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[7] 1.) Numerical approximations for solutions to differential equations are often needed as the solutions to many differential equations cannot be expressed algebraically.

A) True

B) False

[7] 2.) If f is continuous, then f is integrable.

A) True

B) False

[7] 3.) For extremely large positive x, $x^{255} < (1.01)^x$

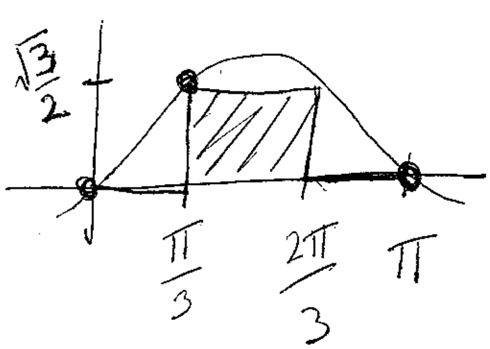
A) True

B) False

[7] 4.) Use 3 inscribed rectangles of equal width to estimate $\int_0^\pi sin(x)dx$.

A) $\frac{\pi}{6}$ B) $\frac{\sqrt{2}\pi}{6}$ C) $\frac{\sqrt{3}\pi}{6}$ D) $\frac{\pi}{2}$ E) π

F) 0 G) $\frac{1}{2}$ H) $\frac{\sqrt{2}}{6}$ I) $\frac{\sqrt{3}}{6}$ J) 1



$$0 + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\pi}{3}\right) + 0$$

$$= \frac{\sqrt{3}\,\pi}{6}$$

[7] 5.) If $f(x) = ln\left(\frac{2e^x - e^{-x}}{e^x}\right)$, then the instantaneous rate of change at at $x = 0$ is

A) Does not exist B) -1 C) -2 D) -3 E) -4

F) 0 G) 1 H) 2 I) 3 J) 4

$$\left[\ln(2e^x - e^{-x}) - \ln(e^x)\right]'$$

$$= \left[\ln(2e^x - e^{-x}) - x\right]'$$

$$= \frac{2e^x + e^x}{2e^x - e^{-x}} - 1$$

$$f'(0) = \frac{2+1}{2-1} - 1$$

$$= 3 - 1 = 2$$

[7] 6.) Find the equation of the tangent line to $f(x) = \frac{x^2+1}{x+1}$, at $x = 0$

A) $y = x$ B) $y = -x$ C) $y = 2x + 1$ D) $y = 2x - 1$ E) $y = -1$

F) $y = x + 1$ G) $y = -x + 1$ H) $y = x - 1$ I) $y = -x - 1$ J) $y = 1$

$$f'(x) = \frac{(x+1)(2x) - (x^2+1)}{(x+1)^2}$$

$$f'(0) = \frac{0-1}{1} = -1 = \text{slope}$$

$$\text{pt}: (0, f(0)) = (0, 1)$$

$$y = -x + 1$$

[7] 7.) Use linearization to approximate $\sqrt[3]{9}$

A) $\frac{8}{3}$ B) $\frac{10}{3}$ C) $\frac{13}{6}$ D) $\frac{23}{12}$ E) $\frac{25}{12}$

F) 1 G) 2 H) 3 I) $\frac{5}{2}$ J) $\frac{9}{4}$

$$f(x) = \sqrt[3]{x} = x^{1/3}$$

$$f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3x^{2/3}}$$

$$f'(8) = \frac{1}{3(8)^{2/3}} = \frac{1}{3(2^2)} = \frac{1}{12} = \text{slope}$$

point on line: $(8, f(8)) = (8, 2)$

$$\frac{y-2}{x-8} = \frac{1}{12} \Rightarrow y - 2 = \frac{1}{12}(x-8)$$

$$y = \frac{1}{12}(x-8) + 2$$

$$\Rightarrow \sqrt[3]{x} \approx \frac{1}{12}(x-8) + 2$$

$$\Rightarrow \sqrt[3]{9} \sim \frac{1}{12}(9-8) + 2 = \frac{1}{12} + 2 = \frac{1+24}{12} = \frac{25}{12}$$

[7] 8.) Suppose the function $f(x) = (sinx)e^{-x}$ represents the concentration of a certain drug in the blood stream during the time period between $x = 0$ and $x = \frac{\pi}{2}$. Find the maximum concentration of drug during this time interval (i.e., in the interval $[0, \frac{\pi}{2}]$).

