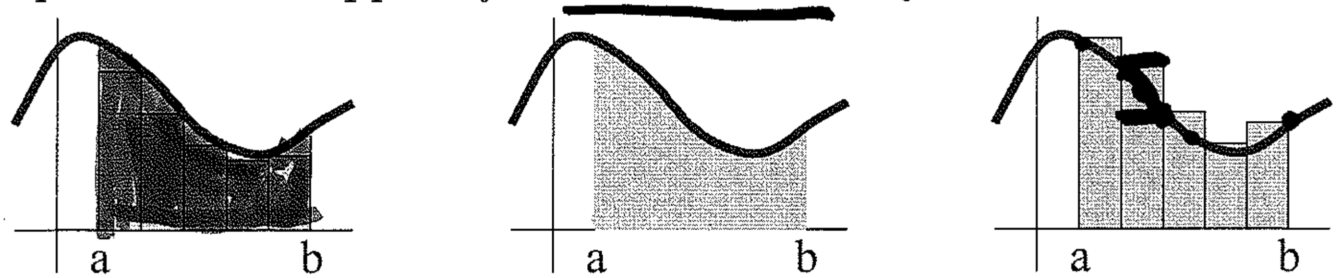
Find the area between $y = 0, \quad y = f(x), \quad x = a, \quad x = b.$
Special case: Suppose f is continuous and $f > 0$.



$$\begin{array}{c}\text{area of } n\\ \text{inscribed}\\ \text{rectangles}\end{array} \leq \text{actual area} \leq \begin{array}{c}\text{area of } n\\ \text{circumscribed}\\ \text{rectangles}\end{array}$$

$$\lim_{n\to\infty}\left(\begin{array}{c}\text{area of } n\\ \text{inscribed}\\ \text{rectangles}\end{array}\right) \leq \text{actual area} \leq \lim_{n\to\infty}\left(\begin{array}{c}\text{area of } n\\ \text{circumscribed}\\ \text{rectangles}\end{array}\right)$$

Theorem: f cont

$$L = \lim_{n\to\infty}\left(\begin{array}{c}\text{area of } n\\ \text{inscribed}\\ \text{rectangles}\end{array}\right) = \lim_{n\to\infty}\left(\begin{array}{c}\text{area of } n\\ \text{circumscribed}\\ \text{rectangles}\end{array}\right) = U.$$

$$\begin{array}{c}\text{area of } n\\ \text{inscribed}\\ \text{rectangles}\end{array} \leq \Sigma_{i=1}^{n} f(x_i)\Delta x \leq \begin{array}{c}\text{area of } n\\ \text{circumscribed}\\ \text{rectangles}\end{array}$$

where x_i could be right end-point, left end-point, mid-point, or etc.

$$\lim_{n\to\infty}\left(\begin{array}{c}\text{area of } n\\ \text{inscribed}\\ \text{rectangles}\end{array}\right) \leq \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x \leq \lim_{n\to\infty}\left(\begin{array}{c}\text{area of } n\\ \text{circumscribed}\\ \text{rectangles}\end{array}\right)$$

Theorem: If f continuous, $f > 0$, actual area $= \displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$

count area is negative below x-axis

Cor: If f is continuous, $\int_a^b f(x)dx =$ NET area $= \displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$

