2.5 Formulas

take the derivative

(1) $(x^n)' = nx^{n-1}$

Ex: $(x)' = \frac{d(x)}{dx} = \frac{dx}{dx} = 1$

obvious graphically (from their graphs)

$= (x^1)' = 1x^{1-1} = 1x^0 = 1$

Ex: $(1)' = (x^0)' = 0 \cdot x^{0-1} = 0$

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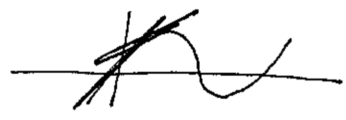
(2) $(\sin x)' = \cos x$



$y = \sin x$

(3) $(\cos x)' = -\sin x$

$y = \cos x$

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(4) $(cf)' = c(f)' = cf'$

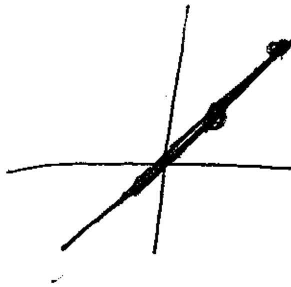
constant

(5) $(f+g)' = f' + g'$

(6) $(fg)' = fg' + f'g$

(7) quotient rule

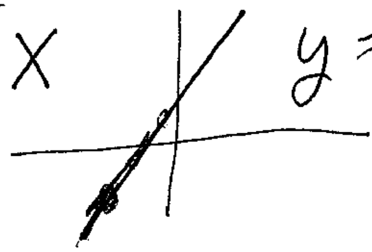
(8) chain rule



$y = x$

$(x)' =$ slope of tangent line $= \underline{1}$

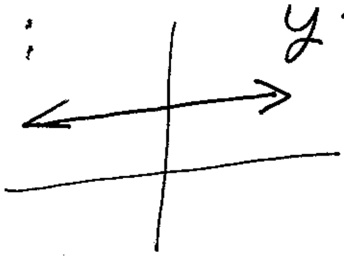
tangent line: $y = x$

Ex  $y = 2x+3$

$(2x+3)' =$ slope of tangent line $= 2$

tangent line: $y = 2x+3$

The best linear (ie line) approximation to the line $y = 2x+3$

Ex:  $y = 3$

slope $= (3)' = 0$

tangent line: $y = 3$

Ex: $(2x^3 - 4x^{-1} + 2)'$

$= 2\cdot(x^3)' - 4\cdot(x^{-1})' + (2)'$

$= 2\cdot(3x^2) - 4\cdot(-1x^{-2}) + 0$

$= \boxed{6x^2 + 4x^{-2}}$

Ex: $\left(\dfrac{3x^2 - \sqrt{x} - 1}{x}\right)'$

SIMPLIFY!!!

$= \left(3x - x^{-1/2} - x^{-1}\right)'$

$= \boxed{3 + \frac{1}{2}x^{-3/2} + x^{-2}}$

Ex: $\left[x^2\sqrt{x}\right]' = \left[x^2 \cdot x^{\frac{1}{2}}\right]'$

$= \left[x^{5/2}\right]' = \frac{5}{2}x^{3/2}$

Ex: $(5x + 2\sin x + 3\cos x)'$

$= 5 + 2\cos x - 3\sin x$

2.7 prodoct rule

$$(f\cdot g)'(x) = \lim_{h\to 0} \frac{(fg)(x+h) - (fg)(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= f(x)\,g'(x) + f'(x)\,g(x)$$

see book

In other notation

$$\frac{d}{dx}(fg) = f\frac{dg}{dx} + \frac{df}{dx}g$$

$$(fg)' = fg' + f'g$$

EX: $(2x \cdot \sin x)'$

$= (2x)(\sin x)' + (2x)'(\sin x)$

$= \boxed{2x\cos x + 2\sin x}$

EX $\left(\frac{\cos x}{\sqrt{x}}\right)' = (x^{-1/2} \cdot \cos x)'$

SIMPLIFY!!!!