A) 0 B) $\frac{1}{2}e^{-\frac{\pi}{6}}$ C) $\frac{\sqrt{2}}{2}e^{-\frac{\pi}{4}}$ D) $\frac{\sqrt{3}}{2}e^{-\frac{\pi}{3}}$ E) $e^{-\frac{\pi}{2}}$

F) $\frac{\sqrt{3}}{2}e^{-\frac{\pi}{6}}$ G) $\frac{1}{2}e^{-\frac{\pi}{4}}$ H) $\frac{1}{2}e^{-\frac{\pi}{3}}$ I) 1 J) Does not exist

$$f'(x) = (\cos x)e^{-x} + \sin x(-e^{-x})$$

$$= e^{-x}(\cos x - \sin x) = 0$$

$$e^{-x} > 0 \qquad \cos x - \sin x = 0$$

$$\cos x = \sin x$$

$$\Rightarrow \boxed{x = \frac{\pi}{4} \quad \text{for } x \in [0, \frac{\pi}{2}]}$$

$\frac{\pi}{4}$

x	y
0	$\sin(0)e^{0} = 0$
$\frac{\pi}{2}$	$\sin(\frac{\pi}{2})e^{-\pi/2} = e^{-\pi/2}$
$\frac{\pi}{4}$	$\sin(\frac{\pi}{4})e^{-\pi/4} = \frac{\sqrt{2}}{2}e^{-\pi/4}$

$+ \nearrow \quad \frac{\pi}{4} \quad \searrow$

$$f'(x) = e^{-x}(\cos x - \sin x)$$

[7] 9.) For the data sets below, graph these points on either semi-log or log-log paper and determine the function from the choices below which best models these data points.

Data set: (1, 1000), (5, 450), (70, 110), (3000, 11)

A) $1000(10^{-\frac{t}{2}})$ B) $1000(10^{-\frac{2t}{3}})$ C) $1000(10^{-t})$ D) $1000(10^{-\frac{3t}{2}})$ E) $1000(10^{-2t})$

F) $1000t^{-\frac{1}{2}}$ G) $1000t^{-\frac{2}{3}}$ H) $1000t$ I) $1000t^{-\frac{3}{2}}$ J) $1000t^{-2}$

