Tuesday at 7:30 am
early morning

in MH AUD
Macbride Hall

Problem Session
MONDAY 10:30 - 12 in 105 MLH
O.H : 1 - 5pm in B1H MLH

Things that will definitely appear on final exam:

log-log plots (but no semi-log plots) -- see log-log problems

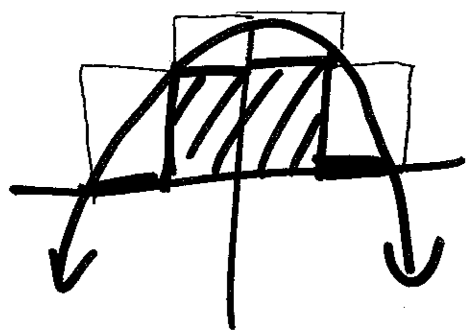
Ch 8: direction fields

Review slope fields (see 8.1 supplemental HW,
http://people.duke.edu/~kfr/Scans/CalcLesson2-4.pdf,
http://people.duke.edu/~kfr/Scans/CalcLesson3-2.pdf)
Review 8.1, 8.3, 8.4 HW
Review TF (including multiple choice slope fields problems)
Also See today's class notes

Note: For sections 8.2, 8.5 you only need to know/understand TF problems.

True/False questions Partial Set 1 , Answers to Set 1
True/False questions Partial Set 2 , Answers to Set 2

Generic Review (i.e. some of the following will appear on your exam and some will not.)



Ch 5:

Fully understand integration:

1.) Definition
Be able to approximate the integral using inscribed or circumscribed rectangles – see class notes, HW problems 5.2: 1- 2 or better examples here $\int_{-4}^{4}(16-x^2)\,dx$ using 4 rectangles
plus answers
2.) Can be used to find actual area, net area, volume - see HW in sections 5.2, 5.3, 5.8, exam 2, quizzes, and class notes.
Also see 5.9: Improper integral -- See class notes and 5.9 HW.

Be able to calculate integrals
-- integration by substitution -- 5.5 HW
-- integration by formula -- 5.7 HW
-- you do not need to know integration by parts

Not on final exam: section 5.6

Ch 4:

Understand exponential decay/growth. Compare 8.4 to 4.3 and 4.4
Know log rules
Log-log plots
Also see below

Not on final exam: semi-log plots

Ch 3:

Fully understand how the derivative (first and second) applies to graphing

3.5: Optimization – Very important application
--Understand relative vs absolute max/min
--Understand Extreme Value Theorem
See 3.5 HW as well as min/max problems in other sections including Ch 4

3.6: Understand that the tangent line to y = f(x) at the point (a, f(a)) is a good approximation to the function y = f(x).
That is if the tangent line to y = f(x) at the point (a, f(a)) is the function y = mx + b, then $\boxed{f(x) \sim mx + b}$ for x close to a. Thus

1.) You can use the tangent line to approximate the function y = f(x). See for example HW problems $\boxed{3.6: 11 – 20.}$ Make sure that you

FULLY understand these problems. $f(x) \sim mx + b$

2.) You can also use the tangent line to approximate solutions to the equation f(x) = 0. By doing multiple rounds of Newtons method, you can get a very good approximation. However, for the final exam you would need to do at most one round. See lecture notes from 12/5 (a) find tangent line (y = mx + b) ~~at appropriate point (b)~~ since f(x) ~ mx + b, instead of solving $\boxed{f(x) = 0}$, solve $\boxed{mx + b = 0}$

Note you only use Newton's method IF asked to solve f(x) = 0

FYI (i.e. not on exam): in real applications, to solve f(x) = k, instead solve f(x) – k = 0.

3.7: Implicit differentiation and related rates – Very important application. See HW, class notes, and this week's double quiz.

Ch 2:

Fully understand the derivative
--slope of tangent line
--instantaneous rate of change vs average rate of change
--limit definition

Be able to calculate the derivative. Practice problems from ch 2, 3 and 4 as well as exams.

Understand and be able to calculate limits: see 2.1, 2.2 and ch 4

Ch 1:

Pre-calculus is assumed. Know sin and cosine values.

When to use log-log paper:

Suppose you suspect your data points satisfy polynomial growth of the form $y = At^m$ for some constants A and m.

$y = At^m$
$log(y) = log(At^m)$
$log(y) = log(A) + m(log(t))$ Let $z = log(y)$ and $x = log(t)$. Then
$z = log(A) + mx$.

$$\boxed{y = 10^b t^m}$$

$z = mx + log(A)$. That is we have the equation of a line where slope $= m$ and z-intercept $= log(A)$.

If $z = mx + b$, then $log(A) = b$. Hence $A = 10^{log(A)} = 10^b$. $y = At^m$
10 10

Hence to determine the constants A and m in $y = At^m$, graph (t, y) on log-log paper ~~(note this is the same~~ as taking $z = log(y)$ and $x = log(t)$), and determine equation of best fit line, $\boxed{z = mx + b. \text{ Then } y = 10^b t^m.}$

However if the data points do not satisfy a best fit line, then the data points do NOT satisfy polynomial growth of the form $y = At^m$

When to use semi-log paper:

Suppose you suspect your data points satisfy exponential growth of the form $y = Ac^t$ for some constants A and c.

$y = Ac^t$
$log(y) = log(Ac^t)$
$log(y) = log(A) + tlog(c)$. Let $z = log(y)$. Then
$z = log(A) + tlog(c)$.

$$y = Ac^t$$

$z = [log(c)]t + log(A)$. I.e. we have the equation of a line where slope $= log(c)$ and z-intercept $= log(A)$.

If $z = mt + b$, then (i) $log(A) = b$. Hence $A = 10^{log(A)} = 10^b$. (ii) $log(c) = m$. Hence $c = 10^m$.

Hence to determine the constants A and c in $y = Ac^t$, graph (t, y) on semi-log paper (note this is the same as taking $z = log(y)$), and determine equation of best fit line, $z = mx + b$. Then $y = 10^b(10^m)^t$. I.e., $y = 10^b(10^{mt})$

However if the data points do not satisfy a best fit line, then the data points do NOT satisfy polynomial growth of the form $y = Ac^t$

Semi-log and log-log plots problems (not HW, but highly recommended).

For each of the data sets below, graph these points on either semi-log or log-log paper and determine the function which best models these data points from the choices below.

