6.2: Combinations with repetitions.

The number of integral solutions to $\Sigma_{n=1}^{5} x_i = 20$
where $-2 \leq x_i \leq 7 \ \forall i$

= The number of integral solutions to $\Sigma_{n=1}^{5} y_i = 30$
where $0 \leq y_i \leq 9 \ \forall i$

Pf: Let $y_i = x_i + 2$

The number of 30-combinations of the
multiset $\{9 \cdot a_1, 9 \cdot a_1, 9 \cdot a_2, 9 \cdot a_3, 9 \cdot a_4, 9 \cdot a_5\} =$

Let S = the set of integral solutions to $y_1 + y_2 + y_3 + y_4 + y_5 = 30$
where $0 \leq y_i \ \forall i$

Then $|S|$ = the number of permutations of $\{30 \cdot 1, 4 \cdot +\} =$

For $i = 1, 2, 3, 4, 5$,
let A_i = the set of integral solutions to $y_1 + y_2 + y_3 + y_4 + y_5 = 30$
where $10 \leq y_i$

Ex: $(10, 5, 5, 5, 5) \in A_1$, $(0, 20, 7, 2, 1) \in A_2$,
$(0, 0, 10, 10, 10) \in A_3 \cap A_4 \cap A_5$

Then $\overline{\cup_{i=1}^{5} A_i}$ = the set of of integral solutions to $\Sigma_{n=1}^{5} y_i = 30$
where $0 \leq y_i \leq 9 \ \forall i$