2.9 Higher Order Derivatives

$$y = f(x)$$

$$\underbrace{f' = y' = \frac{dy}{dx} = \frac{df}{dx}}_{\text{Take derivative once}}$$

$\boxed{\text{velocity}}$

$$\underbrace{f'' = (f')' = y'' = \frac{d^2y}{dx^2} = \frac{d^2f}{dx^2}}_{\text{Take derivative twice}}$$

$$= \frac{d\left(\frac{df}{dx}\right)}{dx}$$

$\boxed{\text{acceleration}}$

Ex: $(x^2)'' = (2x)' = 2$

$$x^2 \longrightarrow 2x \longrightarrow 2$$

Ex: $[\sin(3x)]'''$

$$\sin(3x) \xrightarrow{①} [\cos(3x)] \cdot (3)$$

②

$$[-\sin(3x)] \cdot (3) \cdot (3)$$

③

$$[-\cos(3x)]\,(3) \cdot (3) \cdot (3)$$

$$= -27\cos(3x)$$

EX: $\left[\sin(3x)\right]$

$$\downarrow$$

$$3\cos(3x)$$

$$\downarrow$$

$$-9\sin(3x)$$

$$\downarrow$$

$$-27\cos(3x)$$

$$\downarrow$$

$$81\sin(3x)$$

$$\downarrow$$

$$3^5\cos(3x)$$

EX: $(x)^{(4)} = x'''' = x^{(iv)}$

take 4th derivative

$$x^4 \neq x^{(4)}$$

$$x \to 1 \to 0 \to 0 \to 0$$

$$x^{(4)} = 0$$

4th derivative of x

EX: $(x^4)^{(4)} = 24$

$$x^4 \xrightarrow{1} 4x^3 \xrightarrow{2} 12x^2 \xrightarrow{3} 24x \xrightarrow{4} 24$$

$$(x^4)^{(n)} = 0$$

$$n \geq 5$$

EX: $\left[\sin(\cos(2x))\right]''$

$$\sin(\cos(2x)) \to$$

$$\to \cos(\underline{\cos(2x)}) \cdot (-\sin(\underline{2x})) \cdot 2$$

$$-2\left[\cos(\cos(2x))\right] \cdot \left[\sin(2x)\right]$$

$\downarrow$ take derivative again
product rule first

$$-2\Big\{\cos(\cos(2x)) \cdot [\sin(2x)]'$$
$$+ [\cos(\cos(2x))]' \cdot \sin(2x)\Big\}$$

$$= -2\Big\{\cos(\cos(2x)) \cdot (\cos(\underline{2x})) \cdot 2$$
$$+ \left[-\sin(\underline{\cos(2x)})(-\sin(\underline{2x}))\,2\right] \cdot \sin(2x)\Big\}$$

$$= -4\left\{\left[\cos(\cos(2x))\right]\cdot\left[\cos(2x)\right] + \left[\sin(\cos(2x))\right]\cdot\left[\sin^2(2x)\right]\right\}$$

EX: $\left[\sqrt{\sin(4x^2+1)}\right]''$

$$\left[\sin(4x^2+1)\right]^{1/2}$$

$$\rightarrow \frac{1}{2}\left[\sin(4x^2+1)\right]^{-1/2} \cdot \cos(4x^2+1) \cdot 8x$$

$$\underset{\text{simplify}}{=} \frac{4x\cos(4x^2+1)}{\left[\sin(4x^2+1)\right]^{1/2}}$$

For 2^{nd} derivative now use quotient rule (or leave as product)

Quotient rule: $\left(\frac{H}{L}\right)' = \frac{LH' - HL'}{L^2}$

$$\frac{[\sin(4x^2+1)]^{1/2} \cdot [4x \cdot \cos(4x^2+1)]' - 4x\cos(4x^2+1)[\sin(4x^2+1)]^{1/2}}{\sin(4x^2+1)}$$

$$\frac{[\sin(4x^2+1)]^{1/2}\cdot\left[4x\cdot(-\sin(4x^2+1)\cdot 8x) + 4\cos(4x^2+1)\right] - 4x\cos(4x^2+1)\left[\frac{1}{2}(\sin(4x^2+1))^{-1/2}(\cos(4x^2+1))\cdot 8x\right]}{\sim}$$

$$= \frac{4[\sin(4x^2+1)]^{1/2}\left[-8x^2\sin(4x^2+1)+\cos(4x^2+1)\right] - 16x^2\cos^2(4x^2+1)[\sin(4x^2+1)]^{-1/2}}{\sin(4x^2+1)}$$