$lim_{x\to 3}\ \frac{x^2-1}{x+3}$

$lim_{x\to 3}\ \frac{x^2-1}{x-3}$

$lim_{x\to 3}\ \frac{(x^2-1)(x-3)}{x-3}$

$lim_{x\to 3}\ \frac{x-3}{x^2-1}$

$lim_{x\to 3}\ \frac{(x-4)^2}{x^5(x-8)^9(x-3)^3}$

$lim_{x\to 3}\ \frac{(x-4)^2(x-3)}{x^5(x-8)^9(x-3)^3}$

Challenge example: $g(x) = xsin\frac{1}{x}$

$$-|x| \leq xsin\frac{1}{x} \leq |x|$$

$$lim_{x\to 0}\big(-|x|\big) = 0,\ lim_{x\to 0}\big(|x|\big) = 0.$$

Hence , $lim_{x\to 0}\big(xsin\frac{1}{x}\big) = 0$

Standard example:

Suppose $f(x) = \sqrt{x}$. Find $lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ where $x > 0$

Suppose $c \in \mathcal{R}$ and suppose $lim_{x \to a} f(x)$ and $lim_{x \to a} g(x)$ exist. Then

$$lim_{x \to a}[f(x) + g(x)] = lim_{x \to a} f(x) + lim_{x \to a} g(x)$$

$$lim_{x \to a}[cf(x)] = c\ lim_{x \to a} f(x)$$

$$lim_{x \to a}[f(x)g(x)] = lim_{x \to a} f(x)\ lim_{x \to a} g(x)$$

$$lim_{x \to a} \frac{f(x)}{g(x)} = \frac{lim_{x \to a} f(x)}{lim_{x \to a} g(x)} \text{ if } lim_{x \to a} g(x) \neq 0$$

Defn: f is continuous at a if $lim_{x \to a} f(x) = f(a)$
(i.e., if $lim_{x \to a} f(x) = f(lim_{x \to a} x)$)

In other words, f is continuous at a if
1.) $f(a)$ exists,
2.) $lim_{x \to a} f(x)$ exists, and
3.) $lim_{x \to a} f(x) = f(a)$

Defn: f is continuous is f is continuous at a for every a in the domain of f.

Examples:

Ex: Polynomial, rational, root, trigonometric, inverse trigonometric, exponential, logarithmic functions are continuous functions.

If f, g continuous at a, $c \in \mathcal{R}$, then $f+g$, fg, cf, f/g (if $g(a) \neq 0$) are continuous at a.

If g continuous at a and f continuous at $g(a)$, then $f \circ g$ continuous at a.

Ex: $lim_{x \to 0} \frac{x^2 - e^{x^3}}{cos(x)} =$

Ex: $lim_{x \to 9} e^{\sqrt{x}} - 2\sqrt{x} + 4 =$

Ex: $lim_{x \to 0} cos(sin(x)) =$

Ex: $lim_{x \to 0} cos(\frac{sin(x)}{x}) =$

Ex: $lim_{h \to 0} (h)tan(x)csc(h) =$