Section 4.2

Rolle's theorem: Suppose
1.) f continuous on $[a, b]$
2.) f differentiable on (a, b)
3.) $f(a) = f(b)$.

Then there exists $c \in (a, b)$ such that $f'(c) = 0$

Mean Value Theorem: Suppose
1.) f continuous on $[a, b]$
2.) f differentiable on (a, b)

Then there exists $c \in (a, b)$ Ruth that $f'(c) = \frac{f(b)-f(a)}{b-a}$

Proof: Let $h(x) = f(x) -$ secant line.

$h(x) =$

Applications of MVT (including Rolle's)

Ex 1: Show $x^5 + 3x^3 + 2x - 4 = 0$ has exactly one real root.

Step 1: Show there exists a root (review IVT)

Step 2: Show there is at most one root.

Ex 2: Suppose $f(1) = 2$ and $f'(x) \leq 3$. How large can $f(4)$ be?

Ex 3: If $f'(x) = 0$ for all $x \in (a, b)$, then $f(x) = c$ for some constant c.

Ex 4: If $f'(x) = g'(x)$ for all $x \in (a, b)$, then $f(x) = g(x) + c$ for some constant c.

Section 4.4: Indeterminate forms:

"$\infty - \infty$" "$\infty \cdot 0$" "$\frac{\infty}{\infty}$" "$\frac{0}{0}$"

"∞^0" "0^0" "1^∞"

L'Hospital's Rule: Suppose f' and g' exist, $g'(x) \neq 0$ near a except possibly at a where $a \in \mathcal{R} \cup \{+\infty, -\infty\}$ and if either

1.) $lim_{x \to a} f(x) = lim_{x \to a} g(x) = 0$

or

2.) $lim_{x \to a} f(x) = \pm\infty$ and $lim_{x \to a} g(x) = \pm\infty$

and if $lim_{x \to a} \frac{f'(x)}{g'(x)}$ exists, then

$$lim_{x \to a} \frac{f(x)}{g(x)} = lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Ex 1) $lim_{x \to +\infty} \frac{-2x^2+3}{5x^2+4x}$

Old method: $lim_{x \to +\infty} \frac{-2x^2+3}{5x^2+4x} = lim_{x \to +\infty} \frac{x^2(-2+\frac{3}{x^2})}{x^2(5+\frac{4}{x})} =$

New method:

$lim_{x \to +\infty} \frac{-2x^2+3}{5x^2+4x}$

$lim_{x \to 1} \frac{1-x+ln(x)}{x^3-3x+2}$