1.) (1, 10), (8, 40), (32, 100), (8000, 4000)

~~2.) (1, 10000), (2, 3900), (6, 620), (7, 290)~~

3.) (1, 10000), (5, 400), (15, 50), (73, 2)

~~4.) (0, 1), (1.4, 3), (4.4, 30), (8, 480)~~

~~5.) (0, 100), (2, 45), (3.2, 7), (4, 2)~~

6.) (1, 1), (60, 4), (200, 6), (3200, 15), (8000, 20)

7.) (1, 1000), (5, 200), (20, 50), (515, 2)

~~8.) (0, 10), (0.6, 40), (1.8, 605), (2, 1000)~~

9.) (1, 100), (35, 600), (400, 2000), (8100, 9000)

polynomial

exponential

A) $y = 0$ **B)** $y = t^{\frac{1}{3}}$ **C)** $y = t^{\frac{1}{2}}$ **D)** $y = t^{\frac{2}{3}}$ **E)** $y = 10t$ **F)** $y = t^{\frac{3}{2}}$ **G)** $y = t^2$

H) $y = 1$ **I)** $y = t^{-\frac{1}{3}}$ **J)** $y = t^{-\frac{1}{2}}$ **K)** $y = t^{-\frac{2}{3}}$ **L)** $y = t^{-1}$ **M)** $y = t^{-\frac{3}{2}}$ **N)** $y = t^{-2}$

0) $y = 10$ **P)** $y = 10t^{\frac{1}{3}}$ **Q)** $y = 10t^{\frac{1}{2}}$ **R)** $y = 10t^{\frac{2}{3}}$ **S)** $y = 10t$ **T)** $y = 10t^{\frac{3}{2}}$ **U)** $y = 10t^2$

V) $y = 10t^{-\frac{1}{3}}$ **W)** $y = 10t^{-\frac{1}{2}}$ **X)** $y = 10t^{-\frac{2}{3}}$ **Y)** $y = 10t^{-1}$ **Z)** $y = 10t^{-\frac{3}{2}}$ **ZZ)** $y = 10t^{-2}$

a) $y = 100$ **b)** $y = 100t^{\frac{1}{3}}$ **c)** $y = 100t^{\frac{1}{2}}$ **d)** $y = 100t^{\frac{2}{3}}$ **e)** $y = 100t$ **f)** $y = 100t^{\frac{3}{2}}$ **g)** $y = 100t^2$

h) $y = 100t^{-\frac{1}{3}}$ **i)** $y = 100t^{-\frac{1}{2}}$ **j)** $y = 100t^{-\frac{2}{3}}$ **k)** $y = 100t^{-1}$ **l)** $y = 100t^{-\frac{3}{2}}$ **m)** $y = 100t^{-2}$

n) $y = 1000t^{\frac{1}{3}}$ **o)** $y = 1000t^{\frac{1}{2}}$ **p)** $y = 1000t^{\frac{2}{3}}$ **q)** $y = 1000t$ **r)** $y = 1000t^{\frac{3}{2}}$ **s)** $y = 1000t^2$

t) $y = 1000t^{-\frac{1}{3}}$ **u)** $y = 1000t^{-\frac{1}{2}}$ **v)** $y = 1000t^{-\frac{2}{3}}$ **x)** $y = 1000t^{-1}$ **y)** $y = 1000t^{-\frac{3}{2}}$ **z)** $y = 1000t^{-2}$

B) $y = 10^{\frac{t}{3}}$ **C)** $y = 10^{\frac{t}{2}}$ **D)** $y = 10^{\frac{2t}{3}}$ **E)** $y = 10(10^t$ **F)** $y = 10^{\frac{3t}{2}}$ **G)** $y = 10^{2t}$

I) $y = 10^{-\frac{t}{3}}$ **J)** $y = 10^{-\frac{t}{2}}$ **K)** $y = 10^{-\frac{2t}{3}}$ **L)** $y = 10^{-t}$ **M)** $y = 10^{-\frac{3t}{2}}$ **N)** $y = 10^{-2t}$

P) $y = 10(10^{\frac{t}{3}})$ **Q)** $y = 10(10^{\frac{t}{2}})$ **R)** $y = 10(10^{\frac{2t}{3}})$ **S)** $y = 10(10^t)$ **T)** $y = 10(10^{\frac{3t}{2}})$ **U)** $y = 10(10^{2t})$

W) $y = 10(10^{-\frac{t}{2}})$ **X)** $y = 10(10^{-\frac{2t}{3}})$ **Y)** $y = 10(10^{-t})$ **Z)** $y = 10(10^{-\frac{3t}{2}})$ **ZZ)** $y = 10(10^{-2t})$

c) $y = 100(10^{\frac{t}{2}})$ **d)** $y = 100(10^{\frac{2t}{3}})$ **e)** $y = 100(10^t)$ **f)** $y = 100(10^{\frac{3t}{2}})$ **g)** $y = 100(10^{2t})$

i) $y = 100(10^{-\frac{t}{2}})$ **j)** $y = 100(10^{-\frac{2t}{3}})$ **k)** $y = 100(10^{-t})$ **l)** $y = 100(10^{-\frac{3t}{2}})$ **m)** $y = 100(10^{-2t})$

o) $y = 1000(10^{\frac{t}{2}})$ **p)** $y = 1000(10^{\frac{2t}{3}})$ **q)** $y = 1000(10^t)$ **r)** $y = 1000(10^{\frac{3t}{2}})$ **s)** $y = 1000(10^{2t})$

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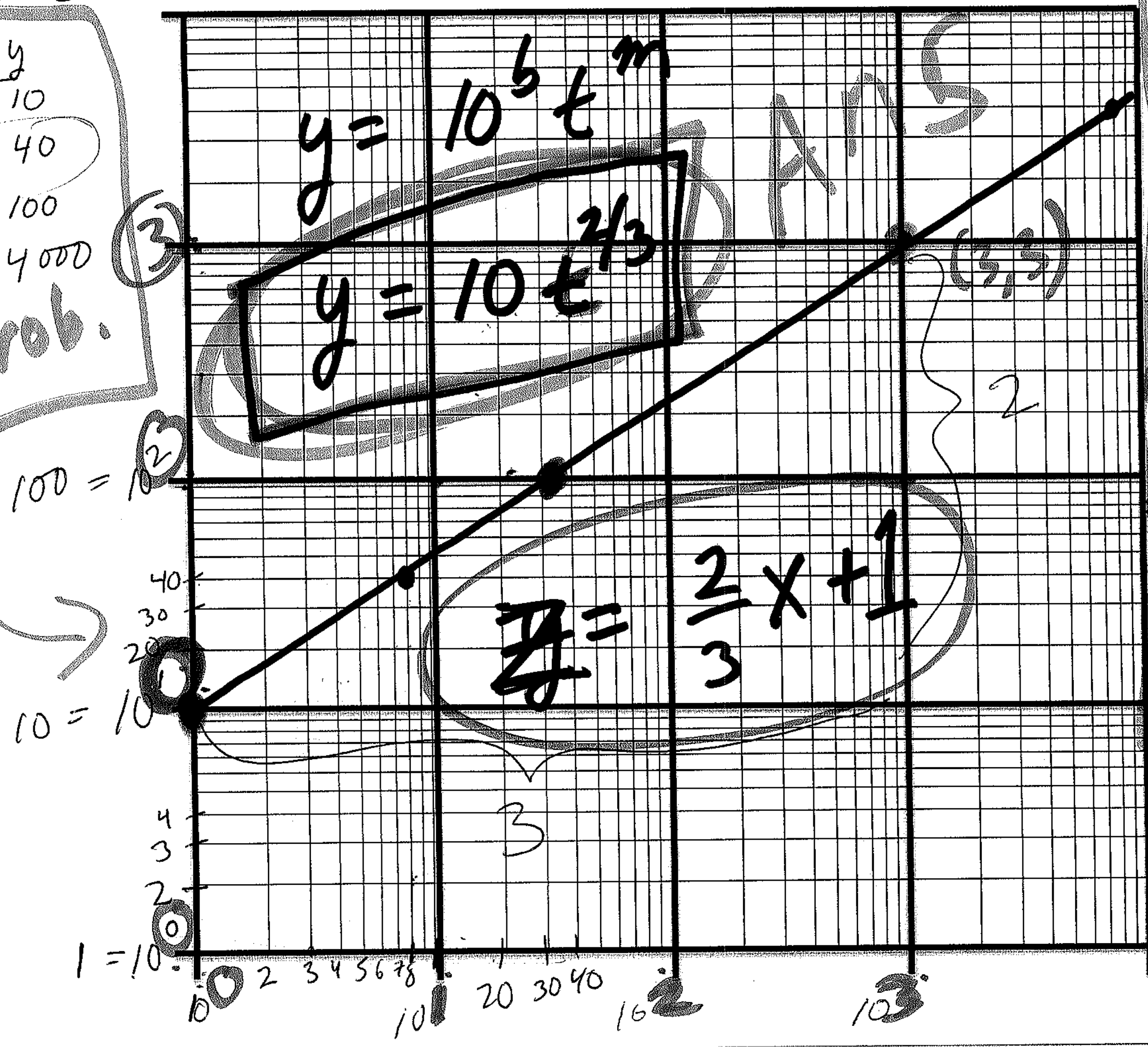
$y = 10^b t^m$

