Math 150 Exam 1
September 10, 2009

Choose 6 from the following 8 problems. Circle your choices: 1 2 3 4 5 6 7 8
You may do more than 6 problems in which case one of your two unchosen problems can replace your lowest problem at 2/3 the value (or more) as discussed in class.

1.) Determine which of the following sequences are inversion sequences. For each inversion sequence, determine its corresponding permutation. State whether the permutation is even or odd. If the sequence is not an inversion sequence, state why you know it is not.

0321401

2103000

51023020

0243100

2a.) Determine the inversion sequence for 526314

2b.) Which permutation of $\{1, 2, 3, 4, 5, 6\}$ follows 526314 in using the algorithm described in Section 4.1? Explain.

3a.) Determine the number of linear permutations of the multiset $\{1 \cdot a,\ 5 \cdot b,\ 10 \cdot c,\ 20 \cdot d\}$.

3b.) Determine the number of circular permutations of the multiset $\{1 \cdot a,\ 5 \cdot b,\ 10 \cdot c,\ 20 \cdot d\}$.

3c.) Determine the number of linear 5-permutations of the set $\{1, 2, ..., 20\}$.

3d.) Determine the number of circular 5-permutations of the set $\{1, 2, ..., 20\}$.

4a.) In how many ways can 16 blue rooks be placed on a 30-by-30 chessboard in non-attacking position?

4b.) In how many ways can 16 blue rooks be placed on a 30-by-30 chessboard in non-attacking position so that the first column is NOT empty?

4c.) In how many ways can 10 blue rooks, 5 green rooks, and 1 red rook be placed on a 30-by-30 chessboard in non-attacking position so that the first column is NOT empty?

5a.) How many sets of 4 numbers can be formed from the numbers $\{1, 2, ..., 310\}$ if no two consecutive numbers are to be in a set?

5b.) There are 310 identical sticks lined up in a row occupying 310 distinct places. 4 of the sticks are to be chosen. How many choices are there if no two of the chosen sticks can be consecutive?

6.) Prove that the number of nonnegative integral solutions to $x_1 + x_2 = 8$ is the same as the number of permutations of the multiset $\{8 \cdot 1, +\}$

7.) $r(s,2) =$ ________________. Prove this **equality**.

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8) Show that given m integers $a_1, a_2, ..., a_m$, there exists k and l with $0 \leq k < l \leq m$ such that $a_{k+1} + a_{k+2} + ... + a_l$ is divisible by m.

Hint: consider the m objects $a_1,\ \ a_1+a_2,\ \ a_1+a_2+a_3,\ \ ...,\ \ a_1+...+a_m$ and use remainders.