Byzantine Computability and Combinatorial Topology

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University of Rochester

Applied Algebraic Topology Research Network
November 10, 2015

joint work with Maurice Herlihy, Christine Tasson, done at Brown University

Outline

1. Introduction
2. Asynchronous Byzantine Systems
3. Synchronous Byzantine Systems
4. Conclusion & Future Work

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Introduction

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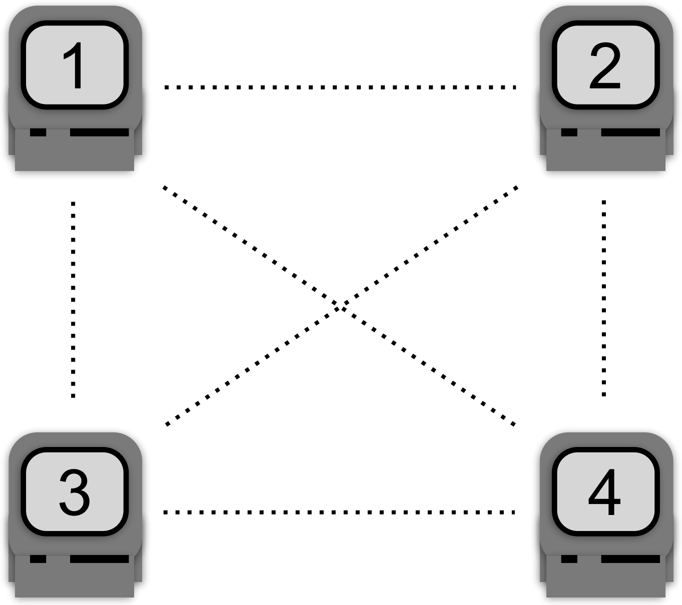

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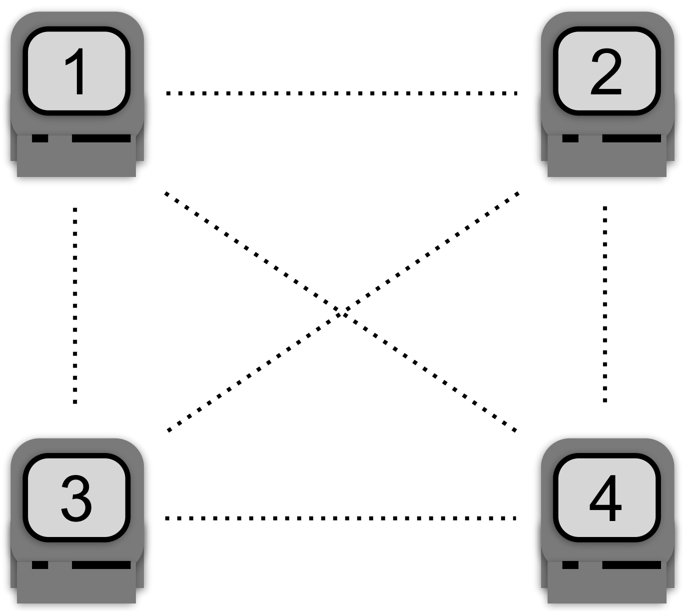

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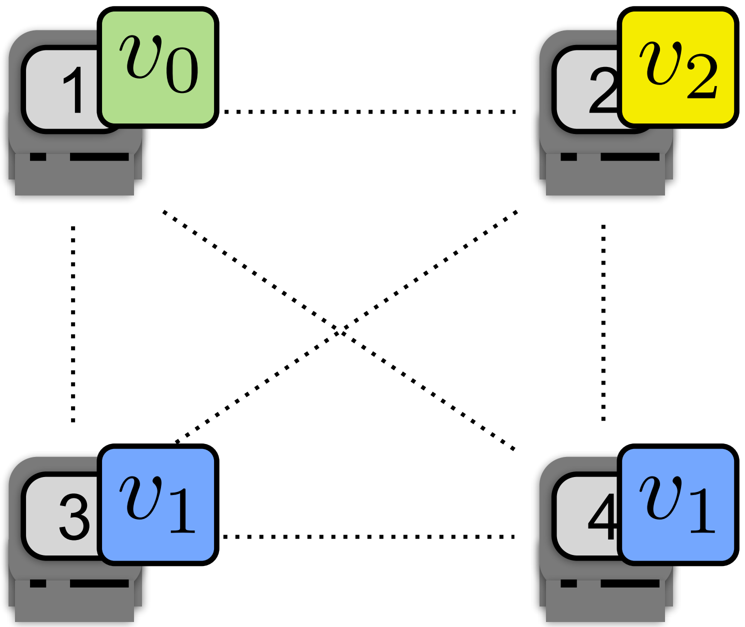

