$p_{ij} = P(O_t = u_j \mid O_{t-1} = u_i)$
$=$ probability that the outcome at time t is u_j given that the outcome at time $t-1$ is u_i
$=$ State at time t is u_j given that the state at time $t-1$ is u_i.

$p_{ij}^{(t)} = P(O_t = u_j \mid O_0 = u_i)$
$=$ probability that state at time t is u_j given that the chain starts in state u_i (at time 0).
$=$ probability that you go from state u_i to u_j in t steps given that you are at u_j.
$=$ i,j entry of P^t where P is the transition matrix.

A set C is closed if $p_{ij} = 0$ for all $u_i \in C$ and $u_j \notin C$

A set E is ergodic if it is a minimal closed set (i.e. E is closed and no proper subset of E is closed) $=$ strong component which is closed.

Transient set $=$ strong component which is not ergodic.

A state is ergodic if it is in an ergodic strong component.
A state is transient if it is in a transient strong component.

If an ergodic strong component consists of only a single vertex u_i, then u_i is an absorbing state.

A Markov chain is ergodic iff it is strongly connected

A Markov chain is absorbing if each of its ergodic strong components have only one element.

Entering an absorbing state is called absorption.

Every Markov chain has an ergodic set.

Thm 5.4: In any (finite) Markov chain, the probability after t steps that the process is in an ergodic state approaches 1 as t approaches ∞.

Thm 5.3: A Markov chain is absorbing if and only if it has at least one absorbing state and from every nonabsorbing state it is possible to reach some absorbing state

Corollary to 5.4: In an absorbing Markov chain, the probability of absorption is 1.

Questions:

2.) The expected number of times the process will be in a given non-absorbing state u_j starting from a given non-absorbing state u_i

2.5) The expected number of steps before absorption starting from a given non-absorbing state u_i

Cor 5.5: If I is the identity matrix and Q is any square matrix of real numbers such that $lim_{n \to \infty} Q^n = 0$, the zero matrix, then $I - Q$ is invertible and $(I - Q)^{-1} = I + Q + Q^2 + ... = \Sigma_{n=0}^{\infty} Q^n$.