$$\frac{f}{g} = f \cdot g^{-1}$$ −1 exponent

$$\left(\frac{f}{g}\right)' = \left(f \cdot g^{-1}\right)'$$

$$\left(\frac{f(x)}{g(x)}\right)' = \left(f(x) \cdot [g(x)]^{-1}\right)'$$

$$= f'(x) \cdot [g(x)]^{-1} + f(x) \cdot \left([g(x)]^{-1}\right)'$$

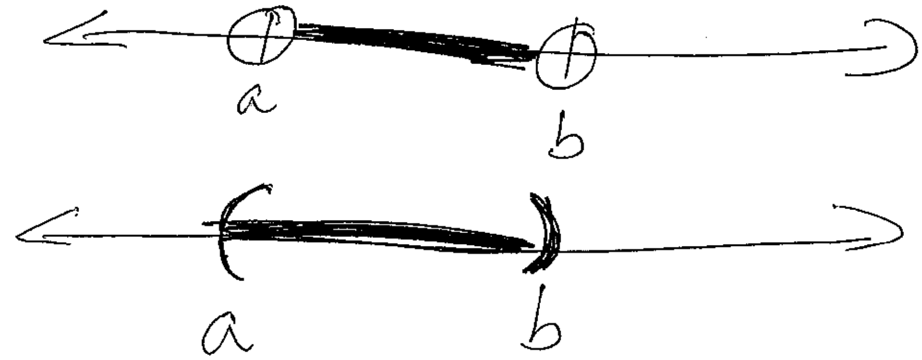
$$= f'(x) \cdot [g(x)]^{-1} + f(x)\left(-[g(x)]^{-2} g'(x)\right)$$

$$= \left(f'(x) \cdot [g(x)]^{-1} - f(x) g'(x) [g(x)]^{-2}\right) \frac{[g(x)]^2}{[g(x)]^2}$$

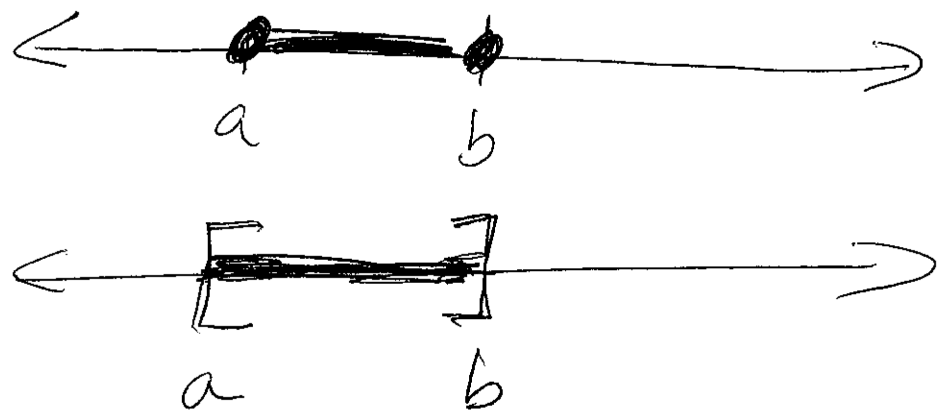
$$= \frac{f'(x) \cdot g(x) - f(x) g'(x)}{[g(x)]^2}$$