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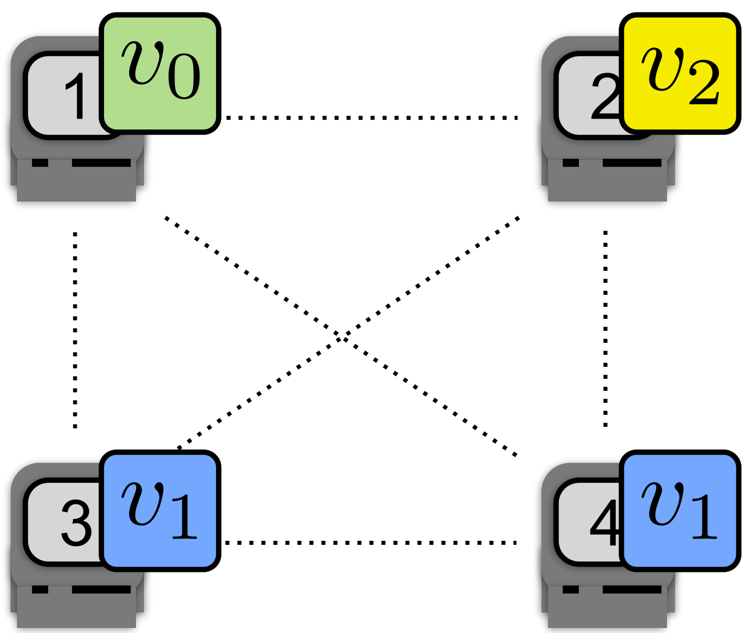

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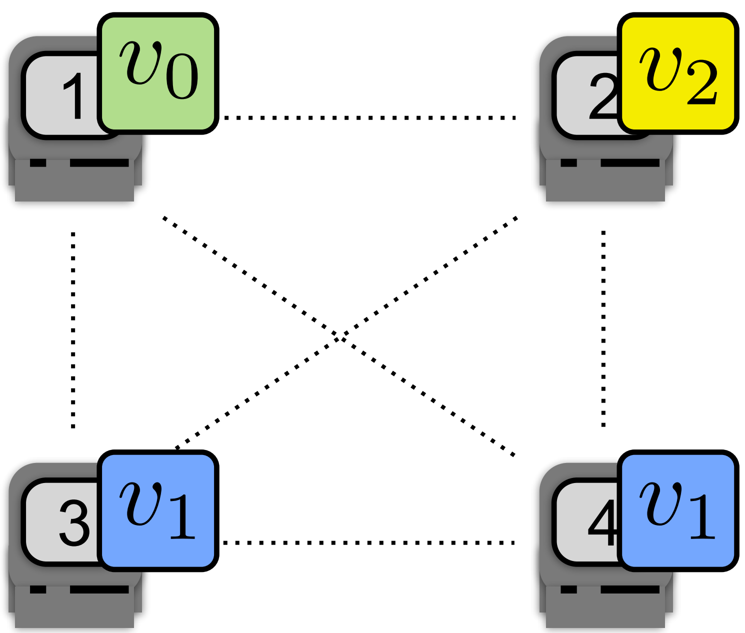

