Ex: Solve the recurrance relation: $h_n + h_{n-2} = 0$

$h_0 = 3, h_1 = 5$

$q^n + q^{n-2} = 0$

$q^2 + 1 = 0$

$q = \pm i$

$h_n = c_1 i^n + c_2(-i)^n$

$c_1 + c_2 = 3$ implies $c_2 = 3 - c_1$

$c_1 i - c_2 i = 5$

$-c_1 + c_2 = 5i$

$-c_1 + 3 - c_1 = 5i$

$-2c_1 + 3 = 5i$

$c_1 = \frac{3-5i}{2}$

$c_2 = 3 - (\frac{3-5i}{2}) = \frac{3+5i}{2}$

$h_n = (\frac{3-5i}{2})i^n + (\frac{3+5i}{2})(-i)^n$

$h_{2j} = (\frac{3-5i}{2})i^{2j} + (\frac{3+5i}{2})(-i)^{2j}$

$h_{2j+1} = (\frac{3-5i}{2})i^{2j+1} + (\frac{3+5i}{2})(-i)^{2j+1}$

$= (\frac{3i+5}{2})(-1)^j + (\frac{-3i+5}{2})(-1)^j$

Ex: Solve the recurrance relation, $h_n - 2h_{n-1} + h_{n-3} - h_{n-4} = 0$
$h_0 = 3, h_1 = 3, h_2 = 7, h_3 = 15,$

$(x-1)^3(x+1)$

$= (x^3 - 3x^2 + 3x - 1)$

$= (x^4 - 2x^3 + 2x - 1)$

q = 1 , 1, 1, -1

$h_n = c_1 + c_2 n + c_3 n^2 + c_4(-1)^n$

$h_0 = c_1 + c_4$

$h_1 = c_1 + c_2 + c_3 - c_4$

$h_2 = c_1 + 2c_2 + 4c_3 + c_4$

$h_3 = c_1 + 3c_2 + 9c_3 - c_4$

$$\begin{array}{ccccc} 1 & 0 & 0 & 1 & a \\ 1 & 1 & 1 & -1 & b \\ 1 & 2 & 4 & 1 & c \\ 1 & 3 & 9 & -1 & d \end{array}$$

$$\begin{array}{ccccc} 1 & 0 & 0 & 1 & a \\ 0 & 1 & 1 & -2 & b-a \\ 0 & 2 & 4 & 0 & c-a \\ 0 & 3 & 9 & -2 & d-a \end{array}$$

$$\begin{array}{ccccc} 1 & 0 & 0 & 1 & a \\ 0 & 1 & 1 & -2 & b-a \\ 0 & 0 & 2 & 4 & c-a-2(b-a) \\ 0 & 0 & 6 & 4 & d-a-3(b-a) \end{array}$$

$$\begin{array}{ccccc} 1 & 0 & 0 & 1 & a \\ 0 & 1 & 1 & -2 & b-a \\ 0 & 0 & 2 & 4 & c-a-2(b-a) \\ 0 & 0 & 0 & -8 & d-a-3(b-a)-3[c-a-2(b-a)] \end{array}$$

$$\begin{array}{ccccc} 1 & 0 & 0 & 1 & a \\ 0 & 1 & 1 & -2 & \\ 0 & 0 & 2 & 4 & c-a \\ 0 & 0 & 0 & -8 & d-a-3[c-a] \end{array}$$

$$\begin{array}{ccccc} 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 2 & 4 & 4 \\ 0 & 0 & 0 & -8 & d-3-12 \end{array}$$

$h_0 = 3, h_1 = 3, h_2 = 7, h_3 = 15,$

$$\begin{array}{ccccc} 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{array}$$

$$\begin{array}{ccccc} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{array}$$

$$\begin{array}{ccccc} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{array}$$

$$= \qquad (x^3 - 3x^2 + 3x - 1)$$

$$= (x^4 - 3x^3 + 3x^2 - x)$$