3.1) Interval defn's

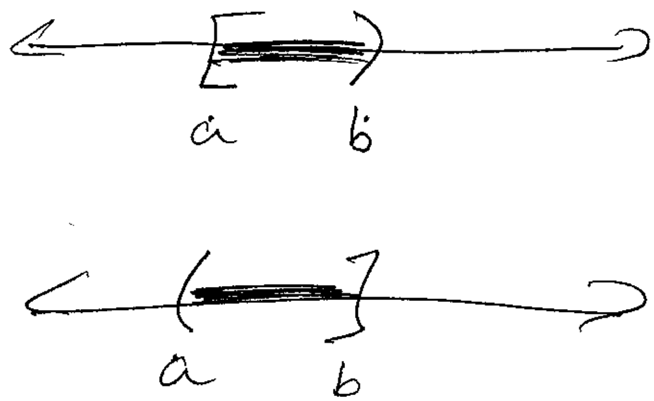
open: $(a,b) = \{x \mid a < x < b\}$



closed: $[a,b] = \{x \mid a \le x \le b\}$



half open half closed

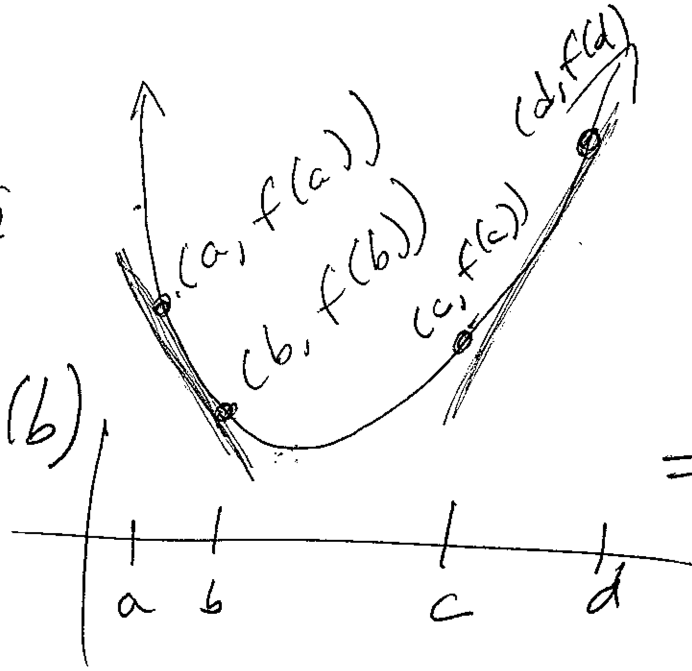


FYI: for all = $\forall$

there exists = $\exists$

f is **decreasing** if $a < b$

$\Rightarrow f(a) > f(b)$



f is **increasing** if ~~a<b~~ $c < d$

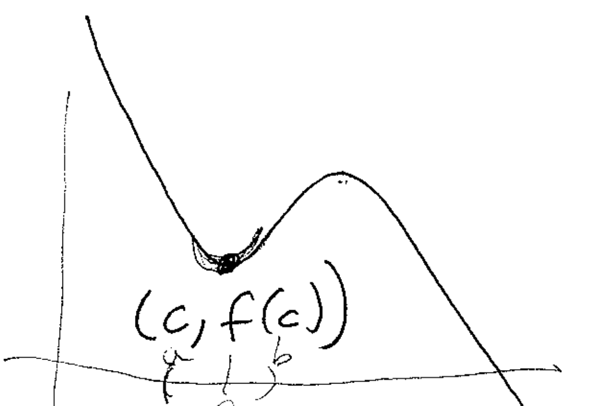
$\Rightarrow f(c) < f(d)$

Thm 1: $f'(x) < 0$ for all x in an interval I

