Arithmetic sequence: $h_0, h_0 + q, h_0 + 2q, ...$

$$h_n = h_{n-1} + q = h_0 + nq,\ n \geq 0$$

Geometric sequence: $h_0, qh_0, q^2 h_0, ...$

$$h_n = qh_{n-1} = q^n h_0,\ n \geq 0$$

Ex: $1, 2, 2^2, ...$

$h_n = 2^n =$ number of combinations of an n-element set.

Partial sums: $s_n = \Sigma_{k=0}^{n} h_k$

Arithmetic sequence: $s_n = \Sigma_{k=0}^{n} h_0 + kq = (n+1)h_0 + \frac{qn(n+1)}{2}$

Geometric sequence: $s_n = \Sigma_{k=0}^{n} q^k h_0 = \begin{cases} \frac{q^{n+1}-1}{q-1} h_0 & q \neq 1 \\ (n+1)h_0 & q = 1 \end{cases}$

Fibonacci:

Suppose a pair of rabbits of the opposite sex give birth to a pair of rabbits of opposite sex every month starting with their second month. If we begin with a pair of newly born rabbits, how many rabbits are there after one year.

Let $f_n =$ number of rabbits at the beginning of month $n =$ end of month $n - 1$.

$f_0 =$ $\quad f_1 =$ $\quad f_2 =$ $\quad f_3 =$ $\quad f_4 =$ $\quad f_5 =$

Hence $f_n =$
where

Lemma: $s_n = \Sigma_{k=0}^{n} f_n = f_{n-2} - 1$

Proof by induction on n.

Lemma: f_n is even iff $3|n$.

Proof by induction on n.

Note that $f_0 = 0$ is even, $f_1 = 1$ is odd, and $f_2 = 1$ is odd.

Suppose f_{3n} is even, f_{3n+1} is odd, and f_{3n+2} is odd.

Then $f_{3n+3} = f_{3n+2} + f_{3n+1}$. Since odd + odd is even, f_{3n+3} is even.

Then $f_{3n+4} = f_{3n+3} + f_{3n+2}$. Since even + odd is odd, f_{3n+4} is odd.

Then $f_{3n+5} = f_{3n+4} + f_{3n+3}$. Since odd + even is odd, f_{3n+5} is odd.

Thm 7.1.2: $f_n = \Sigma_{k=0}^{n} \begin{pmatrix} n-1-k \\ k \end{pmatrix}$

Proof: Check if $g(n) = \Sigma_{k=0}^{n} \begin{pmatrix} n-1-k \\ k \end{pmatrix}$ satisfies $g(n) = g(n-1) + g(n-2)$ and $g(0) = 0$ and $g(1) = 1$

Thm 7.1.1: $f_n = \frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^n - \frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^n$