Thm 2.1.1. Pigeonhole Principle (weak form): If you have $n+1$ pigeons in n pigeonholes, then at least one pigeonhole with be occupied by 2 or more pigeons.

If $f : A \to B$ is a function and $|A| = n+1$, and $|B| = n$, then f is not 1:1.

If $f : A \to B$ is a function and A is finite and $|A| > |B|$, then f is not 1:1.

$id : \{1, ..., n\} \to \{1, ..., n\}$, $id(k) = k$ is 1:1.

$c : \{1, ..., n\} \to \{1, ..., n\}$, $id(k) = 1$ is not 1:1.

Thm 2.2.1 Pigeonhole Principle (strong form): Let $q_1, q_2, ..., q_n$ be positive integers. If $q_1 + q_2 + ... + q_n - n + 1$ objects are put into n boxes, then for some i the ith box contains at least q_i objects

Cor: $q_i = 1$ for all i implies Thm 2.1.1.

Cor: If $q_i = r$ for all i, then if $n(r-1)+1$ objects are put into n boxes, then there exists a box containing at least r objects.

Cor: If $\frac{m_1 + ..., m_n}{n} > r - 1$, then there exists an i such that $m_i \geq r$.

Cor: If $\frac{m_1 + ..., m_n}{n} < r$, then there exists an i s. t. $m_i < r$.

Appl 7: If you have an arbitrary number of apples, bananas and oranges, what is the smallest number of these fruits that one needs to put in a basket in order to guarantee there are at least 8 apples or at least 6 bananas or at least 9 oranges in the basket.

Appl 9: Show that every sequence $a_1, a_2, ..., a_{n^2+1}$ contains either an increasing or decreasing subsequence of length $n+1$.

Example ($n = 2$):

$a_1 =$, $a_2 =$, $a_3 =$, $a_4 =$, $a_5 =$

$m_1 =$

$m_2 =$

$m_3 =$

$m_4 =$

$m_5 =$

Proof:

Let $m_k =$ length of largest increasing subsequence beginning with a_k.

Example of a Ramsey theorem: In a group of 6 people, there are either 3 who know each other or 3 who are strangers to each other.

Ramsey number $= r(s,t) = min\{n \mid$ if the edges of K_n are colored red and blue, then there exists either a red K_s or a blue $K_t\}$

$r(3,3) = 6$ $\qquad$ $r(s,t) = r(t,s)$ $\qquad$ $r(s,2) = r(2,s) = s$

Thm (Erdos and Szekeres): $r(s,t)$ is finite for all $s,t \geq 2$. If $s > 2$, $t > 2$, then

$$r(s,t) \leq r(s-1,t) + r(s,t-1)$$

$$r(s,t) \leq \binom{s+t-2}{s-1}$$

$r = r(s_1, ..., s_k)$: using k colors, there exist an i such that K_r contains an i colored K_{s_i}

Hypergraph: (V, E), $E \subset \mathcal{P}(V)$

$X^{(t)}$ = set of all t-tuples of X.

A coloring of edges: $c : X^{(t)} \to \{red,\ blue\}$

$Y \subset X$ is a red n set if $|Y| = n$ and $c(Y^{(t)}) =$ red.

$R_t(n_1, n_2) = min\{m \mid |X| = m$ implies $X^{(t)}$ has a red n_1 set or a blue n_2 set $\}$

Ex: If $X = \{a, b, c, d\}$ then
$X^{(2)} = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\} = K_4$

$R_2(s, t) = R(s, t)$

$X^{(3)} = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

$X^{(4)} = \{\{a, b, c, d\}\}$

Have you had 22M50 (the pre-req for this course)?

How comfortable are you with writing proofs?

How comfortable are you with permutations/combinations?

Is the pace of the class too slow, about right, or too fast?

Do you have any special interests in Discrete Mathematics?

Any other comments?