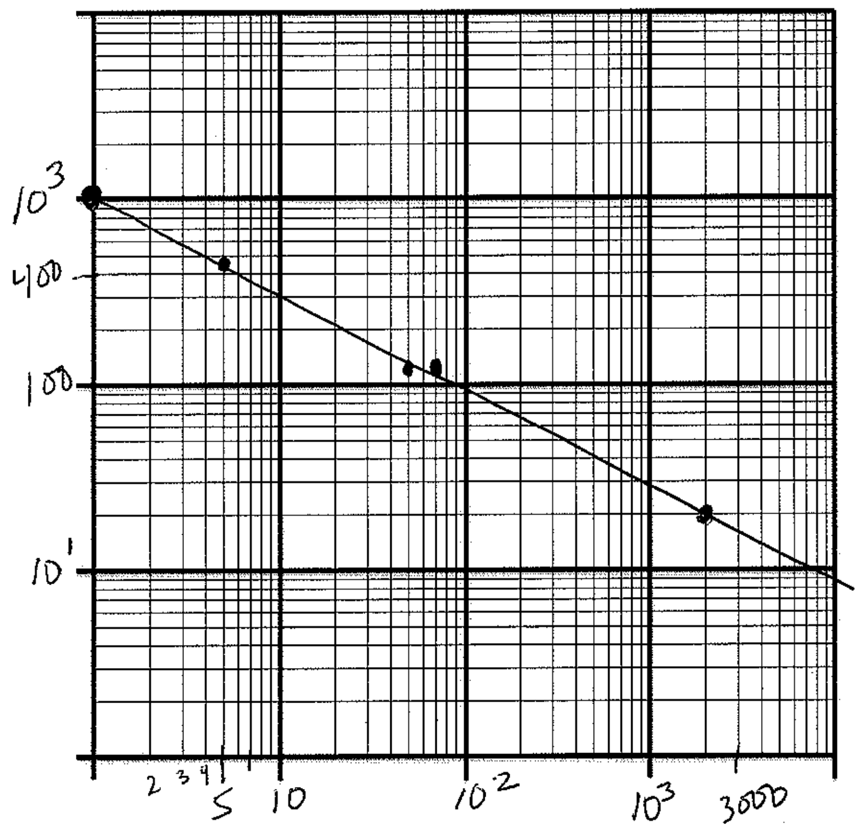
$y = -\frac{1}{2}x + 3$

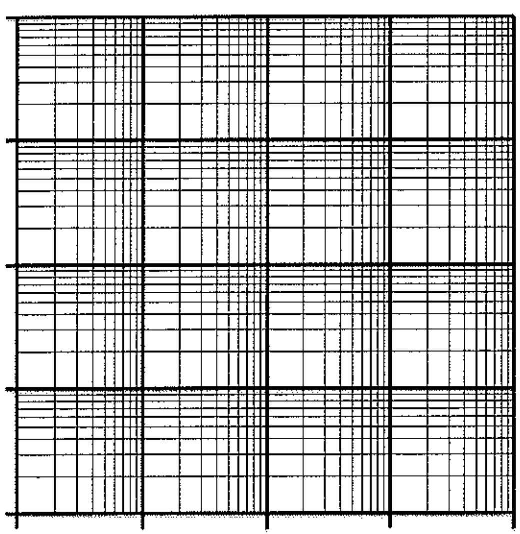
$m = -\frac{1}{2}$

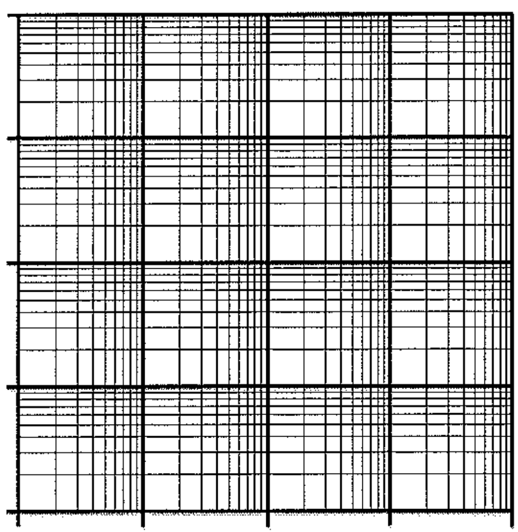
$A = 10^{b}$

$y = 10^{b} t^{m}$

$= 1000 t^{-1/2}$







[7] 10.) $\int_0^\infty e^{-x}dx$

A) $\frac{e}{2}$ **B)** $\frac{e-1}{2}$ **C)** $\frac{1-e}{2}$ **D)** $-\frac{e}{2}$ **E)** Does not exist (Divergent)

F) 1 **G)** $\frac{1}{2}$ **H)** 0 **I)** $-\frac{1}{2}$ **J)** -1

$$\int_0^\infty e^{-x}dx = \lim_{b\to\infty}\int_0^b e^{-x}dx$$

$$= \lim_{b\to\infty}\left[-e^{-x}\right]\Big|_0^b$$

$$= \lim_{b\to\infty}\left(-e^{-b} - (-e^{-0})\right)$$

$$= \lim_{b\to\infty}\left(-e^{-b} + 1\right)$$

$$= \underline{1}$$

[7] 11.) Find the area of the region bounded by $y = 2x$ and $y = \sqrt{x}$

A) $\frac{1}{48}$ B) $\frac{3}{32}$ C) $\frac{1}{4}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$

