Section 3.3:

Motivation: Graph $f(x) = \frac{x^2}{(x-2)(x+2)}$

To find vertical asymptotes,
find all $a \in \mathcal{R}$ such that
$lim_{x \to a^-} f(x) = \pm\infty$ and/or $lim_{x \to a^+} f(x) = \pm\infty$

Ex: $f(x) = \frac{1}{(x+2)(x-3)^2}$

Horizontal asymptotes/limits at infinity

To find horizontal asymptotes:
calculate $lim_{x \to +\infty} f(x)$ and $lim_{x \to -\infty} f(x)$

IF $lim_{x \to +\infty} f(x) = L$ where L is a finite real number, then $y = L$ is a horizontal asymptote.

IF $lim_{x \to -\infty} f(x) = K$ where K is a finite real number, then $y = K$ is a horizontal asymptote.

Ex: $f(x) = \frac{2x^3 - x^2 + 1}{8x^3 + x + 3}$

$lim_{x \to +\infty} \frac{2x^3 - x^2 + 1}{8x^3 + x + 3} =$

Similarly, $lim_{x \to -\infty} \frac{2x^3 - x^2 + 1}{8x^3 + x + 3} =$

Horizontal asymptote(s):

Ex: $f(x) = \frac{x^2+1}{2x^5+x^2-3}$

$lim_{x\to+\infty} \frac{x^2+1}{2x^5+x^2-3} =$

Similarly, $lim_{x\to-\infty} \frac{x^2+1}{2x^5+x^2-3} =$

Horizontal asymptote(s):

Ex: $f(x) = \frac{2x^5+x^2-3}{x^2+1}$

$lim_{x\to+\infty} \frac{2x^5+x^2-3}{x^2+1} =$

Also, $lim_{x\to-\infty} \frac{2x^5+x^2-3}{x^2+1} =$

Horizontal asymptote(s):

Ex: $f(x) = \frac{2x}{\sqrt{x^2+1}}$

$lim_{x\to+\infty}\frac{2x}{\sqrt{x^2+1}} =$

$lim_{x\to-\infty}\frac{2x}{\sqrt{x^2+1}} =$

Horizontal asymptote(s):

Ex: $f(x) = x^2 - x^3$

$lim_{x \to +\infty} x^2 - x^3 =$

$lim_{x \to -\infty} x^2 - x^3 =$

Horizontal asymptote(s):

Ex: $f(x) = x^{\frac{2}{3}} - x$

$lim_{x \to +\infty} x^{\frac{2}{3}} - x =$

$lim_{x \to -\infty} x^{\frac{2}{3}} - x =$

Horizontal asymptote(s):