t	y
1	10
8	40
32	100
8000	4000

prob.



$z = mx + b$

x	z
$\log 1 \approx 0$	$\log 10 \approx 1$
$\log 8 \approx 0.9$	$\log 40 \approx 1.6$
$\log 32 \approx 1.5$	$\log 100 = 2$
$\log 8000 \approx 3.9$	$\log 4000 \approx 3.6$

FYI

8.1 supplemental HW $\int \frac{1}{t}\,dt = \ln|t| + C$

1.) Which of the following could be the general solution to the differential equation whose directi field is given below:

lines

A) $y = t + C$ B) $y = 2t + C$

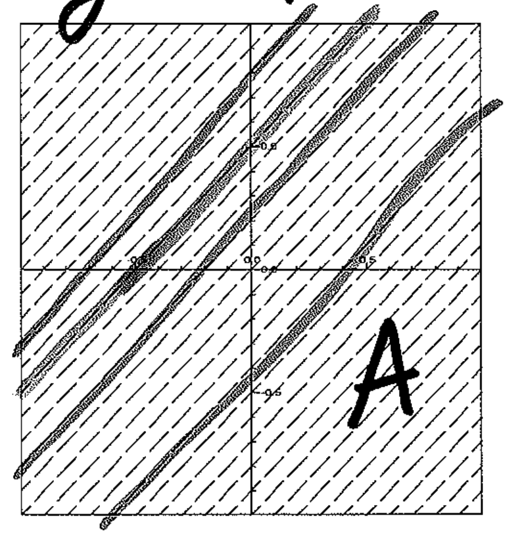
C) $y = \frac{1}{2}t + C$ D) $y = -\frac{1}{2}t + C$

E) $y = -t + C$ F) $y = -2t + C$

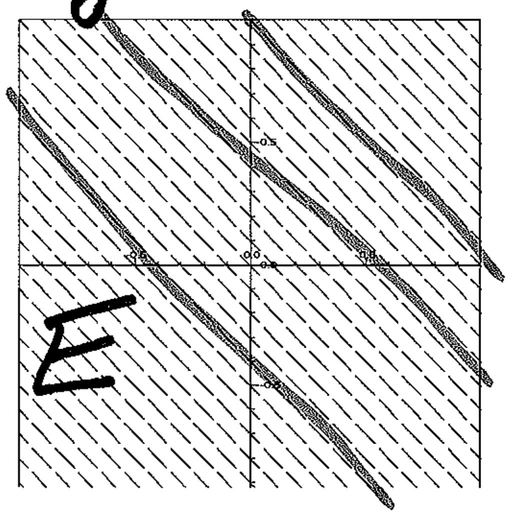
G) $y = ln|t| + C$ H) $\cancel{y = C}$

$\boxed{\text{I) } y = \frac{Ct^3}{3} \quad (0,0) \text{ lines on all solns}}$ J) $y = \frac{t^3}{3} + C$

