfrac{1}{2}(6) + 4](\tfrac{1}{2})$$

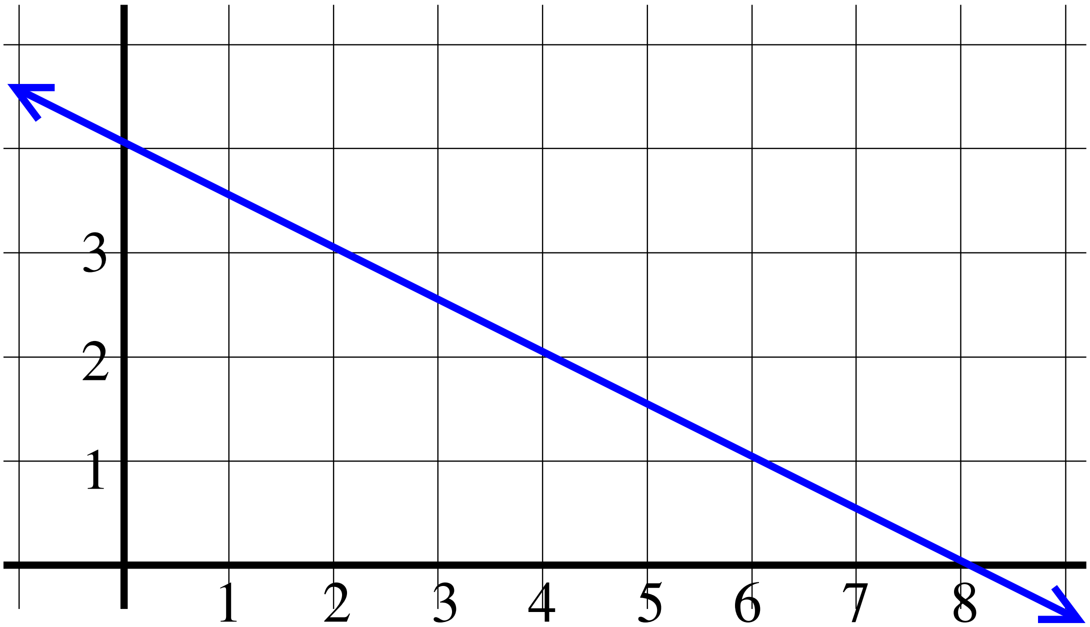
$$= \tfrac{11}{4}(\tfrac{1}{2}) + \tfrac{5}{2}(\tfrac{1}{2}) + \tfrac{9}{4}(\tfrac{1}{2}) + 2(\tfrac{1}{2}) + \tfrac{7}{4}(\tfrac{1}{2}) + \tfrac{3}{2}(\tfrac{1}{2}) + \tfrac{5}{4}(\tfrac{1}{2}) + 1(\tfrac{1}{2})$$

$$= \tfrac{15}{2}$$

Inscribed rectangles with $\Delta x = \frac{6-2}{n} = \frac{4}{n}$:



Circumscribed rectangles with $\Delta x = 1$:



$$f(2)(1) + f(3)(1) + f(4)(1) + f(5)(1) =$$

$$= [-\tfrac{1}{2}(2) + 4](1) + [-\tfrac{1}{2}(3) + 4](1) + [-\tfrac{1}{2}(4) + 4](1) + [-\tfrac{1}{2}(5) + 4](1)$$

$$= 3 + \tfrac{5}{2}(1) + 2(1) + \tfrac{3}{2}(1) = 9$$

Defn: $\int_a^b f(x)dx = lim_{n\to\infty}\Sigma_{i=1}^n f(x_i)\Delta x$

If f is continuous, can use inscribed rectangles, circumscribed rectangles, all left-hand endpoints, all right-hand endpoints, or all midpoints, etc.

If $\Delta x = \frac{b-a}{n}$ and if right-hand endpoints are used, then $x_i = a + i\Delta x = a + \frac{(b-a)i}{n}$

$$\int_a^b f(x)dx = lim_{n\to\infty}\Sigma_{i=1}^n f(a + \frac{(b-a)i}{n})(\frac{b-a}{n})$$