NET area can be negative

What you need to Know

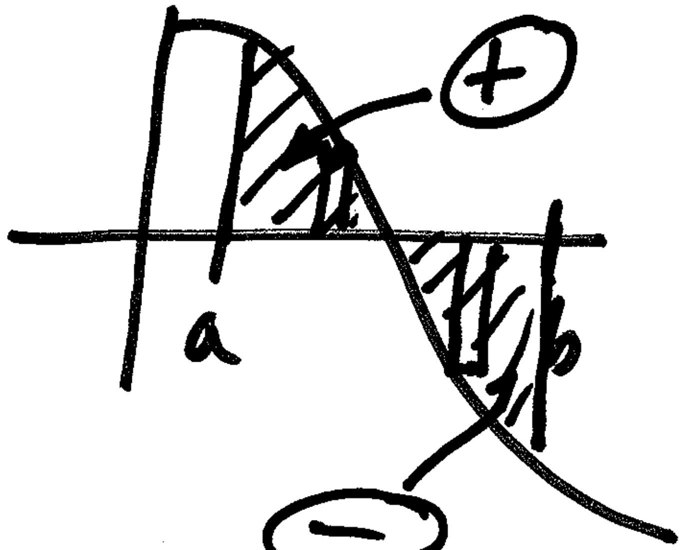
- Estimate area using shapes such as rectangles
  - underestimate via inscribed rectangles
  - over-estmate via circumscribed rectangles
  - all right-handpts

- limit defn for finding net area btwn x-axis

$y = f(x)$

$x = a$

$x = b$



$y = f(x)$

$$\lim_{n \to \infty} \underbrace{\sum_{i=1}^{n} \underbrace{\underbrace{f(x_i)}_{\text{height}} \underbrace{\Delta x}_{\text{width}}}_{\text{area}}}_{\text{add up all } n \text{ areas}}$$

$$= \text{net area} = \int_a^b f(x)\,dx$$

Understand how distance & area are related.

If constant velocity v_0 distance traveled in time Δt

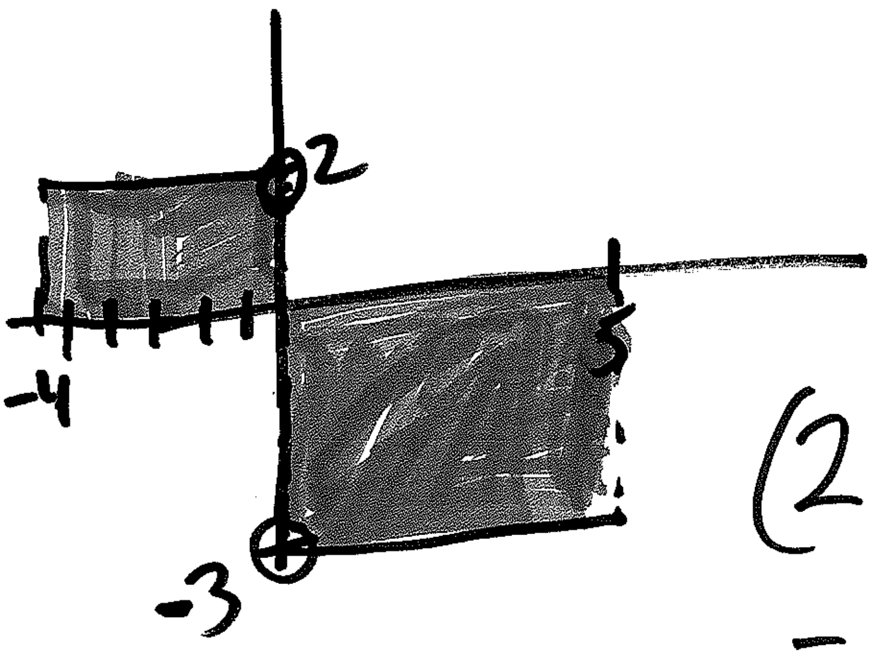
$= v_0 \Delta t$

$=$ area of rectangle with height v_0 width Δt

If velocity not constant, in the limit $\int_a^b v(t)\,dt =$ net distance traveled real area

1.) Find the area between the curve $f(x) = \begin{cases} 2 & x < 0 \\ -3 & x > 0 \end{cases}$,

and the x-axis, and between $x = -4$ and $x = 5$.