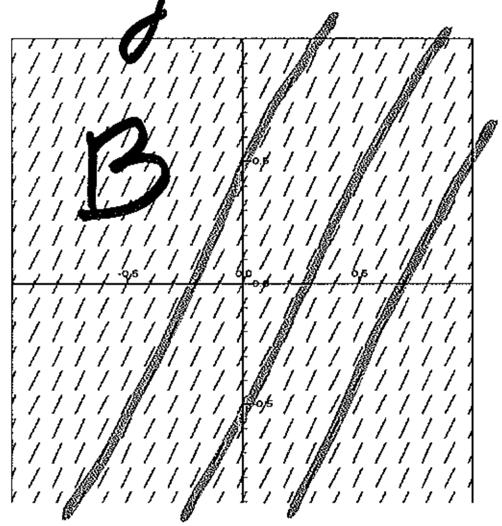
$y' = 1$



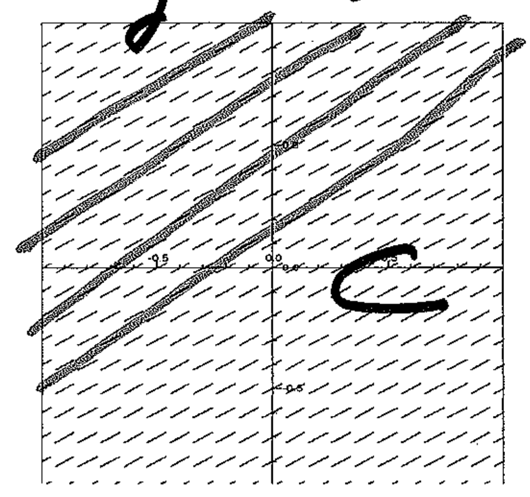
$y' = -1$



$y' = 2$



$y' = \frac{1}{2}$



8.1 supplemental HW

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D) $y = -\frac{1}{2}t + C$

E) $y = -t + C$

F) $y = -2t + C$

G) $y = ln|t| + C$

H) $y = C$

I) $y = \frac{Ct^3}{3}$

J) $y = \frac{t^3}{3} + C$

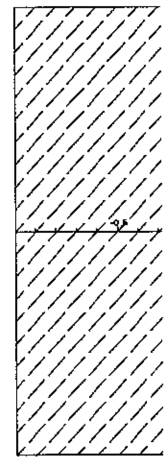
K) $y = e^t + C$

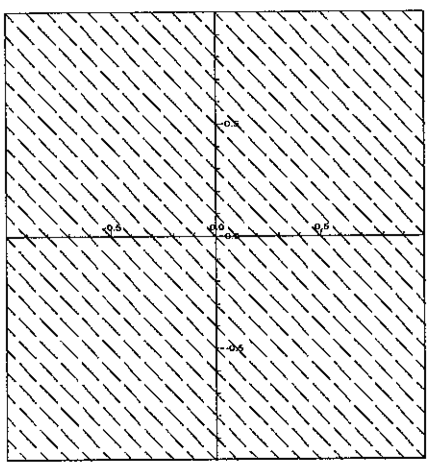
e^{-t}

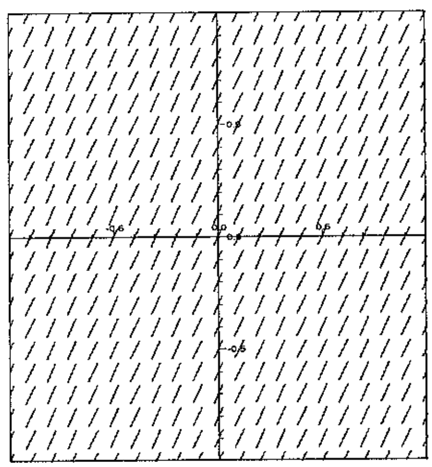
t^2

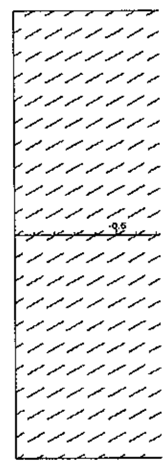
sin t

cos t









2.) Circle the differential equation whose direction field is given below:

A) $y' = t^2$

B) $y' = \frac{1}{2}$

C) $y' = 1$

D) $y' = -1$

E) $y' = y + 1$

F) $y' = y - 2 \Rightarrow y = 2$

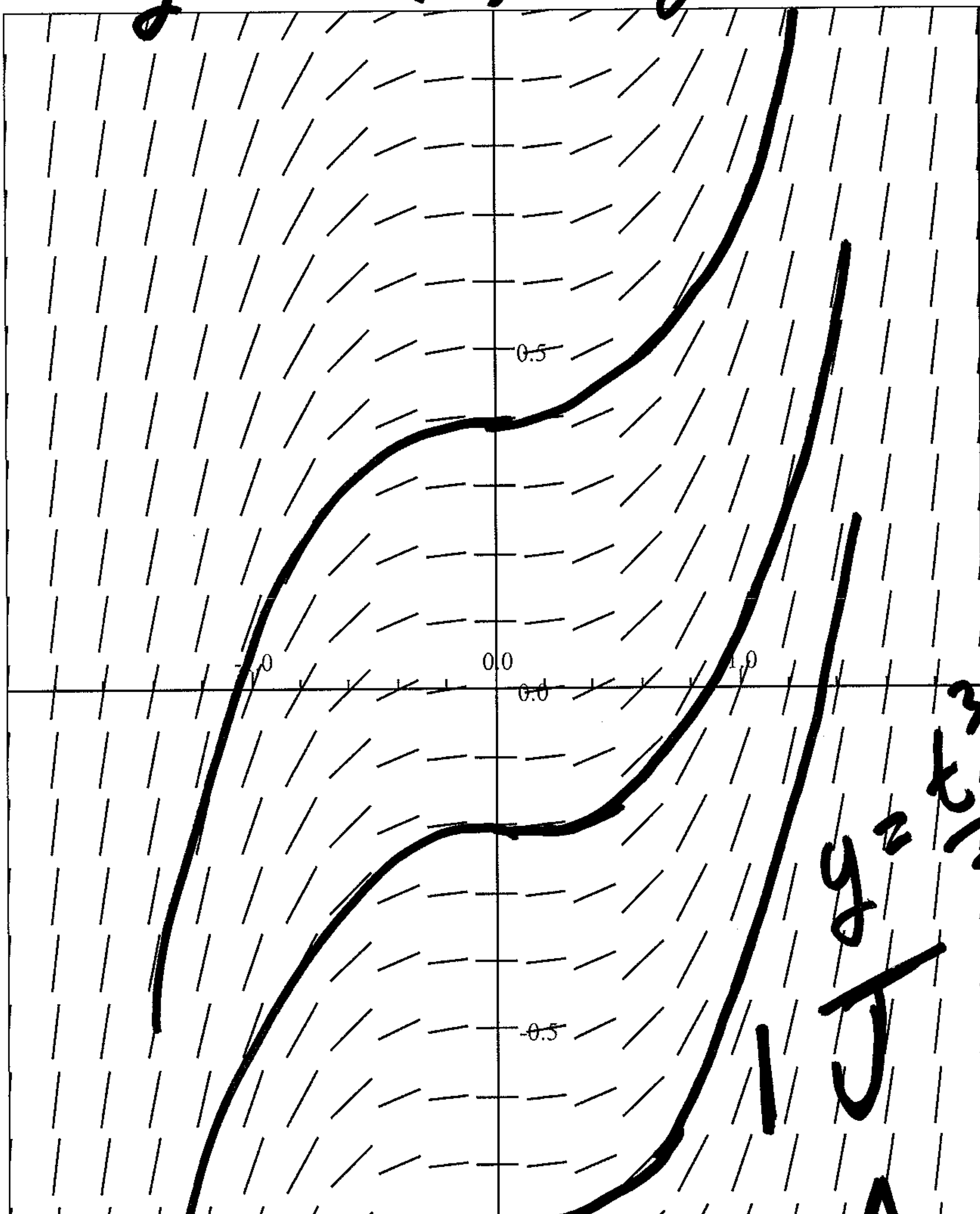
G) $y' = (y+1)(y-2)$

H) $y' = (y+1)^2(y-2)^2$

I) $y' = (y+1)(y-2)^2$

J) $y' = (y+1)^2(y-2)$

$y' = f(t) \Rightarrow y = F(t) + C$



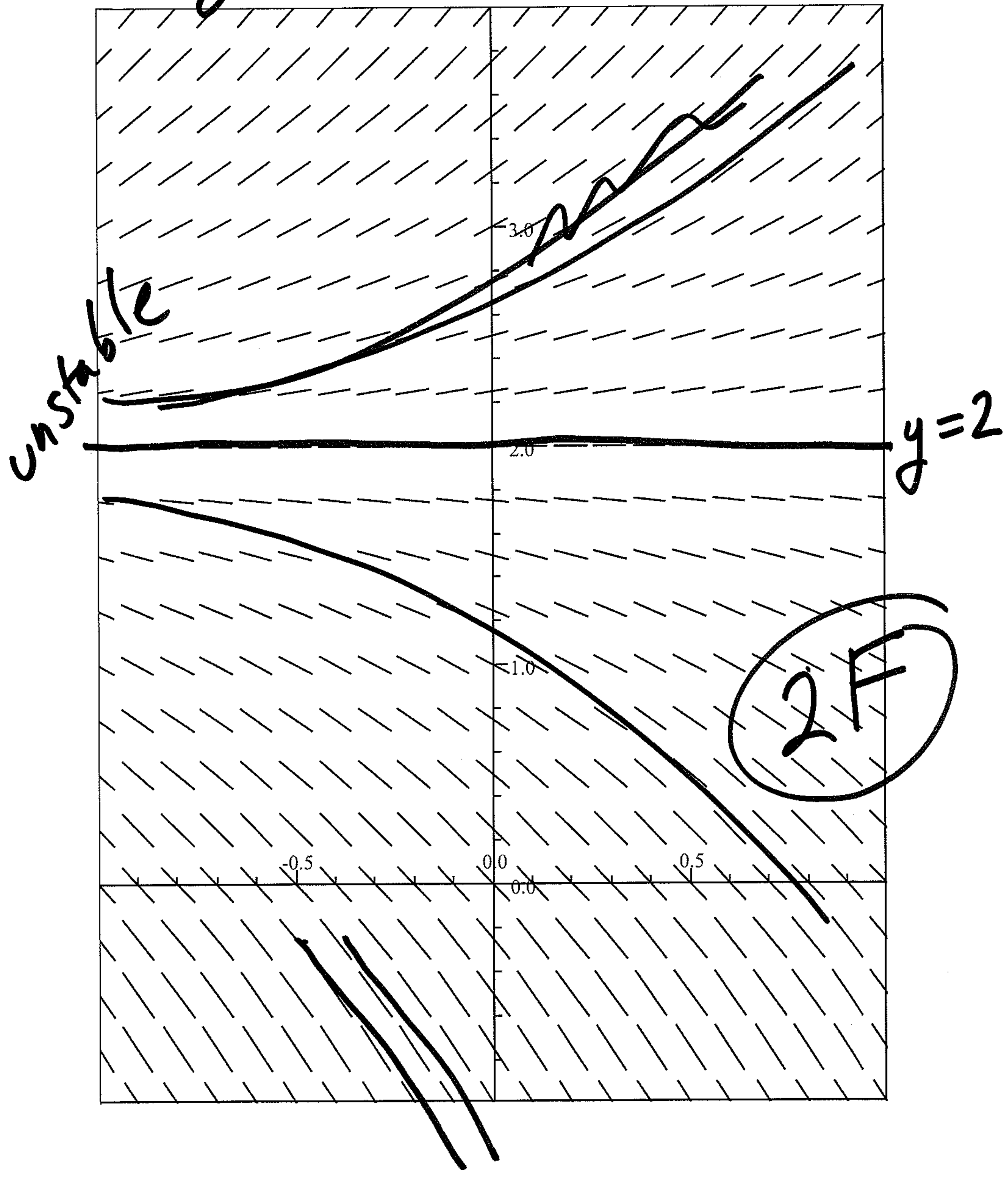
$y = \frac{t^3}{3} + C$

$1\ J$

$2\ A$

$y' = t^2$

8.3 $y' = f(y)$



8.1 supplemental HW

1.) Which of the following could be the general solution to the differential equation whose direction field is given below:

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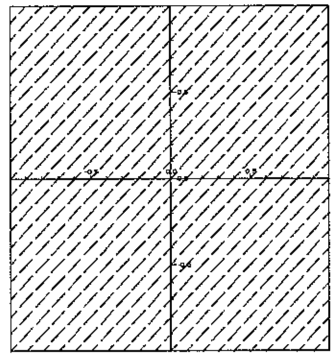
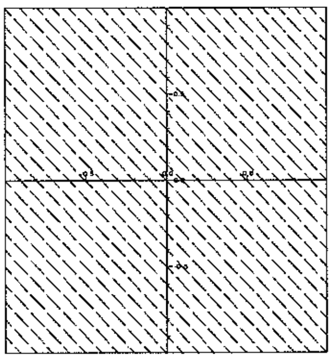
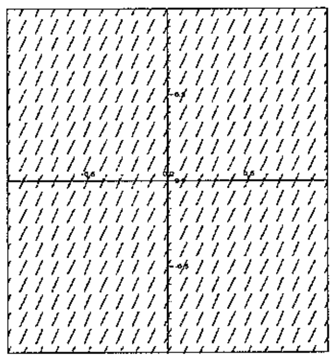
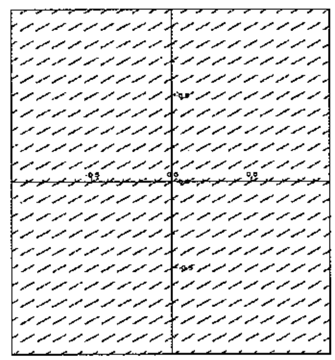
I) $y = \frac{Ct^3}{3}$ J) $y = \frac{t^3}{3} + C$









2.) Circle the differential equation whose direction field is given below:

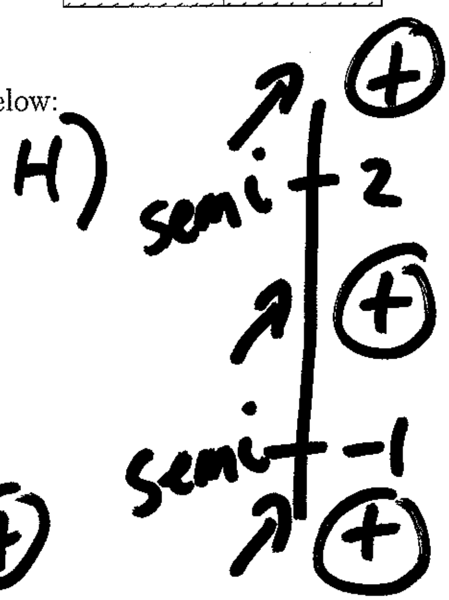
A) $y' = t^2$ B) $y' = \frac{1}{2}$

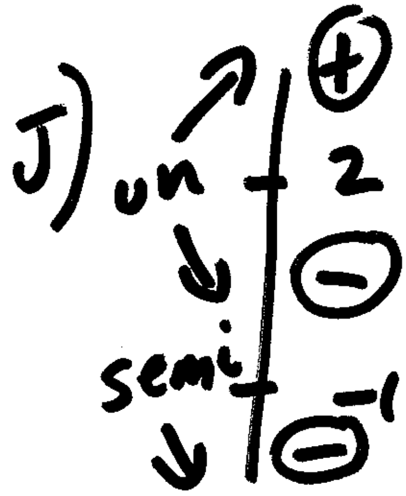
C) $y' = 1$ D) $y' = -1$

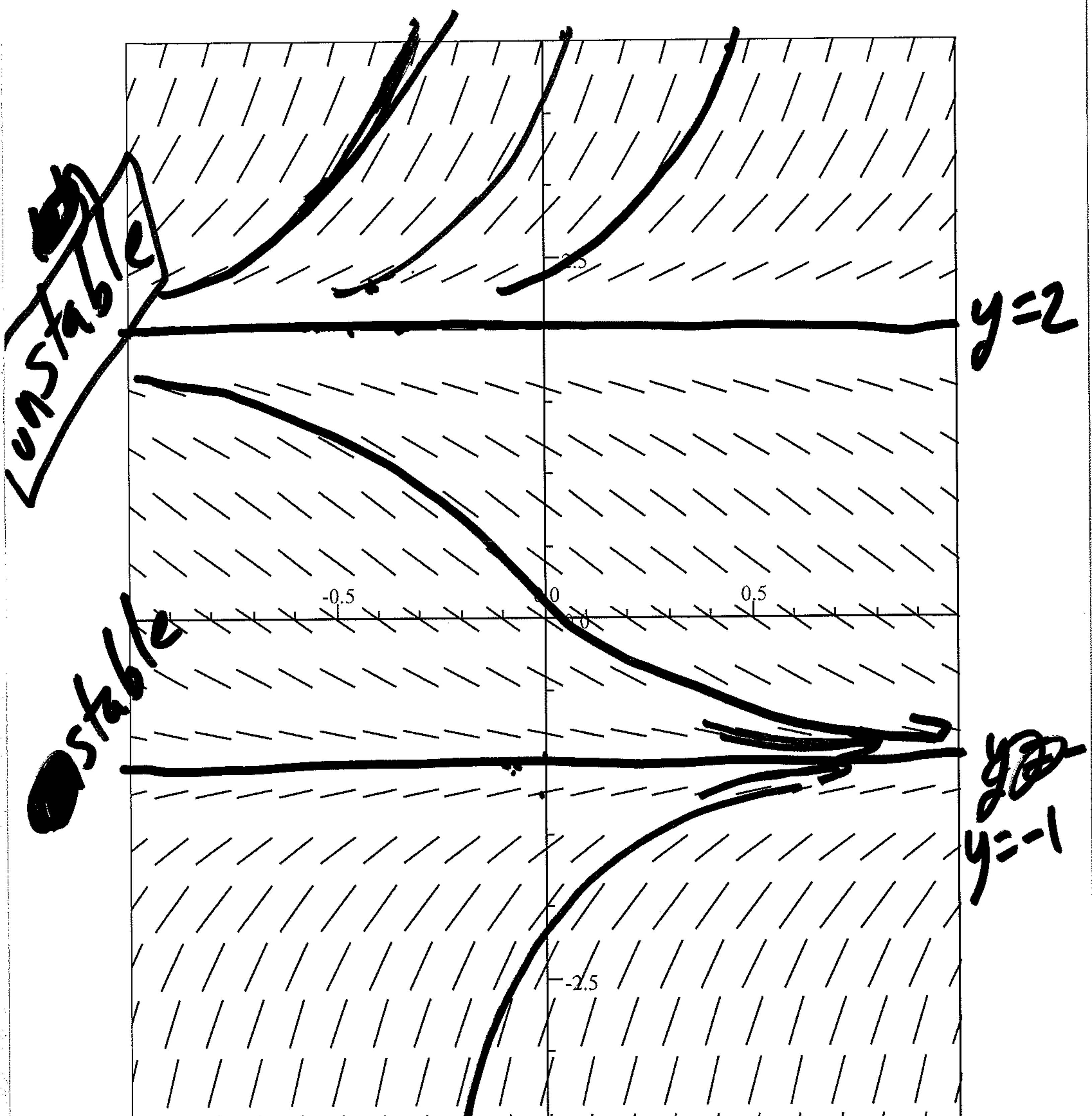
E) $y' = y + 1$ F) $y' = y - 2$

G) $y' = (y+1)(y-2)$ H) $y' = (y+1)^2(y-2)^2$

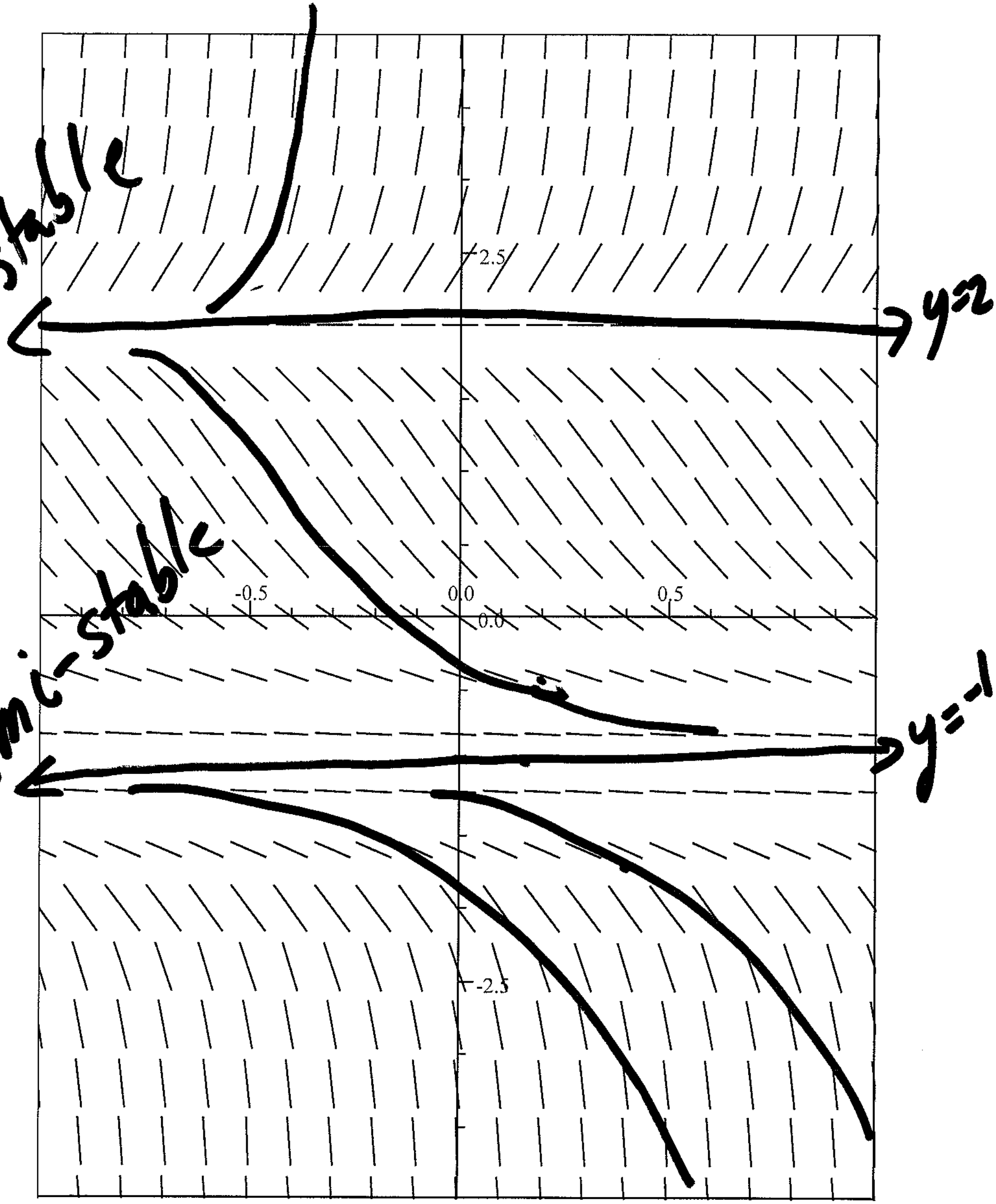
I) $y' = (y+1)(y-2)^2$ J) $y' = (y+1)^2(y-2)$