$\Rightarrow$ f is decreasing over I

$f'(x) > 0$ for all x in an interval I

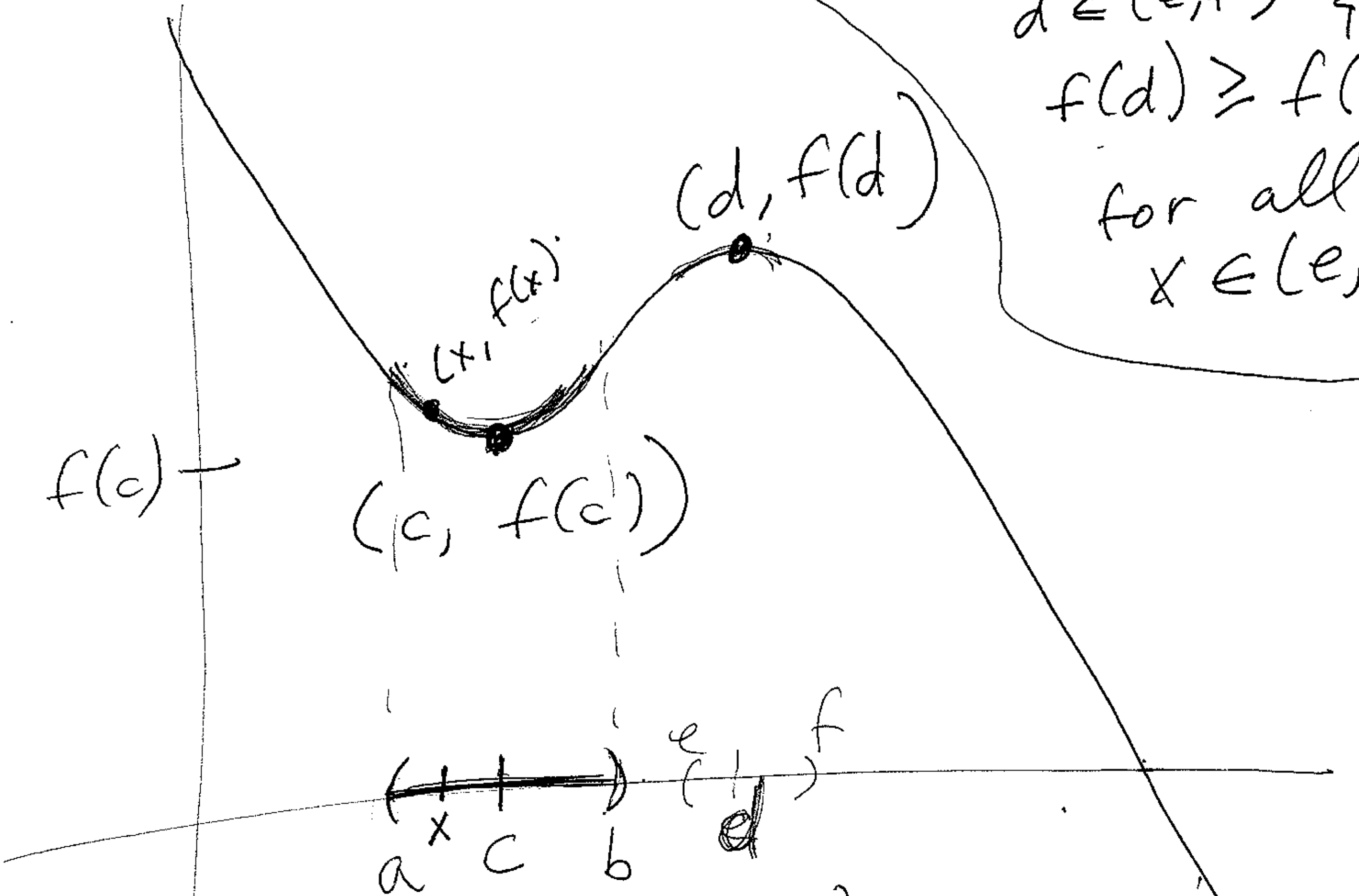
$\Rightarrow$ f is increasing over I



$f(c)$ is a **relative minimum** if there exists (a,b) s.t. $c \in (a,b)$ & $f(x) \geq f(c)$ for all $x \in (a,b)$

$f(c)$ & $f(d)$ are <u>relative extrema</u>

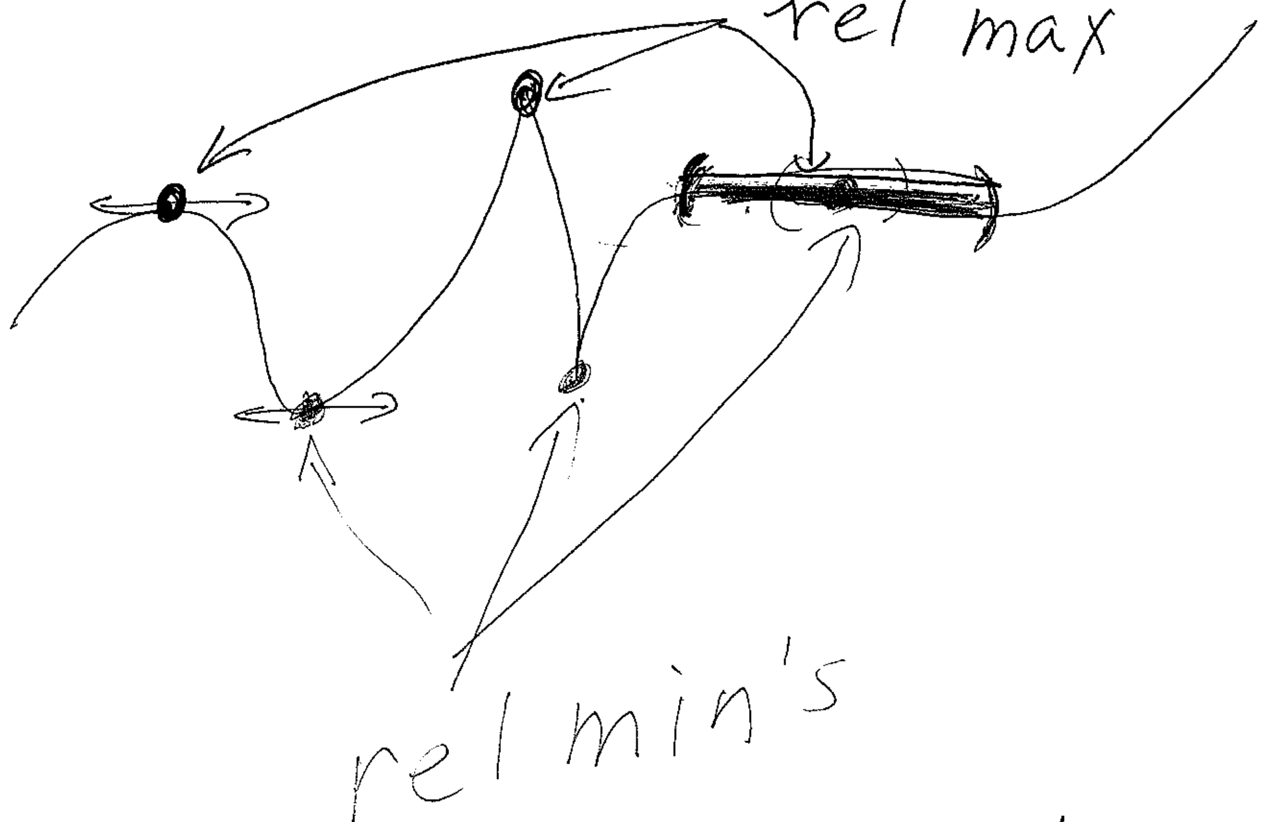
$f(d)$ is a <u>relative maximum</u> if there exist (e,f) st $d \in (e,f)$ & $f(d) \geq f(x)$ for all $x \in (e,f)$



