

Thm 5.3.1: Let n be a positive integer. The sequence of binomial coefficients is a unimodal sequence. In particular

if n is even,

$$\binom{n}{0} < \binom{n}{1} \cdots < \binom{n}{\frac{n}{2}}$$

$$\binom{n}{\frac{n}{2}} > \cdots > \binom{n}{n-1} > \binom{n}{n}$$

and if n is odd

$$\binom{n}{0} < \binom{n}{1} \cdots < \binom{n}{\frac{n-1}{2}} = \binom{n}{\frac{n+1}{2}}$$

$$\binom{n}{\frac{n+1}{2}} > \cdots > \binom{n}{n-1} > \binom{n}{n}$$

Proof idea: Look at $\frac{\binom{n}{k}}{\binom{n}{k-1}} = \frac{n-k+1}{k}$

5.4: Multinomial thm

Define
$$\binom{n}{n_1 n_2 \dots n_t} = \frac{n!}{n_1! n_2! \dots n_t!}$$

Thm 5.4.1: Let $n \in \mathcal{Z}_+$. Then

$$(x_1 + x_2 + \dots x_t)^n = \sum \binom{n}{n_1 n_2 \dots n_t} x_1^{n_1} x_2^{n_2} \dots x_t^{n_t}$$

where the summation extends over all nonnegative integral solutions to $n_1 + n_2 + \dots + n_t = n$

5.5: Newton's Binomial Theorem

Let $r \in \mathcal{R}$, $k \in \mathcal{Z}$.

Define
$$\binom{r}{k} = \begin{cases} \frac{r(r-1)\dots(r-k+1)}{k!} & \text{if } k \geq 1 \\ 1 & \text{if } k = 0 \\ 0 & \text{if } k \leq -1 \end{cases}$$

Thm 5.5.1: Let $\alpha \in \mathcal{R}$. Then if $0 \leq |x| < |y|$,

$$(x + y)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k y^{\alpha-k}$$