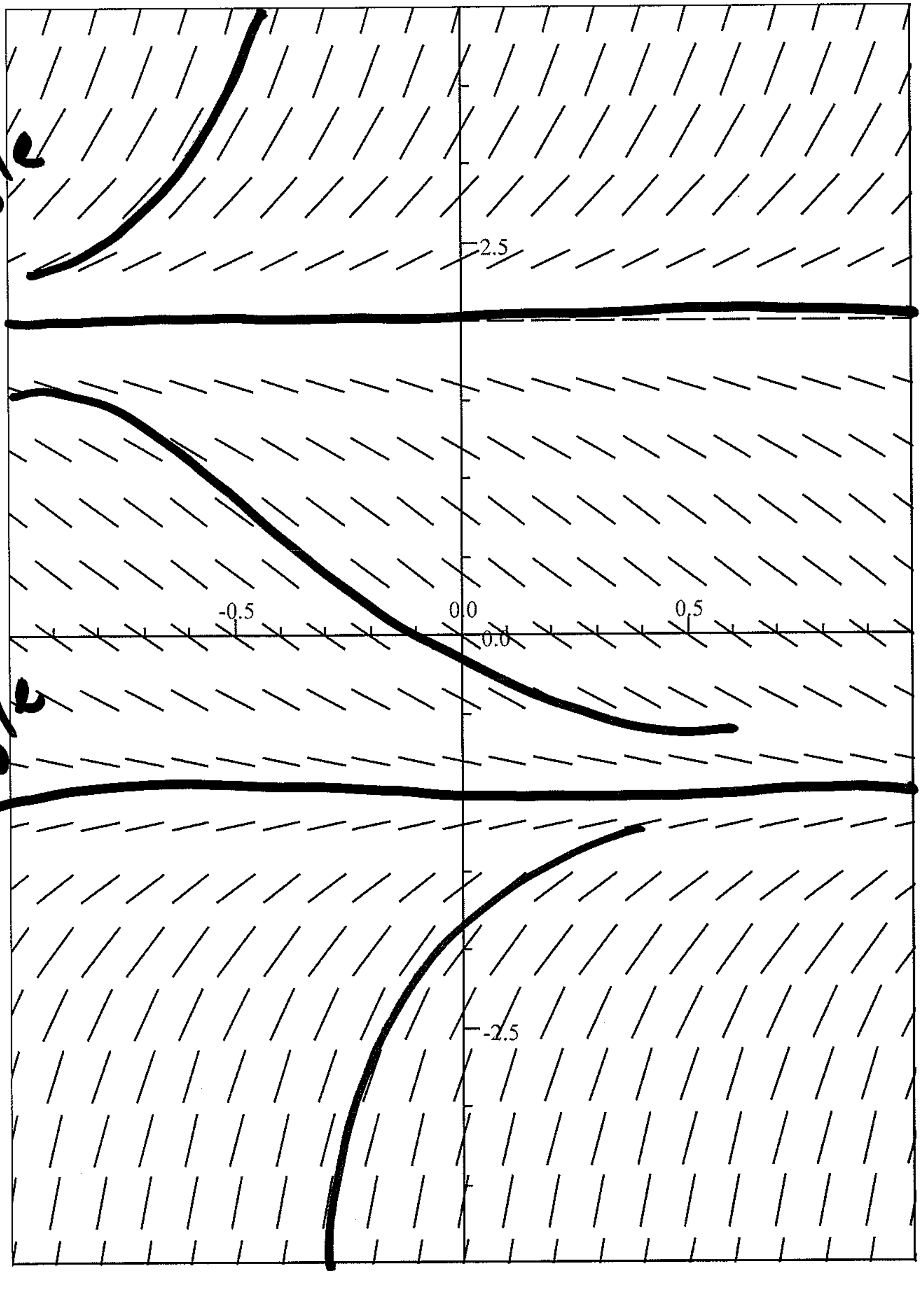






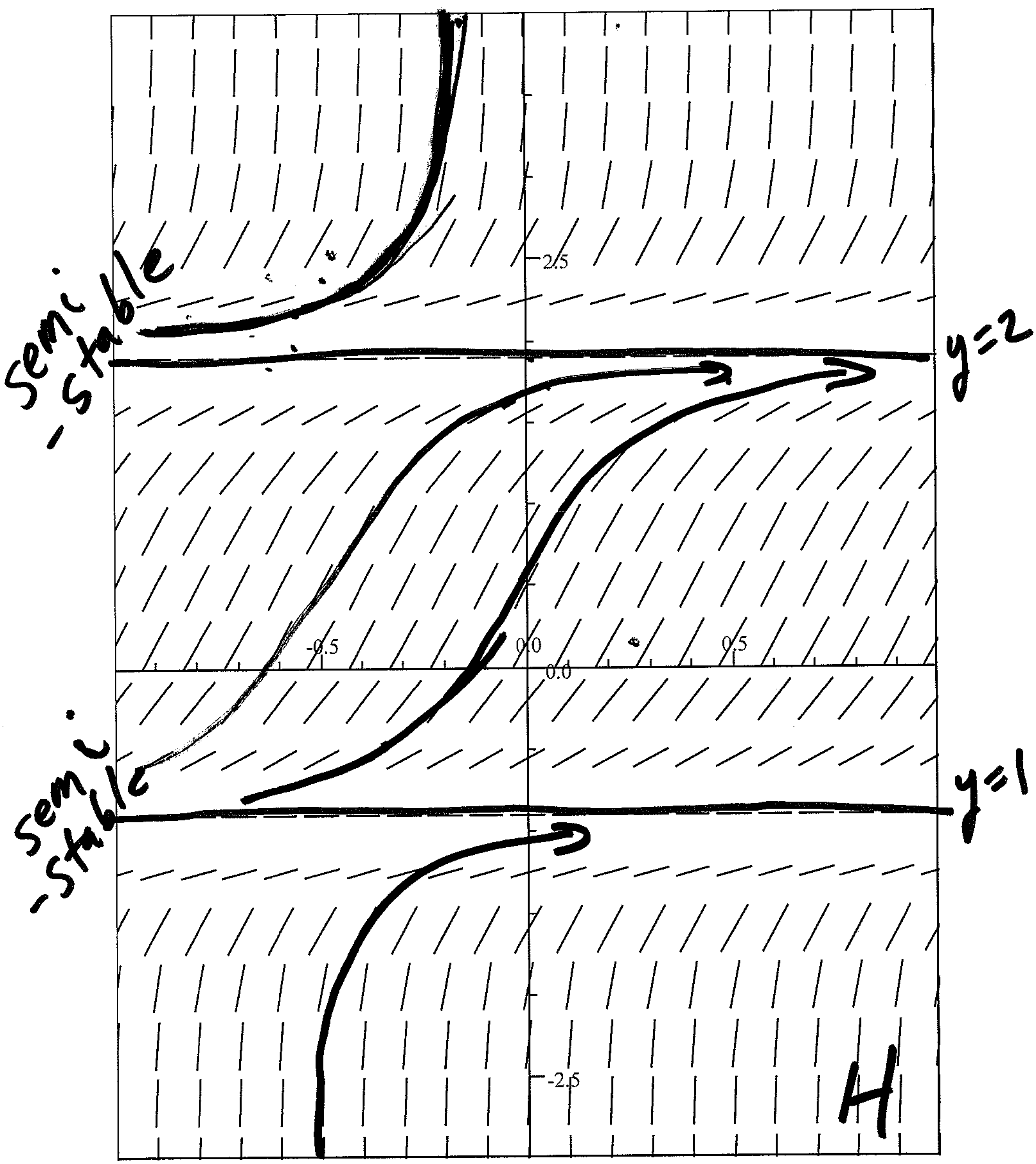
unstable
stable
y=2
y=-1
-0.5
0
0.5
2.5
-2.5