$(2)(4) + (3)(5)$

$= 8 + 15 = 23$

actual area

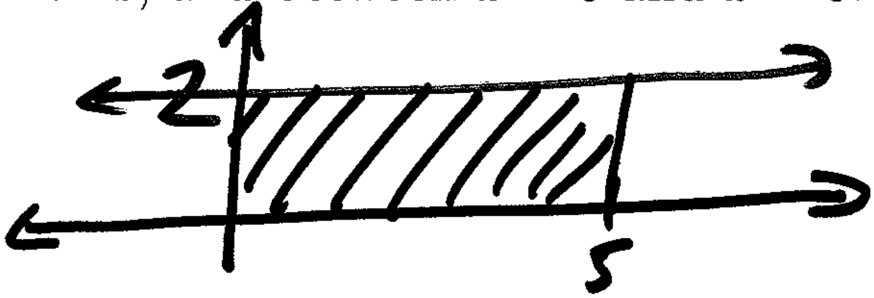
2a.) $\int_{-4}^{5} |f(x)| dx =$ 23

net area

2b.) $\int_{-4}^{5} f(x) dx =$ $8 - 15 = -7$

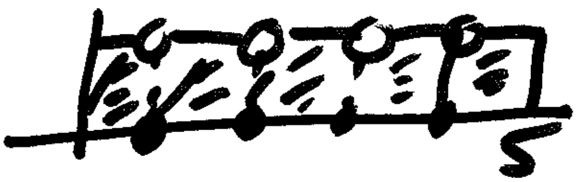
FYI: Side Note:

A.) Find the area under the curve $f(x) = 2$, above the x-axis, and between $x = 0$ and $x = 5$.



$(2)(5) = 10$

B.) Find the area under the curve $h(x) = \begin{cases} 2 & x \neq 1, 2, 3, 4 \\ 0 & x = 1, 2, 3, 4 \end{cases}$, [redacted] above the x-axis, and between $x = 0$ and $x = 5$.



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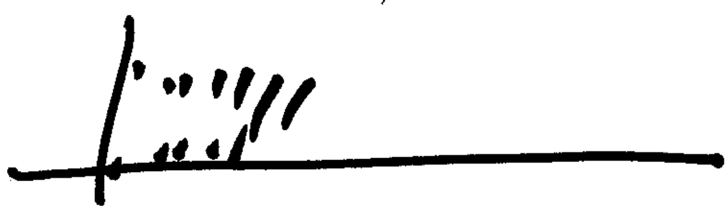
C.) Find the area under the curve $h(x) = \begin{cases} 2 & x \text{ irrational} \\ 0 & x \text{ rational} \end{cases}$, [redacted] above the x-axis, and between $x = 0$ and $x = 5$.



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D.) Find the area under the curve $h(x) = \begin{cases} 2 & x \text{ rational} \\ 0 & x \text{ irrational} \end{cases}$, [redacted] above the x-axis, and between $x = 0$ and $x = 5$.

0

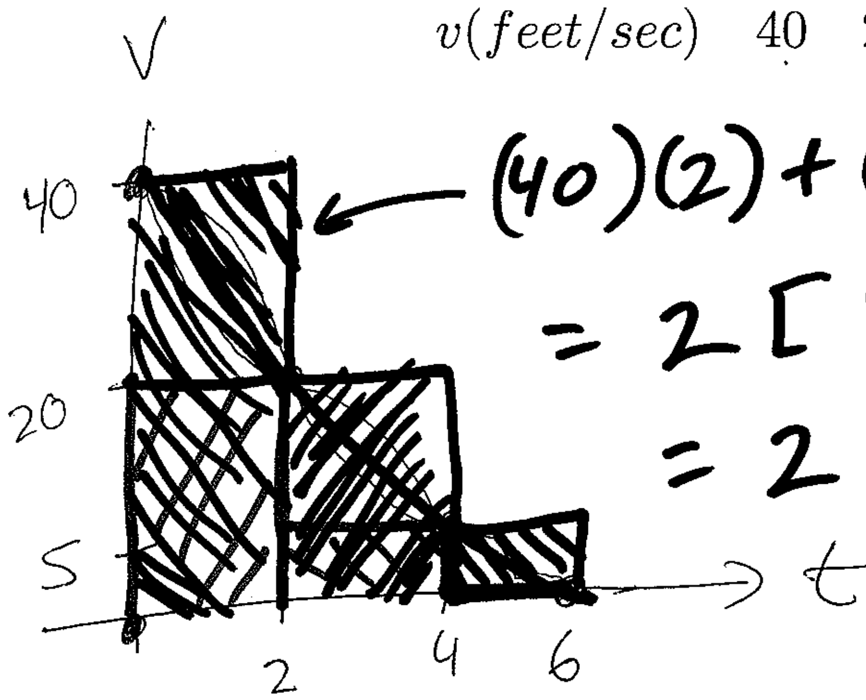
$\frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{1}{3}, \frac{3}{1}, \frac{1}{4}, \frac{2}{3}, \frac{3}{2}, \frac{4}{1}, \ldots$

