Table of Integrals

$$log_a(x) = \frac{lnx}{lna} \tag{1}$$

Basic Forms

$$\int u\,dv = uv - \int v\,du \tag{2}$$

Integrals with Roots

$$\int x\sqrt{x-a}\,dx = \frac{2}{3}a(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2} \tag{3}$$

$$\int \sqrt{\frac{x}{a-x}}\,dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a} \tag{4}$$

$$\int \sqrt{\frac{x}{a+x}}\,dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x}+\sqrt{x+a}\right] \tag{5}$$

$$\int x\sqrt{ax+b}\,dx = \frac{2}{15a^2}(-2b^2+abx+3a^2x^2)\sqrt{ax+b} \tag{6}$$

$$\int \sqrt{x(ax+b)}\,dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)}\right.$$
$$\left.-b^2\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right|\right] \tag{7}$$

$$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \pm \frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right| \tag{8}$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} \tag{9}$$

$$\int \frac{1}{\sqrt{x^2\pm a^2}}\,dx = \ln\left|x+\sqrt{x^2\pm a^2}\right| \tag{10}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a} \tag{11}$$

Integrals with Logarithms

$$\int \ln x\,dx = x\ln x - x \tag{12}$$

$$\int \ln(ax+b)\,dx = \left(x+\frac{b}{a}\right)\ln(ax+b) - x, a\neq 0 \tag{13}$$

$$\int \ln(x^2+a^2)\,dx = x\ln(x^2+a^2) + 2a\tan^{-1}\frac{x}{a} - 2x \tag{14}$$

$$\int \ln(x^2-a^2)\,dx = x\ln(x^2-a^2) + a\ln\frac{x+a}{x-a} - 2x \tag{15}$$

$$\int x\ln(ax+b)\,dx = \frac{bx}{2a} - \frac{1}{4}x^2$$
$$+ \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax+b) \tag{16}$$

Integrals with Exponentials

$$\int a^x\,dx = \frac{a^x}{lna}, \ a>0, a\neq 1 \tag{17}$$

$$\int \sqrt{x}e^{ax}\,dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\text{erf}\left(i\sqrt{ax}\right),$$
$$\text{where erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt \tag{18}$$

$$\int xe^x\,dx = (x-1)e^x \tag{19}$$

$$\int xe^{ax}\,dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} \tag{20}$$

$$\int x^2 e^x\,dx = \left(x^2 - 2x + 2\right)e^x \tag{21}$$

$$\int x^2 e^{ax}\,dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax} \tag{22}$$

$$\int x^3 e^x\,dx = \left(x^3 - 3x^2 + 6x - 6\right)e^x \tag{23}$$

$$\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx \tag{24}$$

$$\int x^n e^{ax}\,dx = \frac{(-1)^n}{a^{n+1}}\Gamma[1+n, -ax],$$
$$\text{where } \Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}\,dt \tag{25}$$

$$\int e^{ax^2}\,dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}}\text{erf}\left(ix\sqrt{a}\right) \tag{26}$$

$$\int e^{-ax^2}\,dx = \frac{\sqrt{\pi}}{2\sqrt{a}}\text{erf}\left(x\sqrt{a}\right) \tag{27}$$

$$\int x^2 e^{-ax^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}\text{erf}(x\sqrt{a}) - \frac{x}{2a}e^{-ax^2} \tag{28}$$

Integrals with Trigonometric Functions

$$\int \sec^2 x\,dx = \tan x \tag{29}$$

$$\int \tan x\,dx = -\ln\cos x \tag{30}$$

Products of Trigonometric Functions and Exponentials

$$\int e^{bx}\cos ax\,dx = \frac{1}{a^2+b^2}e^{bx}(a\sin ax + b\cos ax) \tag{31}$$

$$\int xe^x\sin x\,dx = \frac{1}{2}e^x(\cos x - x\cos x + x\sin x) \tag{32}$$

$$\int xe^x\cos x\,dx = \frac{1}{2}e^x(x\cos x - \sin x + x\sin x) \tag{33}$$

Definite integral lacking closed-form antiderivatives

$$\int_\infty^\infty e^{-x^2} = \sqrt{\pi} \tag{34}$$