2.5
-0.5
0.0
0.5
0.0
-2.5
unstable
y=2
semi-stable
y=-1



unstable

stable


2.5
-0.5
0.0
0.5
0.0
-2.5
semi-stable
y=2
semi-stable
y=1
H

Things that will definitely appear on final exam:

log-log plots (but no semi-log plots) -- see log-log problems

Ch 8:

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Note: For sections 8.2, 8.5 you only need to know/understand TF problems.

True/False questions Partial Set 1 , Answers to Set 1
True/False questions Partial Set 2 , Answers to Set 2

Generic Review (i.e. some of the following will appear on your exam and some will not.)

Ch 5:

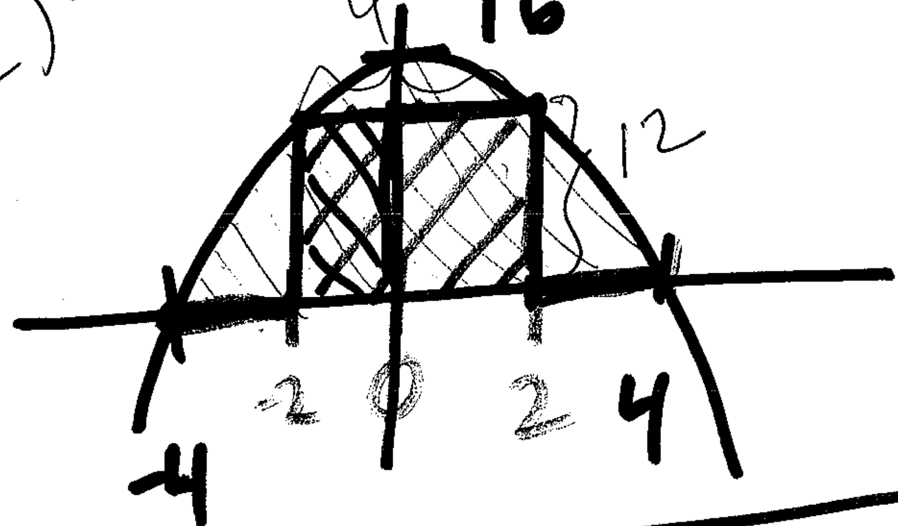
Fully understand integration:

1.) Definition
Be able to approximate the integral using inscribed or circumscribed rectangles – see class notes, HW problems 5.2: 1- 2 or better example here $\int_{-4}^{4} (16 - x^2)\,dx$ using 4 rectangles
plus answers
2.) Can be used to find actual area, net area, volume - see HW in sections 5.2, 5.3, 5.8, exam 2, quizzes, and class notes.
Also see 5.9: Improper integral -- See class notes and 5.9 HW.

Estimate $\int_{-4}^{4}(16-x^2)\,dx$

using 4 inscribed rectangles of equal width

$(12)(4)=48$



Long answer:

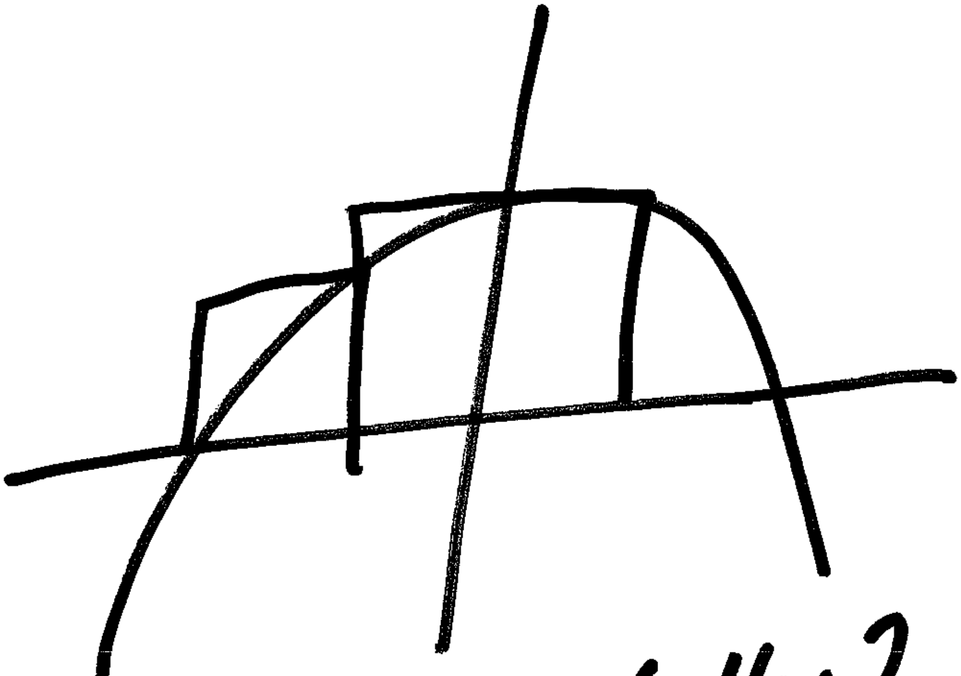
$$0\cdot 2+\underbrace{(16-(-2)^2)}_{\text{height}}\cdot\underbrace{2}_{\text{width}}+(16-(2)^2)\cdot 2+0\cdot 2$$

$= 48 \leftarrow$ under-est.

$$\frac{4-(-4)}{4}=\frac{8}{4}=2$$

$=$ width $=\Delta x$

circum



$$2(16\cdot 2 + 12(2))$$
$$= 2(32+24)$$
$$= 112$$