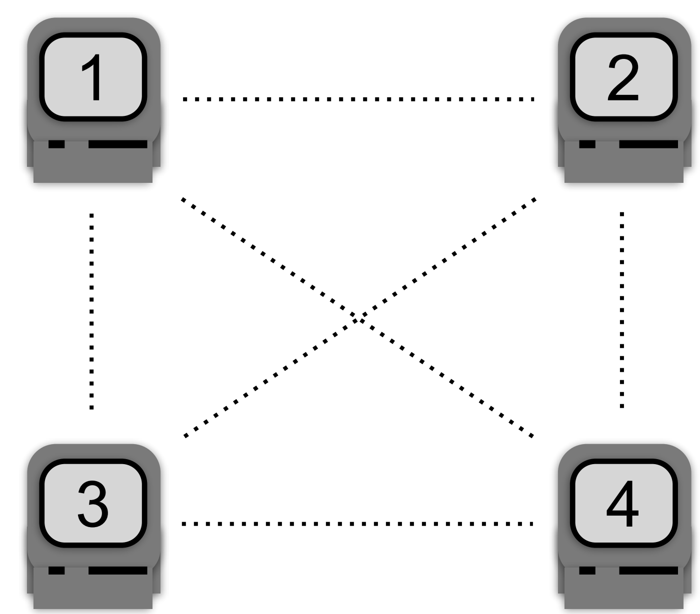

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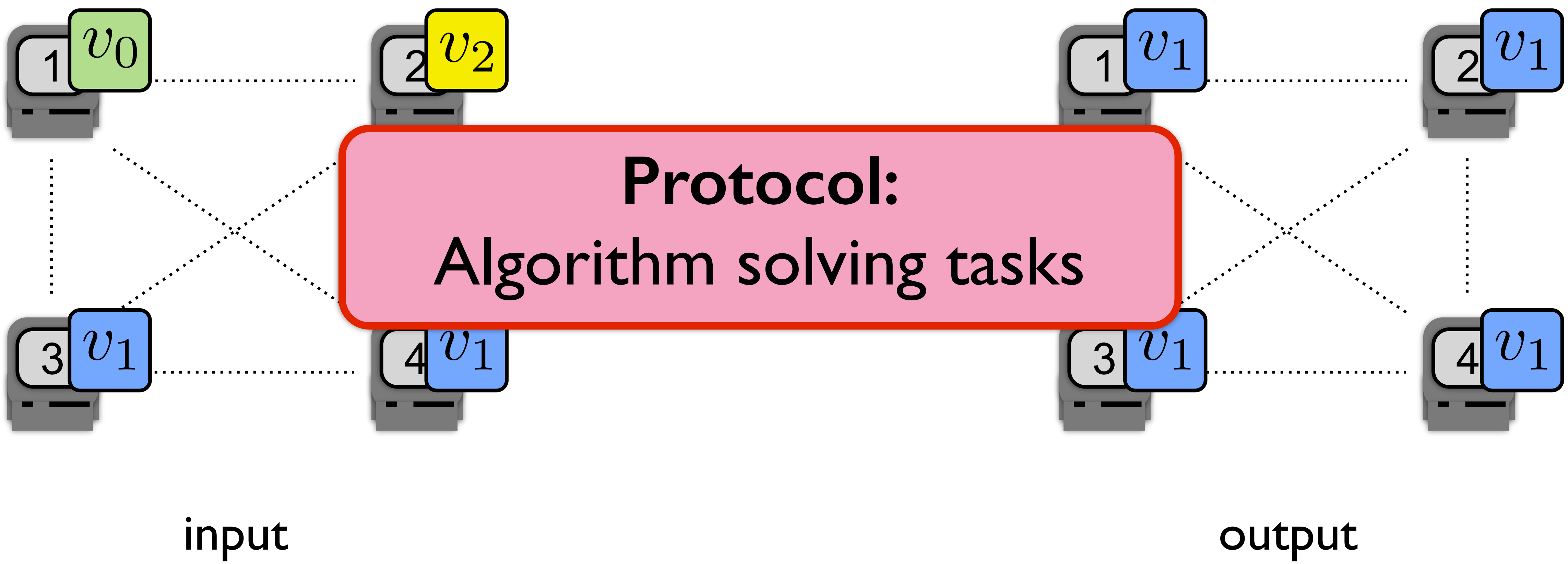