Properties of the definite integral

$\int_a^a f(x)dx = 0$

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

$\int_a^b kf(x)dx = k\int_a^b f(x)dx$

$\int_a^b (f_1 + f_2)(x)dx = \int_a^b f_1(x)dx + \int_a^b f_2(x)dx$

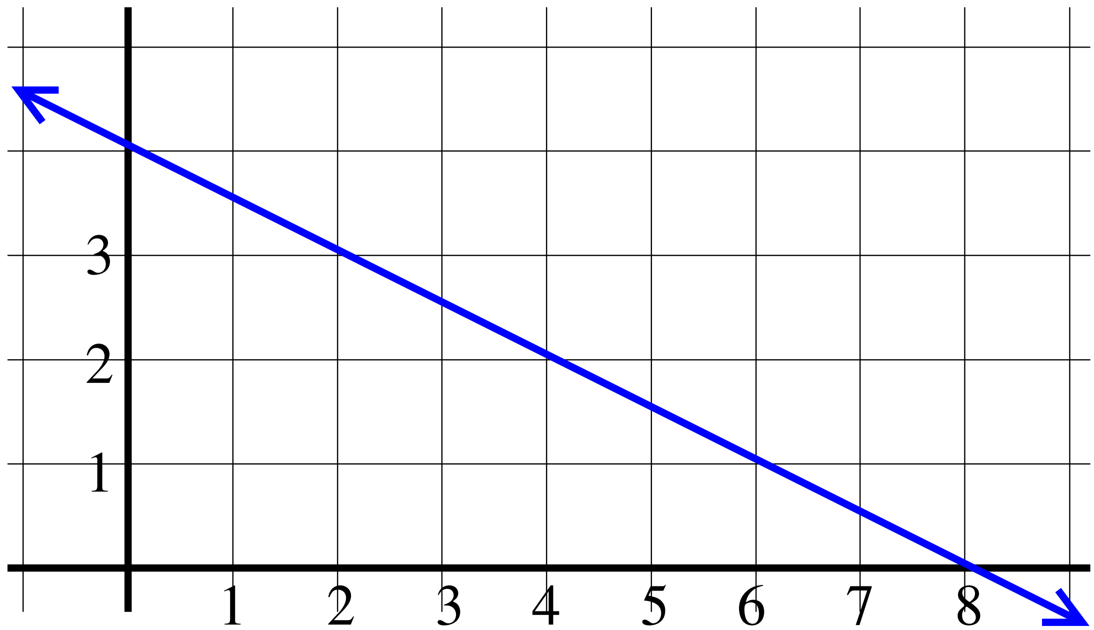
$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$

If $f_1(x) \leq f_2(x)$, then $\int_a^b f_1(x)dx \leq \int_a^b f_2(x)dx$

If $m \leq f(x) \leq M$ then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$

Estimate the distance traveled between $t = 2$ and $t = 6$ if the velocity is given by the function $f(t) = -\frac{1}{2}t + 4$.

Estimate using inscribed rectangles with $\Delta t = 1$:

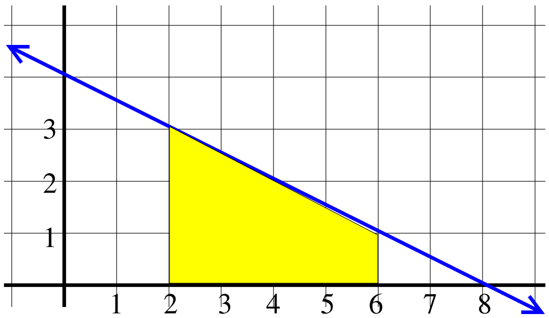


$$f(3)(1) + f(4)(1) + f(5)(1) + f(6)(1) =$$

$$= [-\tfrac{1}{2}(3) + 4](1) + [-\tfrac{1}{2}(4) + 4](1) + [-\tfrac{1}{2}(5) + 4](1) + [-\tfrac{1}{2}(6) + 4](1)$$

$$= \tfrac{5}{2}(1) + 2(1) + \tfrac{3}{2}(1) + 1(1) = 7$$

Find the distance traveled between $t = 2$ and $t = 6$ if the velocity is given by the function $f(t) = -\frac{1}{2}t + 4$.



Method 1: In this case our function is very simple, so we can determine the area without calculus:

Method 2: Use calculus by estimating with rectangles and taking limit.

$$\text{Area} = = lim_{n\to\infty}\Sigma_{i=1}^{n} f(a + \frac{(b-a)i}{n})(\frac{b-a}{n})$$
$$= lim_{n\to\infty}\Sigma_{i=1}^{n} f(2 + \frac{4i}{n})(\frac{4}{n})$$
$$= lim_{n\to\infty}\Sigma_{i=1}^{n}[-\frac{1}{2}(2 + \frac{4i}{n}) + 4](\frac{4}{n}) = 8$$

Method 3 (section 5.3): Use calculus by integrating.

$$\int_2^6 (-\frac{1}{2}t + 4)dt = (-\frac{1}{4}t^2 + 4t)|_2^6$$
$$= (-\frac{1}{4}(6)^2 + 4(6)) - (-\frac{1}{4}(2)^2 + 4(2))$$
$$= -9 + 24 - (-1 + 8) = 15 - 7 = 8$$