Ch 6

15a.) D_7 by definition of D_n.

b.) If it is false that "no man receives his own hat", then at least one man receives his own hat (we only need one counter-example to "no man receives his own hat" for the statement to be false). Since the number of ways to return hats with no restrictions is 7!, The number of ways to return hats so that least one man receives his own hat is $7! - D_7$.

c.) Let S = ways to return hats with no restrictions.

Let A = ways to return hats so that no man receives his own hat.

Let B = ways to return hats so that exactly one man receives his own hat.

Note $A \cap B = \emptyset$. Note $|B| = 7D_6$ since there are 7 choices for the man who receives his own hat, and D_6 ways to return the remaining 6 hats so that none of the 6 remaining men receives their own hat.

The number of ways to return hats so that least two man receives his own hat $= |S| - |A| - |B| = 7! - D_7 - 7D_6$.

27.) If all seats are different: $\Sigma_{i=0}^{8}(-1)^i \begin{pmatrix} 8 \\ i \end{pmatrix} (n-i)!$

Since all the seats are different, we can think of this problem as the number of linear permutations of 8 objects which do not contain the pattern 12, 23, 34, 45, 56, 67, 78, 81 (note we need 81 since the girl who sat in the 1rst seat can't sit in front of the girl who sat in the 8th seat.

S = permutations of 1-8. $|S| = 8!$

Let A_i = permutations containing the pattern $i\ i+1$ where $i \in \{1, ..., 8\}$ and addition is performed mod 8.

Then $|A_i| = (8-1)!$ for $i = 1, ..., 8$.

etc. (similar to calculating Q_n as in section 6.5)

Suppose all seats are identical: $\frac{1}{8}\Sigma_{i=0}^{8}(-1)^i \begin{pmatrix} 8 \\ i \end{pmatrix} (n-i)!$

mod out by the 8 rotations (including the identity).