Let K_n = the complete graph on n vertices.

That is $K_n = (V, E)$, where
V = the vertices of $K_n = \{v_1, ..., v_n\}$,
E = the edges of $K_n = \{\{v_i, v_j\} \mid 1 \le i < j \le n\}$

$K_1 =$ $K_2 =$ $K_3 =$ $K_4 =$

$K_5 =$ $K_6 =$

Example of a Ramsey theorem: In a group of 6 people, there are either 3 who know each other or 3 who are strangers to each other.

Ramsey number $= r(s,t) = min\{n \mid$ if the edges of K_n are colored red and blue, then there exists either a red K_s or a blue $K_t\}$

$r(3,3) = 6$ $r(s,t) = r(t,s)$ $r(s,2) = r(2,s) = s$

Thm (Erdos and Szekeres): $r(s,t)$ is finite for all $s,t \ge 2$. If $s > 2$, $t > 2$, then

$$r(s,t) \le r(s-1,t) + r(s,t-1)$$

$$r(s,t) \le \binom{s+t-2}{s-1}$$

$r(s_1, ..., s_k) = min\{n \mid$ if the edges of K_n are colored using k colors, there exist an i colored $K_{s_i}\}$

Hypergraph: (V, E), $E \subset \mathcal{P}(V)$

$X^{(t)}$ = set of all t-tuples of X.

Ex: If $X = \{a, b, c, d\}$ then

$X^{(2)} = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\} = K_4$

$X^{(3)} = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

$X^{(4)} = \{\{a, b, c, d\}\}$

A coloring of edges: $c : X^{(t)} \to \{red,\ blue\}$

$Y \subset X$ is a red n set if $|Y| = n$ and $c(Y^{(t)})$ = red.

$r_t(n_1, n_2) = min\{m \mid |X| = m$ implies $X^{(t)}$ has a red n_1 set or a blue n_2 set $\}$

$r_2(s, t) = r(s, t)$