circumscibed

3.) The speed of a runner decreased steadily after crossing the finish line. Her speed at 2 second intervals is given in the table. Find lower and upper estimates for the distance that she traveled during these 6 seconds.

inscribed rectangles

$t(seconds)$	0	2	4	6
$v(feet/sec)$	40	20	5	0



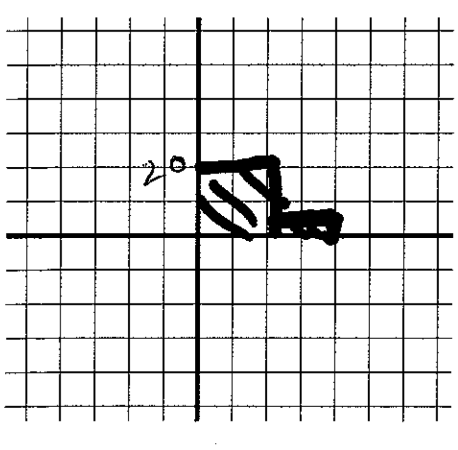
$(40)(2)+(20)(2)+(5)(2)$

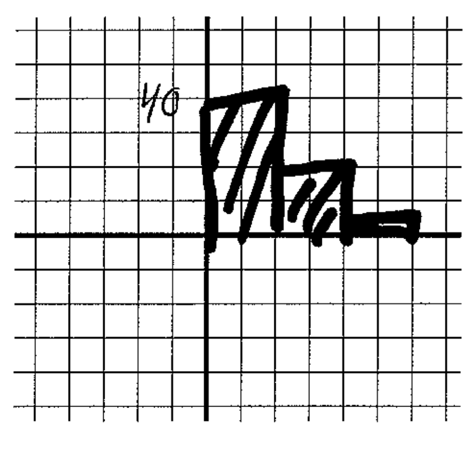
$= 2[40+20+5]$

$= 2(65) = 130$

$(20)(2)+(5)(2)+0 = 40+10 = 50$

Lower estimate: 50 ft, Upper estimate: 130 ft





$$\int_a^b f(t)\,dt = F(b) - F(a)$$

where F is any anti derivative of f

Ex $\int_0^2 x\,dx$

integral

$$= \frac{x^2}{2}\Big|_0^2$$

$$= \frac{2^2}{2} - \frac{0^2}{2} = 2$$

$$\int_0^1 x\,dx = \left.\frac{x^2}{2}\right|_0^1$$

$$= \frac{(1)^2}{2} - \frac{0^2}{2} = \frac{1}{2} - \frac{0}{2}$$

$$= \frac{1}{2}$$

$$\int_2^5 x^3\,dx = \left.\frac{x^4}{4}\right|_2^5$$

$$\frac{5^4}{4} - \frac{2^4}{4} = \frac{609}{4}$$

$$\int_0^3 x^2\,dx = \left.\frac{x^3}{3}\right|_0^3$$

$$= \frac{3^3}{3} - \frac{0^3}{3} = 9$$

$$\int_0^3 x^2\,dx = \left.\frac{x^3}{3} + 4\right|_0^3$$

$$\left[\frac{3^3}{3} + \cancel{4}\right] - \left[\frac{0^3}{3} + \cancel{4}\right] = 9$$

