8.2

Given the sequence $h_0, h_1, h_2, ...$, define its *difference sequence* by

$$\Delta h_0, \Delta h_1, \Delta h_2, ... \text{ where } \Delta h_n = h_{n+1} - h_n$$

and its *pth order difference sequence* by

$$\Delta^p h_0, \Delta^p h_1, \Delta^p h_2, ... \text{ where } \Delta^p h_n = \Delta(\Delta^{p-1} h_n)$$

Ex: $h_n = n^2 - n + 1$

Thm 8.2.1

Suppose $h_n = a_p n^p + a_{p-1} n^{p-1} + ... + a_1 n + a_0$.
Then $\Delta^{p+1} h_n = 0 \ \forall n$

Proof by induction on p:

$p = 0$:

Note that the set of all complex-valued sequences form a vector space and Δ is a linear transformation on this vector space.

Defn: The *0th diagonal* of the difference table for h_n is

$$h_0 = \Delta^0 h_0, \Delta^1 h_0, \Delta^2 h_0, ...$$

Note the 0th diagonal of the difference table for h_n determines h_n

Lemma: Suppose the 0th diagonal of the difference table for h_n is 0, 0, ..., 0, 1, 0, 0, ... where 1 is preceded by exactly p zeros. Then $h_n = \binom{n}{p}$

Thm 8.2.2: Suppose 0th diagonal of the difference table for h_n is $c_0, c_1, ..., c_p, 0, 0, 0, ...$.

Then $h_n = c_0 \binom{n}{0} + c_1 \binom{n}{1} + ... + c_p \binom{n}{p}$.

Thm 8.2.3: Suppose 0th diagonal of the difference table for h_n is $c_0, c_1, ..., c_p, 0, 0, 0, ...$.

Then $\Sigma_{k=0}^{n} h_k = c_0 \binom{n+1}{1} + c_1 \binom{n+1}{2} + ... + c_p \binom{n+1}{p+1}$.

Ex: Find $\Sigma_{k=0}^{n} k^3 =$