Section 2.3

Theorem: If $f(x) \leq g(x)$ near a (except possibly at a) and if $lim_{x\to a}f(x)$ and $lim_{x\to a}g(x)$ exist, then

$$lim_{x\to a}f(x) \leq lim_{x\to a}g(x)$$

Squeeze theorem:
If $f(x) \leq g(x) \leq h(x)$ near a (except possibly at a) and if $lim_{x\to a}f(x) = L$ and $lim_{x\to a}h(x) = L$, then

$$lim_{x\to a}g(x) = L$$

Example: $g(x) = xsin\frac{1}{x}$

$$-|x| \leq xsin\frac{1}{x} \leq |x|$$

$$lim_{x\to 0}\big(- |x|\big) = 0,\ lim_{x\to 0}\big(|x|\big) = 0.$$

Hence , $lim_{x\to 0}\big(xsin\frac{1}{x}\big) = 0$

Section 2.4

Informal Defn: $lim_{x\to a}f(x) = L$ if

x close to a (except possibly at a)
implies $f(x)$ is close to L.

Formal Defn: $lim_{x\to a}f(x) = L$ if

For all $\epsilon > 0$, there exists $\delta > 0$ such that

$0 < |x - a| < \delta$ implies $|f(x) - L| < \epsilon$.

Formal Defn: $lim_{x \to a} f(x) = L$ if

For all $\epsilon > 0$, there exists $\delta > 0$ such that

$0 < |x - a| < \delta$ implies $|f(x) - L| < \epsilon$.

Proof:

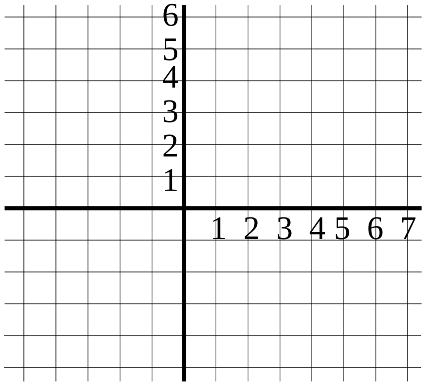
Let $\epsilon > 0$. Choose $\delta =$ ____. Note $\delta =$ ____ > 0.

Suppose $0 < |x - a| < \delta$.

Claim: $|f(x) - L| < \epsilon$.

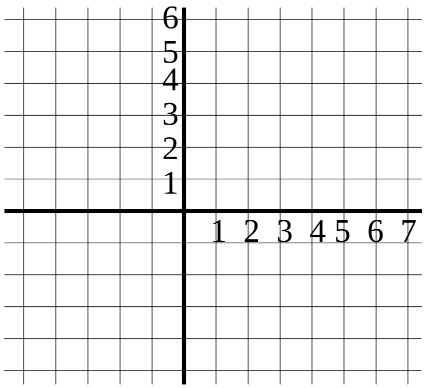
Defn: $lim_{x \to a} f(x) = L$ if
for all $\epsilon > 0$, there exists a $\delta > 0$ such that
$0 < |x - a| < \delta$ implies $|f(x) - L| < \epsilon$

Show $lim_{x \to 1} 2 =$



Defn: $lim_{x \to a} f(x) = L$ if
for all $\epsilon > 0$, there exists a $\delta > 0$ such that
$0 < |x - a| < \delta$ implies $|f(x) - L| < \epsilon$

Show $lim_{x \to 4} 2x + 3 =$



Defn: $lim_{x \to a^-} f(x) = L$ if

x close to a and $x < a$
implies $f(x)$ is close to L.

Defn: $lim_{x \to a^+} f(x) = L$ if

x close to a and $x > a$
implies $f(x)$ is close to L.

Defn: $lim_{x \to a} f(x) = \infty$ if

x close to a (except possibly at a) implies $f(x)$ is large.

Defn: $lim_{x \to a} f(x) = -\infty$ if

x close to a (except possibly at a) implies $f(x)$ is negative and $|f(x)|$ is large.

Section 2.5

Defn: f is continuous at a if $lim_{x \to a} f(x) = f(a)$

(i.e., if $lim_{x \to a} f(x) = f(lim_{x \to a} x)$)

Examples:

Ex: Polynomial, rational, root, trigonometric, inverse trigonometric, exponential, logarithmic functions are continuous functions.

Read left and right continuity

If f, g continuous at a, $c \in \mathcal{R}$, then $f+g$, fg, cf, f/g (if $g(a) \neq 0$) are continuous.

If g continuous at a and f continuous at $g(a)$, then $f \circ g$ continuous at a.

Ex: $lim_{x \to 0} \frac{x^2 - e^{x^3}}{cos(x)} =$

Intermediate value theorem: Suppose f continuous on $[a,b]$, $f(a) \neq f(b)$ and n is between $f(a)$ and $f(b)$, then there exists $c \in (a,b)$ such that $f(c) = N$.

Example: Show that $x^2 - 7x + 1$ has a root between 0 and 1.

Section 2.3: To find vertical asymptotes, find all $a \in \mathcal{R}$ such that $lim_{x \to a^-} f(x) = \pm\infty$ and/or $lim_{x \to a^+} f(x) = \pm\infty$

Ex: $f(x) = \frac{1}{(x+2)(x-3)^2}$

Section 2.6:
Horizontal asymptotes/limits at infinity

To find horizontal asymptotes:
calculate $lim_{x \to +\infty} f(x)$ and $lim_{x \to -\infty} f(x)$

IF $lim_{x \to +\infty} f(x) = L$ where L is a finite real number, then $y = L$ is a horizontal asymptote.

IF $lim_{x \to -\infty} f(x) = K$ where K is a finite real number, then $y = K$ is a horizontal asymptote.

Ex: $f(x) = \frac{2x^3-x^2+1}{8x^3+x+3}$

$lim_{x \to +\infty} \frac{2x^3-x^2+1}{8x^3+x+3} =$

Similarly, $lim_{x \to -\infty} \frac{2x^3-x^2+1}{8x^3+x+3} =$

Horizontal asymptote(s):

Ex: $f(x) = \frac{x^2+1}{2x^5+x^2-3}$

$lim_{x \to +\infty} \frac{x^2+1}{2x^5+x^2-3} =$

Similarly, $lim_{x \to -\infty} \frac{x^2+1}{2x^5+x^2-3} =$

Horizontal asymptote(s):

Ex: $f(x) = \frac{2x^5+x^2-3}{x^2+1}$

$lim_{x \to +\infty} \frac{2x^5+x^2-3}{x^2+1} =$

Also, $lim_{x \to -\infty} \frac{2x^5+x^2-3}{x^2+1} =$

Horizontal asymptote(s):

Ex: $f(x) = \frac{2x}{\sqrt{x^2+1}}$

$lim_{x \to +\infty} \frac{2x}{\sqrt{x^2+1}} =$

$lim_{x \to -\infty} \frac{2x}{\sqrt{x^2+1}} =$

Horizontal asymptote(s):

Ex: $f(x) = x^2 - x^3$

$lim_{x \to +\infty} x^2 - x^3 =$

$lim_{x \to -\infty} x^2 - x^3 =$

Horizontal asymptote(s):

Ex: $f(x) = x^{\frac{2}{3}} - x$

$lim_{x \to +\infty} x^{\frac{2}{3}} - x =$

$lim_{x \to -\infty} x^{\frac{2}{3}} - x =$

Horizontal asymptote(s):