You are in a restaurant. There are four people at each table and five tables. The menu contains six entrée’s, but the entire restaurant has to vote on the entrée’s in order to decide which they get to eat. This is an example of a single winner election, because whatever entrée wins is the entrée that every customer has to eat.

Let’s say that this is the data from the restaurant with each table labeled A…E and each person labeled A1, A2… E3, E4 depending on which table they are seated at.

|  |
| --- |
| Restaurant Data |
|   | Chicken | Steak | Vegetables | Lamb | Noodles | Pork |
| Table | Person |   |
| A | 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 5 | 6 | 2 | 1 | 4 | 3 |
| 3 | 2 | 3 | 1 | 5 | 4 | 6 |
| 4 | 1 | 5 | 4 | 6 | 2 | 3 |
| B | 1 | 3 | 1 | 2 | 4 | 6 | 5 |
| 2 | 6 | 5 | 4 | 3 | 2 | 1 |
| 3 | 1 | 6 | 2 | 5 | 4 | 3 |
| 4 | 5 | 6 | 4 | 2 | 1 | 3 |
| C | 1 | 3 | 4 | 1 | 6 | 5 | 2 |
| 2 | 1 | 4 | 3 | 5 | 6 | 2 |
| 3 | 6 | 2 | 5 | 3 | 4 | 1 |
| 4 | 6 | 2 | 4 | 3 | 5 | 1 |
| D | 1 | 4 | 3 | 5 | 6 | 2 | 1 |
| 2 | 1 | 2 | 5 | 4 | 3 | 6 |
| 3 | 2 | 5 | 4 | 3 | 1 | 6 |
| 4 | 1 | 6 | 2 | 5 | 3 | 4 |
| E | 1 | 3 | 5 | 4 | 6 | 1 | 2 |
| 2 | 6 | 5 | 1 | 2 | 4 | 3 |
| 3 | 4 | 5 | 6 | 1 | 2 | 3 |
| 4 | 6 | 5 | 2 | 1 | 4 | 3 |

If this is the data, who would win for each voting method? If there is a tie simply state so.

1. Plurality voting (ignore rankings, each person only votes for their number 1 choice)
2. Borda count
3. Approval voting (ignore rankings, each person votes for their number 1, 2 and 3 choices)
4. Do you think the Condorcet method could apply to this situation? Explain why or why not.

Teacher Answer Guide

1. Plurality Voting: We can figure this out by tallying up the votes:

Chicken: 6 ones

Steak: 1 one

Vegetables: 3 ones

Lamb: 3 ones

Noodles: 3 ones

Pork: 4 ones

Therefore, the clear winner in plurality voting is Chicken.

We could also solve this using a bipartite graph. Our vertices would be the different entrees (chicken, steak…. pork) on one side and the people (A1, A2… E4) on the other. Our edges would connect the person to their number one choice. For example, A1 would only have an edge with chicken because that was his number one choice. Each person would have exactly one edge connecting them to one of the meal choices on the other side. The winner would be determined by the entrée with the largest degree.

1. In Borda Count voting, we would give each candidate a certain number of points by how they were ranked. In this example, we would give the number one choice 6 points, the number two choice 5 points… etc until we get to the number sixth choice that we give only one point. We can solve this by tallying up the points:

Chicken: 6+2+5+6+4+1+6+2+4+6+1+1+3+6+5+6+4+1+3+1 = 73

Steak: 5+1+4+2+6+2+1+1+3+3+5+5+4+5+2+1+2+2+2+2 = 58

Vegetables: 4+5+… = 76

Lamb: 3+6+… = 65

Noodles: 2+3+… = 72

Pork: 1+4+… = 76

In this case we would have a clear tie between Vegetables and Pork

1. In Approval Voting, we tally up the number of votes but we only count the number 1, 2 and 3 votes for each person. We ignore the preferences.

Chicken: 11 votes

Steak: 7 votes

Vegetables: 10 votes

Lamb: 9 votes

Noodles: 9 votes

Pork: 14 votes

In this case the clear winner would be Pork.

We could also model this with a bipartite graph like we did in number 1, the only difference being that each person has a degree of exactly three. In this case, the winner would again be whichever entrée had the highest degree.

1. Could the Condorcet Method apply to this situation? Yes it most certainly could, the only problem being that it takes a very long time to calculate. I will discuss this more in the Condorcet section under the higher level concepts for teachers and advanced students, but for a brief idea of how to apply this, we could make a Condorcet digraph where our entrée’s are our vertices and our arcs are directed at the entrée that loses each pair-wise election. In this case, our winner would be the vertex that has an out degree of n-1 where n is the number of vertices, so for our example the winner would have an out degree of 5.