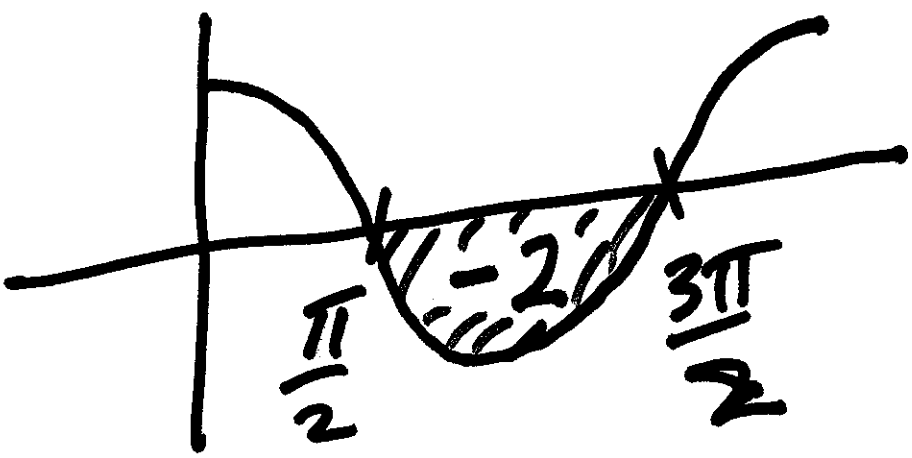
Any anti-derivative works so choose easiest

