

$$\frac{d}{dx} (\cos(\tan(x^3))) = -\sin(\tan(x^3)) \cdot (\tan(x^3))'$$

$$= -\sin(\tan(x^3)) \cdot \sec^2(x^3) \cdot 3x^2$$

$$\frac{d}{dx} [\sin(x)] = \cos(x)$$

$$\frac{d}{dx} [\cos(x)] = -\sin(x)$$

$$\frac{d}{dx} [\tan(x)] = \sec^2(x)$$

$$\frac{d}{dx} (\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\tan^{-1}x) = \frac{1}{1+x^2}$$

Ex:  $\frac{d}{dx} (\cos^{-1}(\tan^{-1}(x^3)))$

$$= \frac{-1}{\sqrt{1 - (\tan^{-1}(x^3))^2}} \cdot (\tan^{-1}(x^3))'$$

$$= \frac{-1}{\sqrt{1 - [\tan^{-1}(x^3)]^2}} \cdot \frac{1}{1 + (x^3)^2} \cdot 3x^2$$

Ex:  $\frac{d}{dx} (\ln(x)) = 1/x$

$y = \ln x$

$$e^y = e^{\ln x}$$

$$e^y = x$$

$$\frac{d}{dx} (e^y) = \frac{d}{dx} (x)$$

$$e^y \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

$$2x^2y - 3y^2 = 4$$

To find  $y''$  first, find  $y'$

$$4x \cdot y + 2x^2 \cdot \frac{dy}{dx} - 6y \cdot \frac{dy}{dx} = 0$$

Solve for  $y'$ :

$$\frac{dy}{dx} (2x^2 - 6y) = -4xy$$

$$\frac{dy}{dx} = \frac{-4xy}{2x^2 - 6y} \frac{(-1)}{(-1)} = \frac{4xy}{6y - 2x^2}$$

Find  $y''$

$$y'' = \frac{d^2y}{dx^2} = \frac{(6y - 2x^2) \left( \frac{4xy}{6y - 2x^2} \right)' - 4xy (6y - 2x^2)'}{(6y - 2x^2)^2}$$

$$= \frac{(6y - 2x^2)(4) \left( y + x \frac{dy}{dx} \right) - 4xy (6 \frac{dy}{dx} - 4x)}{(6y - 2x^2)^2}$$

~~Substitute~~ Get rid of  $\frac{dy}{dx}$  via substitution

$$y'' = \frac{(6y - 2x^2)(4) \left( y + x \left( \frac{4xy}{6y - 2x^2} \right) \right) - 4xy \left( 6 \left( \frac{4xy}{6y - 2x^2} \right) - 4x \right)}{(6y - 2x^2)^2}$$

3.7

Suppose  $s(t) = t^2 + 3t - 1$  represents position at time  $t$ .

Then velocity =  $v(t) = \frac{d}{dt}(s(t)) = s'(t) = 2t + 3$

and acceleration =  $a(t) = \frac{d}{dt}(v(t)) = v'(t) = s''(t) = 2$

↑ 2<sup>nd</sup> derivative

jerk = change in acceleration

=  $D(a(t)) = \frac{d}{dt}(a(t)) = a'(t) = v''(t) = s'''(t) = 0$

↑ 3<sup>rd</sup> derivative

Ex: Find  $\frac{d^{50}}{dx^{50}}(\sin(x)) = -\sin(x)$

$[\sin(x)]' = \cos(x)$

$[\sin(x)]'' = (\cos(x))' = -\sin(x)$

$[\sin(x)]''' = (-\sin(x))' = -\cos(x)$

$[\sin(x)]^{(4)} = (-\cos(x))' = \sin(x)$

$[\sin(x)]^{(5)} = (\sin(x))' = \cos(x)$

$\left( \frac{d}{dt} \left( \frac{d}{dt} \left( \frac{d}{dt} (s(t)) \right) \right) \right)$

$\frac{d^3}{dt^3} (s(t))$

$D^3 (s(t))$

$s'''(t)$

$s^{(3)}(t)$

↑ take 3<sup>rd</sup> derivative

Ex: Find  $y''$  if  $2x^2y - 3y^2 = 4$

$s^{(40)}(t) = 0$

↑ 40<sup>th</sup> derivative

$s^{(n)}(t) = 0$

if  $n \geq 3$