The ij^{th} entry of P^k is the probability that you are in the jth state after exactly k steps given that you started in the ith state.

Suppose that p_i is the probability that you start in state i. Let $p = (p_1, ..., p_n)$. Then $pP^k = (s_1, ..., s_n)$ where s_j is the probability that you are in the jth state after exactly k steps.

If there are transient states:

If $P = \begin{pmatrix} I & 0 \\ R & Q \end{pmatrix}$, then can use Q^k instead of P^k.

If Q represents transient states, then $lim_{n\to\infty} Q^n = 0$

If $N = (I - Q)^{-1} = \Sigma_{n=0}^{\infty} Q^n$, then the ij^{th} entry of N is the expected number of times you are in state j given that you started in state i.

Hence the expected number of steps before absorption is the sum of the ith row of $(I-Q)^{-1}$ given that you started in state i.

If $B = NR = (I-Q)^{-1}R$, then b_{ij} is the probability that absorbed in state j given that you started in state i.

Ergodic (there are NO transient states):

If regular (i.e. P^k is a positive matrix for some k):

P^n is a positive matrix for all large n ($n \geq k$).

$$lim_{n\to\infty} P^n = W = \begin{pmatrix} \mathbf{w} \\ . \\ . \\ . \\ \mathbf{w} \end{pmatrix}$$

$lim_{n\to\infty} pP^n = \mathbf{w}$
$\mathbf{w}P = \mathbf{w}$

If $E = (I - Z + JZ_{dg})D$, then e_{ij} is the expected number of steps from state i to state j (without going through state j in between, i.e., first time getting to/returning to state j).
$$e_{ii} = \frac{1}{w_i}$$

Regular if and only if period = 1.

If not regular

Period > 1

Fix i: d = Period = gcd$\{n \mid$ there is a path from u_i to u_i of length $n\}$

States can be partitioned into d periodic classes, $C_0, ..., C_{d-1}$ such that if you start at a vertex in C_i, then after k steps, you are in a vertex in $C_{i+k(mod d)}$.