F) $\frac{11}{16}$ G) 1 H) 2 I) $\frac{32}{3}$ J) 0

$2x = \sqrt{x}$

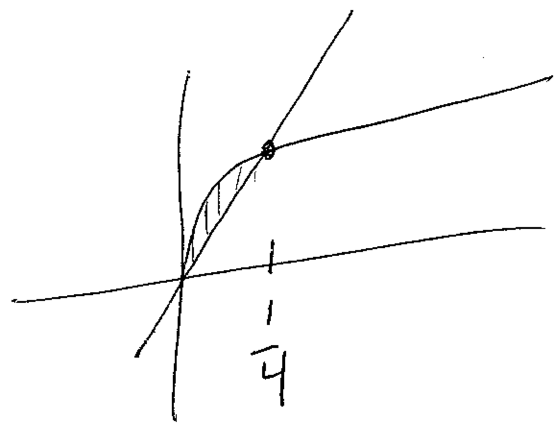
$4x^2 = x$

$4x^2 - x = 0$

$x(4x-1) = 0$

$\Rightarrow x = 0$ & $4x - 1 = 0$

$x = \frac{1}{4}$



$$\int_0^{\frac{1}{4}} (\sqrt{x} - 2x)\,dx = \int_0^{\frac{1}{4}} (x^{1/2} - 2x)\,dx$$

$$= \frac{2}{3}x^{3/2} - x^2 \Big|_0^{\frac{1}{4}} = \frac{2}{3}\left(\frac{1}{4}\right)^{3/2} - \left(\frac{1}{4}\right)^2 - [0-0]$$

$$= \frac{2}{3}\left(\frac{1}{2^3}\right) - \frac{1}{16} = \frac{4-3}{3(16)} = \frac{1}{48}$$

[7] 12.) Polonium-208 is a radioactive element that undergoes exponential decay according to the differential equation: $y' = -ky$. A sample of 10 g of Polonium is placed on a table. Suppose that after 7 years, only 2 g are left. How much Polonium is left after 14 years.

A) $\frac{1}{10}$ g	**B)** $\frac{1}{7}$ g	**C)** $\frac{1}{5}$ g	**D)** $\frac{2}{7}$ g	**E)** $\frac{2}{5}$ g
F) $ln(2)$ g	**G)** $\frac{1}{2}$ g	**H)** 1 g	**I)** $\frac{3}{2}$ g	**J)** $\frac{5}{2}$ g

$$\frac{dy}{dt} = y' = -ky$$

$$\int \frac{dy}{y} = -\int k\,dt$$

$$\ln|y| = -kt + C$$

$$|y| = e^{-kt} e^{C}$$

$$y = Ce^{-kt}$$

$$t = 0,\ y = 10$$

$$10 = C(1)$$

$$y = 10e^{-kt}$$

$$t = 7,\ y = 2$$

$$2 = 10e^{-7k}$$

$$\ln \frac{1}{5} = \ln\left(e^{-7k}\right)$$

$$\ln 1 - \ln 5 = -7K$$

$$-\ln 5 = -7K$$

$$\Rightarrow K = \frac{\ln 5}{7}$$

$t = 14$:

$$y = 10e^{-14K}$$

$$= 10e^{-2\ln 5}$$

$$= 10e^{\ln 5^{-2}}$$

$$= 10(5^{-2})$$

$$= \frac{10}{25} = \frac{2}{5}\text{ g}$$

$$\frac{3xy}{x^2+4} = \frac{3x(x^2+4)^{3/2}}{8(x^2+4)} = \frac{3x(x^2+4)^{1/2}}{8}$$

✓

[7] 13.) Solve the following initial value problem: $y' = \frac{3xy}{x^2+4}$, $y(0) = 1$

A) $y = 0$ **B)** $y = 1$ **C)** $y = \frac{3}{2}x - \frac{1}{2}$ **D)** $y = \frac{1}{4}(x^2+4)$

