2.3 Combinations

$$C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!}$$

$$\Sigma_{i=0}^{n} \binom{n}{i} = 2^n$$

2.4 Permutations of Multisets

Thm 2.4.1: Let $A = \{\infty \cdot 1, \infty \cdot 2, ..., \infty \cdot k\}$

The number of r permutations of $A = k^r$.

Ex: The number of 8-digit numbers with digits $\{1, 2, 3, 4\} = 4^8$

Ex: The number of 9-digit numbers with digits $\{0, 1, 2, 3\} = 3(4^8)$

Thm 2.4.2: Let $B = \{n_1 \cdot 1, n_2 \cdot 2, ..., n_k \cdot k\}$

$n = n_1 + n_2 + ... + n_k$

The number of permutations of $B = \frac{n!}{n_1! n_2! \cdots n_k!}$.

$$\frac{n!}{n_1! n_2! \cdots n_k!} = \binom{n}{n_1} \binom{n - n_1}{n_2} \cdots \binom{n - \Sigma_{i=1}^{k-1} n_i}{n_k}.$$

\# of permutations of $\{n_1 \cdot 1, (n - n_1) \cdot 2\} = \frac{n!}{n_1!(n-n_1)!} = C(n, n_1)$

Thm 2.4.3 If have $n = n_1 + n_2 + ... + n_k$ different objects to be placed in k labeled boxes such that the box B_i contains n_i objects

2.4 Combinations of Multisets

Let $A = \{\infty \cdot 1, \infty \cdot 2, ..., \infty \cdot k\}$

The number of r combinations of $A = \binom{r + k - 1}{r}$

Proof: An r combination is