$= (x^{-1/2})(\cos x)' + (x^{-1/2})'(\cos x)$

$= \boxed{-(x^{-1/2})\sin x + -\frac{1}{2}x^{-3/2}\cos x}$

$-x^{-1/2}\sin x - \frac{x^{-3/2}\cos x}{2}$

2.6

Slope of secant line between $(x_1, f(x_1))$ and $(x_2, f(x_2))$

$$= \text{average rate of change}$$

notation optional

$$= \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$= = \frac{\Delta f(x)}{\Delta x}$$

where Δx = change in $x = x_2 - x_1$

and $\Delta f(x)$ = change in $f(x) = f(x_2) - f(x_1)$

Slope of tangent line to f at x_1 = instantaneous rate of change

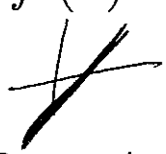
$$= lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= lim_{x_1+h \to x_1} \frac{f(x_1 + h) - f(x_1)}{x_1 + h - x_1}$$

$$= lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}$$

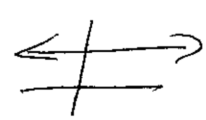
Definition $f'(a)$ = slope of tangent line to f at a

$$= lim_{h\to 0} \frac{f(a+h)-f(a)}{h}.$$

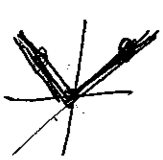


If $f(x) = 2x - 4$, then $f'(8) =$ 2

If $g(x) = 3$, then $g'(1) =$ 0



If $h(x) = |x|$, then $h'(5) =$ 1



and $h'(-5) =$ −1

$h'(0)$ DNE

Definition: Given f, then define the function f' (the derivative of f) as follows:

$f'(x)$ = slope of tangent line to f at $x = lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

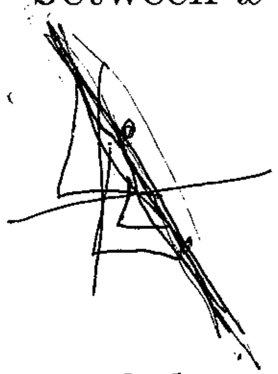
x is in the domain of f' if x is in the domain of f and the above limit exists.

If $f(x) = 2x - 4$, then $f'(x) =$ 2

If $g(x) = 3$, then $g'(x) =$ 0

If $h(x) = |x|$, then $h'(x) =$ $\begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$

Suppose $f(x) = -2x + 12$ represents the distance traveled from home in miles after x hours. Find the average velocity between $x = 1$ and $x = 3$. What are the units?



-2 mph | long method $\frac{f(3)-f(1)}{3-1}$ = etc

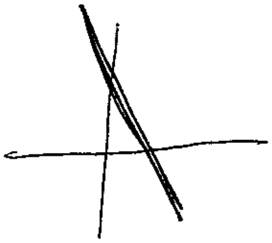
Find the instantaneous velocity at $x = 1$: What are the units?

-2 mph

Find the acceleration at $x = 1$: What are the units?

0 mph ~~miles/hr~~ $0\ \frac{\text{miles}}{\text{hr}^2}$

Suppose $f(x) = -2x + 12$ represents the population of a household x years after 2000. Find the average change in the population between 2001 and 2003 (i.e., $x = 1$ and $x = 3$). What are the units?

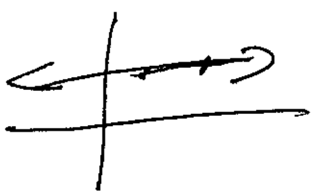


-2 people / year

Find the instantaneous change in the population at $x = 1$: What are the units?

-2 people / yr

Suppose $f(x) = 8$ represents the distance traveled from home in miles after x hours. Find the average velocity between $x = 1$ and $x = 3$. What are the units?



0 mph

Find the instantaneous velocity at $x = 1$: What are the units?

0 mph

Find the acceleration at $x = 1$: What are the units?

0 miles / hr^2

Suppose $f(x) = 8$ represents the population of a household x years after 2000. Find the average change in the population between 2001 and 2003 (i.e., $x = 1$ and $x = 3$). What are the units?

0 people/yr

Find the instantaneous change in the population at $x = 1$: What are the units?

0 people/yr

Suppose $f(x) = \frac{x+3}{4x+1} + 2$ represents the distance traveled from home in miles after x hours. Find the average velocity between $x = 1$ and $x = 3$. What are the units?

$$\frac{f(3) - f(1)}{3 - 1} =$$

Find the instantaneous velocity at $x = 1$: What are the units?

$$f'(x) = \frac{-11}{(4x+2)^2}$$

from previous

$$f'(1) = \frac{-11}{36} \text{ mph}$$