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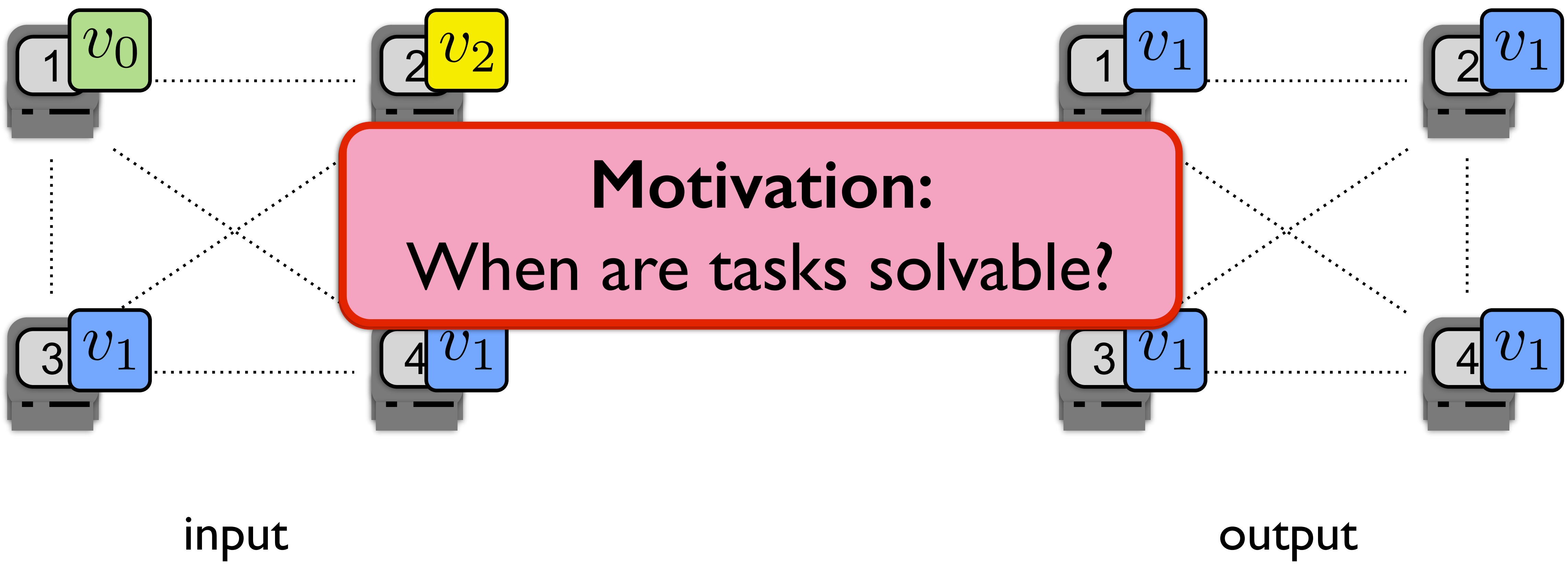

Motivation:
When are tasks solvable?