$$\int_{\frac{\pi}{2}}^{3\pi/2} \cos x \, dx$$

$$= \sin x \Big|_{\frac{\pi}{2}}^{3\pi/2}$$

$$= \sin\left(\frac{3\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)$$

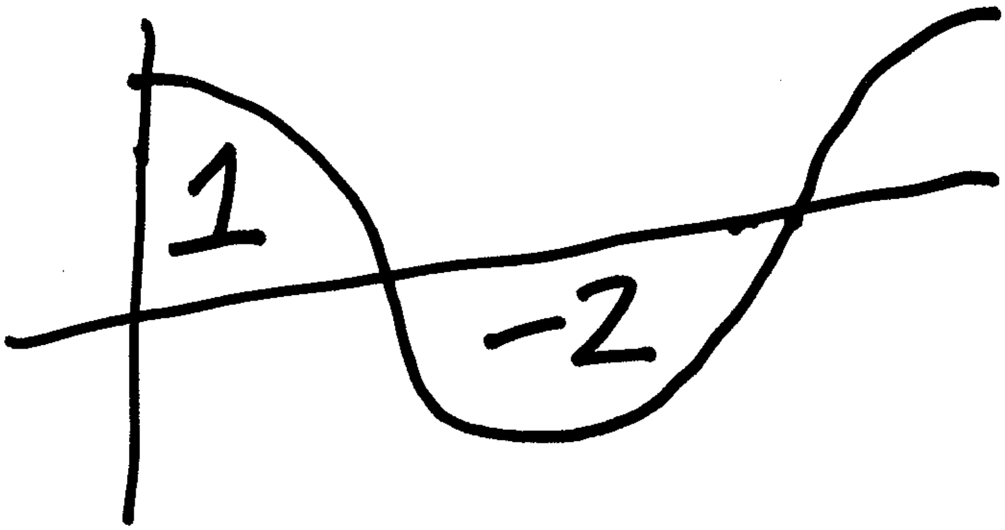
$$= -1 - 1 = -2$$



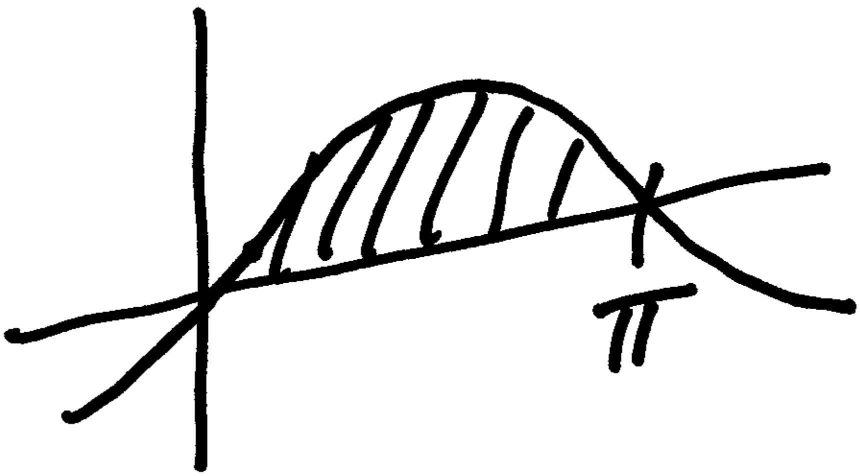
$$\int_0^{\pi/2} \cos x \, dx$$

$$= \sin x \Big|_0^{\pi/2}$$

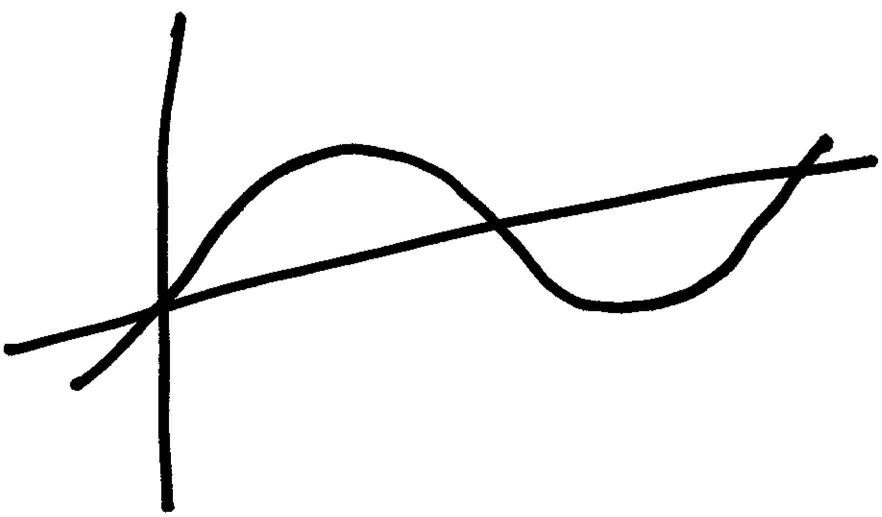
$$= \sin\left(\frac{\pi}{2}\right) - \sin(0) = 1$$



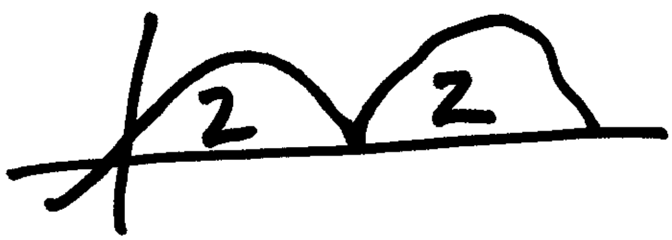
$$\int_0^{\pi} \sin x \, dx = 2$$



$$\int_0^{2\pi} \sin x \, dx = 0$$



$$\int_0^{2\pi} |\sin x|\,dx = 4$$



$$\int_0^2 e^{3x}\,dx =$$

$$\left.\frac{e^{3x}}{3}\right|_0^2 = \frac{e^6}{3} - \frac{e^0}{3}$$

$$= \frac{e^6}{3} - \frac{1}{3}$$