E) $y = -\frac{3}{2}x(x^2+4)^{-2} + 1$ **F)**$y = -\frac{3}{2}(x^2+4)^{-2} + 4$ **G)** $y = -\frac{3}{2}(x^2+4)^{-2} + \frac{35}{32}$

H) $y = \frac{3}{2}(x^2+4) - 5$ **I)** $y = (x^2+4)^{\frac{3}{2}} - 7$ **J)** $y = \frac{1}{8}(x^2+4)^{\frac{3}{2}}$

$$y' = \frac{3}{16}(x^2+4)^{\frac{1}{2}} \cdot 2x = \frac{3x}{8}(x^2+4)^{1/2}$$

$$y' = \frac{dy}{dx} = \frac{3xy}{x^2+4}$$

$$\int \frac{dy}{y} = \int \frac{3x\,dx}{x^2+4}$$

Let $u = x^2+4$

$\frac{du}{2} = \frac{2x\,dx}{2}$

$$\ln|y| = \int \frac{3\,du}{2u}$$

$$\ln|y| = \frac{3}{2}\ln|u| + C$$

$e \qquad e$

$$|y| = e^{\ln|u|^{3/2}} e^{c}$$

$$y = |u|^{3/2} C = y = C|x^2+4|^{3/2}$$

$1 = C(8) \Rightarrow C = \frac{1}{8}$

$y = \frac{1}{8}|x^2+4|^{3/2}$

[7] 14.) Which of the following could be the general solution to the differential equation whose direction field is given below:

A) $y = t + C$

B) $y = t^2 + C$

C) $y = Ct$

D) $y = Ct^2$

E) $y = ln|t| + C$

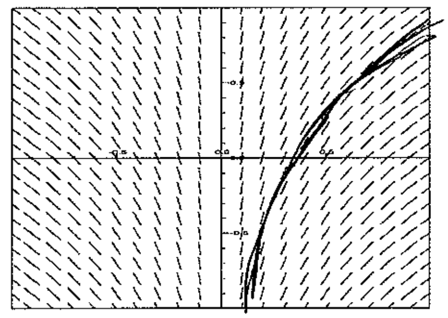
F) $y = C$

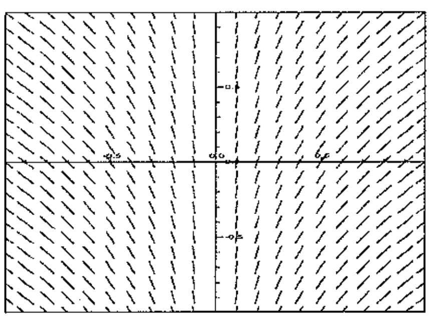
G) $y = Ce^t$

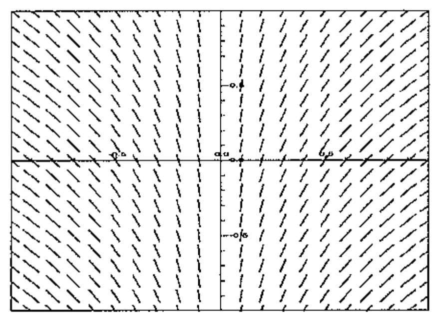
H) $y = Ce^{-t}$

I) $y = cos(t) + C$

J) $y = sin(t) + C$







[7] 15.) Determine the equilibrium solutions (values) to the differential equation $y' = y^2(y-2)$. Determine if these solutions are stable, unstable, or semi-stable.

A) $y = 0$ is stable; $y = 2$ is stable

B) $y = 0$ is stable; $y = 2$ is semi-stable

C) $y = 0$ is stable; $y = 2$ is unstable

D) $y = 0$ is semi-stable; $y = 2$ is stable

E) $y = 0$ is semi-stable; $y = 2$ is semi-stable

F) $y = 0$ is semi-stable; $y = 2$ is unstable

G) $y = 0$ is unstable; $y = 2$ is stable

H) $y = 0$ is unstable; $y = 2$ is semi-stable

I) $y = 0$ is unstable; $y = 2$ is unstable

J) There are no equilibrium solutions

$y^2(y-2) = 0 \Rightarrow y = 0, 2$