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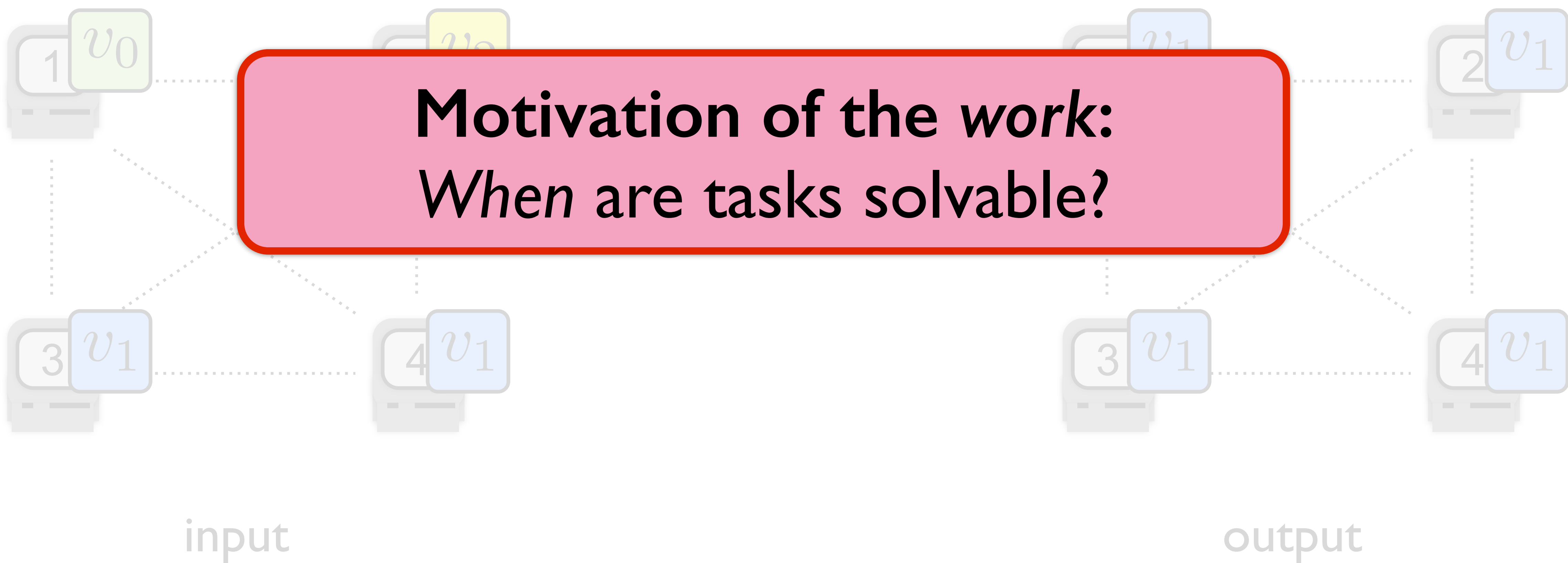

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Motivation of the *work*:

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Motivation of the *talk*:

Overview the research area for the AATRN

Create collaboration bridges

Introduction

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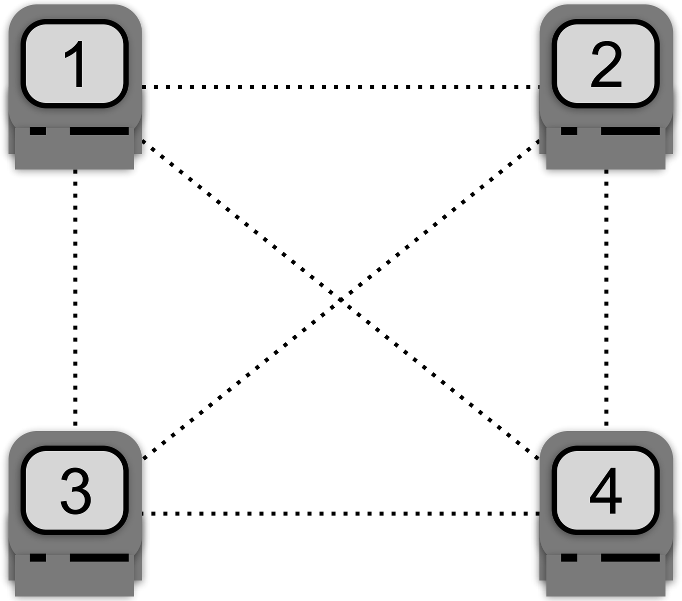

- Message-passing
  - Complete communication graph
  - Senders reliably identified
- FIFO delivery for each pair

