$G' = (V', E')$ is a subgraph of $G = (V, E)$ if $V' \subset V$, $E' \subset E$, and G' is a graph.

$G[V'] = (E', V')$ the subgraph of G induced or spanned by V' if $E' = \{xy \in E \mid x, y \in V'\}$.

$G' = (V', E')$ is a spanning subgraph of $G = (V, E)$ if $V' = V$.

$G - W = G[V - W]$, $G - E' = (V, E - E')$, $G + xy = (V, E \cup \{xy\})$ where x, y are nonadjacent vertices in V.

$|G| =$ order of $G = |V(G)| =$ number of vertices.

$e(G) =$ size of $G = |E(G)| =$ number of edges.

G^n is a graph of order n, $G(n, m)$ is a graph of order n and size m.

$E(U, V) =$ set of $U - V$ edges $=$ set of all edges in $E(G)$ joining a vertex in U to a vertex in V where $U \cap V = \emptyset$.

The complement of $G = (V, E) = \overline{G} = (V, V^{(2)} - E)$

$K_n =$ complete graph on n vertices. $E_n = \overline{K_n} =$ empty graph with n vertices. $K_1 = E_1$ is trivial.

$\Gamma(x) = \Gamma_G(x) = \{y \mid xy \in E(G)\}$.

$d(x) = d_G(x) = deg(x) =$ degree of $x = |\Gamma(x)|$.

$\delta(G) = min\{d(x) \mid x \in V(G)\}$.

$\Delta(G) = max\{d(x) \mid x \in V(G)\}$.

v is an isolated vertex if $d(v) = 0$.

$\Sigma_{x \in V} d(x) = 2e(G)$.

A walk in a graph, $W = v_0, e_1, v_1, e_2, ..., e_n, v_n$, where $v_i \in V$ and $e_i = v_{i-1}v_i \in E$.

length of $W = n$.

trail = walk with distinct edges.

circuit = closed trail.

path = walk with distinct vertices (= trail with distinct vertices).

cycle = circuit with distinct vertices.

A set of vertices (edges) is independent if no two elements in the set are adjacent

A set of paths is independent if no two paths share an interior vertex.

$d(x, y)$ = length of shortest $x - y$ path. If there is no $x - y$ path, then $d(x, y) = \infty$.

A graph is connected if given any pair of distinct vertices, x, y, there is an $x - y$ path.

A component of a graph = a maximal connected subgraph.

A cutvertex = a vertex whose deletion increases the number of components.

A bridge = an edge whose deletion increases the number of components.

A forest = an acyclic graph = a graph without any cycles.

A tree = a connected forest.

$G = (V, E)$ is bipartite if there exists vertex classes, V_1, V_2, such that $V_1 \cup V_2 = V$, $V_1 \cap V_2 = \emptyset$, and $xy \in E$, $x \in V_i$ implies $y \notin V_i$ (i.e., no edge joins two vertices in the same class).

$K(n_1, ..., n_r)$ = complete r-partite graph. $K_{p,q} = K(p, q)$, $K_r(t) = K(t, t, ..., t)$

Thm 3: Suppose that $C = (W, E')$ is the component of $G = (V, E)$ containing the vertex x. Then

$W = \{y \in V \mid G \text{ contains an } x - y \text{ path }\} = \{y \in V \mid G \text{ contains an } x - y \text{ trail }\}$
$= \{y \in V \mid d(x, y) < \infty\}$ = equivalence class of x where we take the smallest equivalence relation on V such that u is equivalent to v if $uv \in E$.

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