(or local)

$f(c)$ is a <u>relative minimum</u> if if there exists (a,b) st $c \in (a,b)$ [ie $a < c < b$] and $f(x) \geq f(c)$ for all $x \in (a,b)$

c is a critical point of f if $f'(c) = 0$ or DNE



~~Note the converse of~~

SIDENOTE: C SHOULD ALSO BE IN DOMAIN OF f (but you can ignore that & focus on all points of interest ie $f'(c) = 0$, DNE

Thm 2: If $f(c)$ is a relative extrema then c is a critical point of f

---

Note the converse is false

Thm 2 is <u>NOT</u> an if and only if

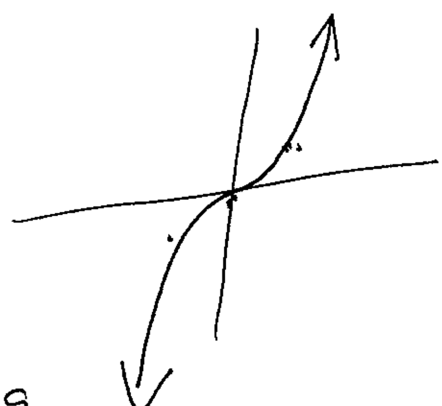
Ex: $f(x) = x^3$

$f'(x) = 3x^2$

$3x^2 = 0 \Rightarrow x = 0$

$x = 0$ is a critical point

but $f(0)$ is not a relative extrema

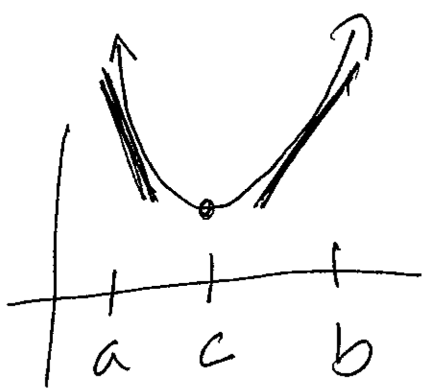


f is an increasing function even though $f'(0) = 0$

Thm 3: 1rst derivative test for finding extrema

Suppose f is cont on (a,b)

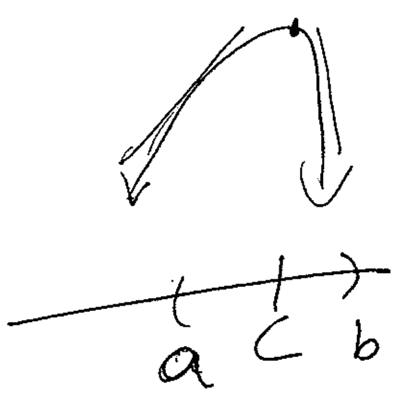
1) If $f'(x) < 0$ on (a,c)
$f'(x) > 0$ on (c,b)



$\Rightarrow$ f has rel min at c

2) If $f'(x) > 0$ on (a,c)
$f'(x) < 0$ on (c,b)

$\Rightarrow$ f has rel max at c



(3) $f'(x) > 0$ on (a, c) & (c, b)

$\Rightarrow$ no rel ext at c

$f'(x) < 0$ on (a, c) & (c, b)

$\Rightarrow$ no rel ext at c

$f(x) = x^2 + x - 6$

$f'(x) = 2x + 1$

$2x + 1 = f'(x)$

rel. min

$-$ $+$

$-\frac{1}{2}$