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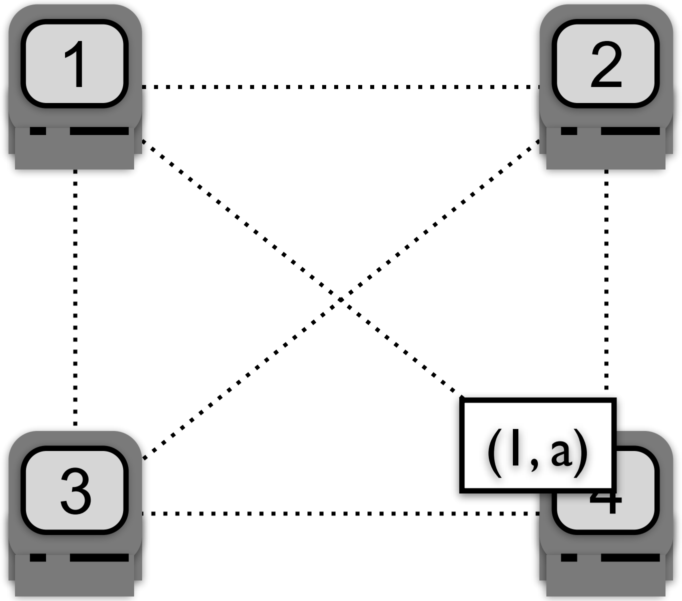

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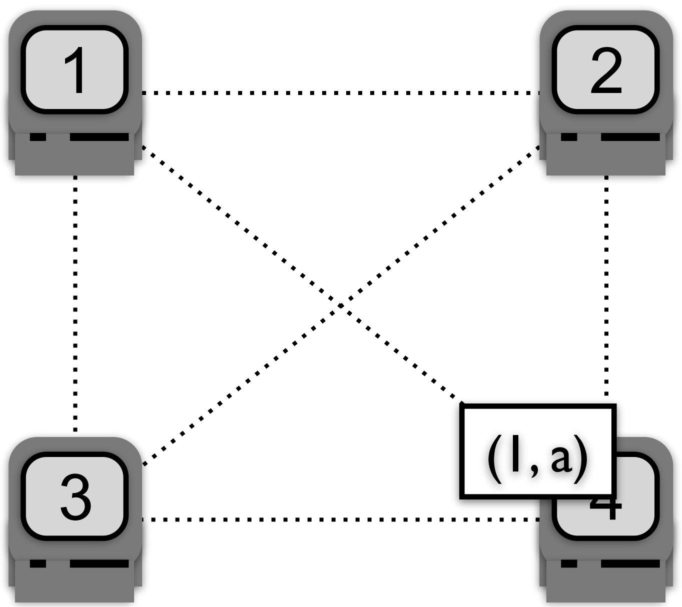

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Trivial?

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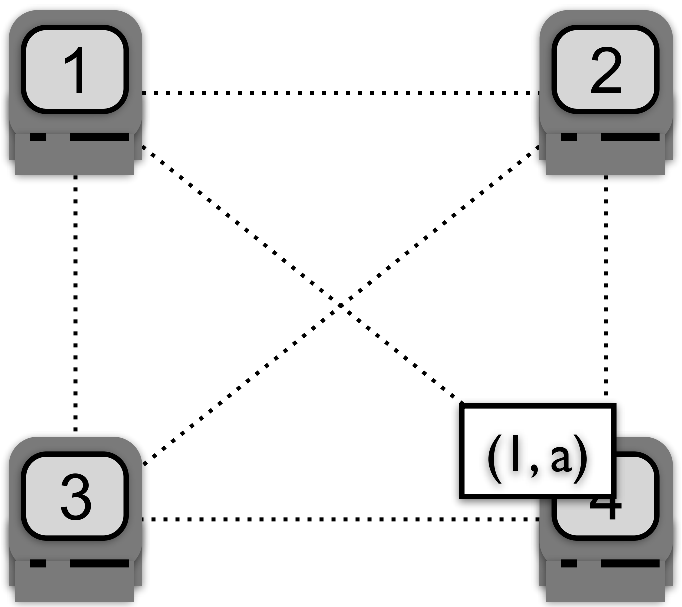

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Failures and (a)synchrony are the difficulties

Failures & Synchrony

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Processes subject to failures

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Asynchrony + Failures

Asynchronous Systems,
Byzantine Failures

$n = 4, t = 1$

Asynchrony + Failures

of processes

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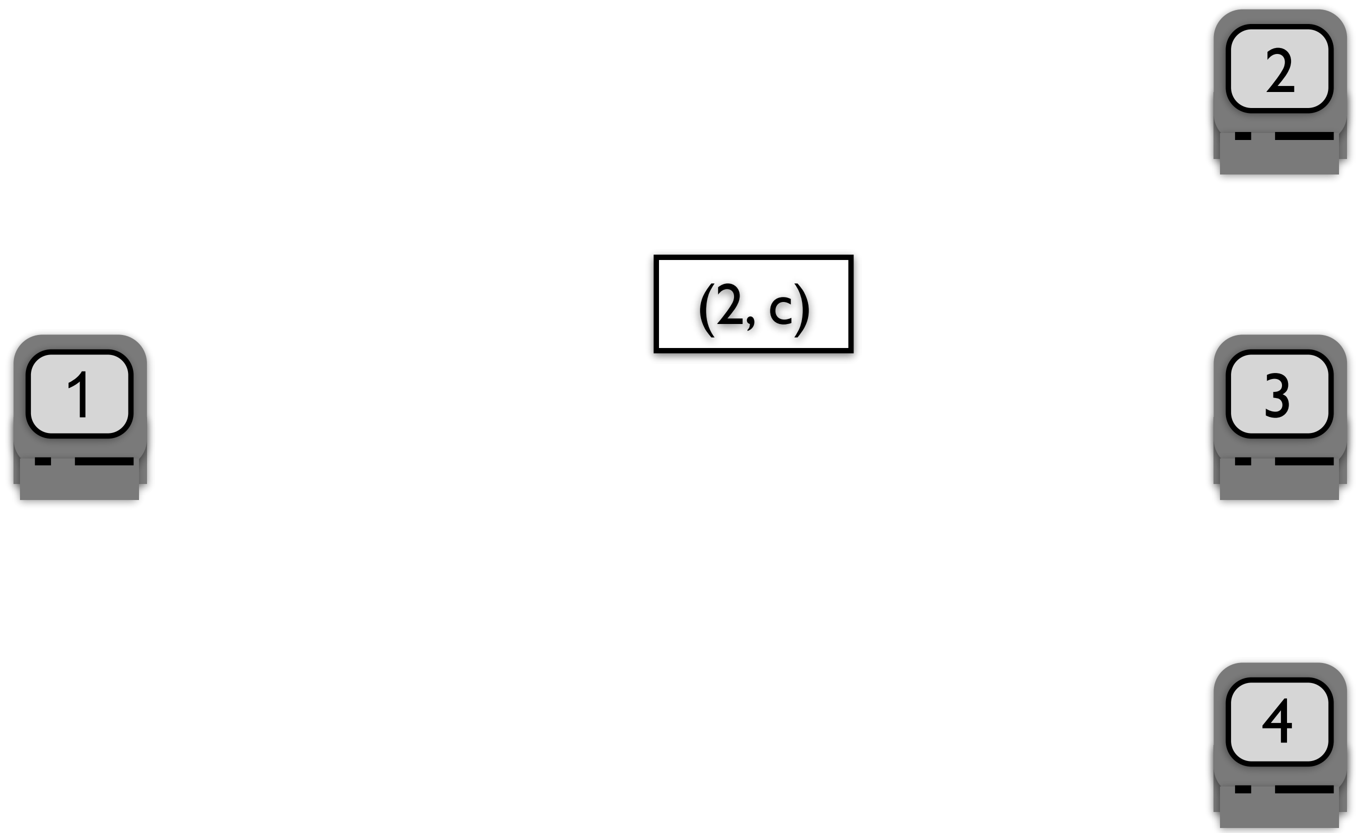

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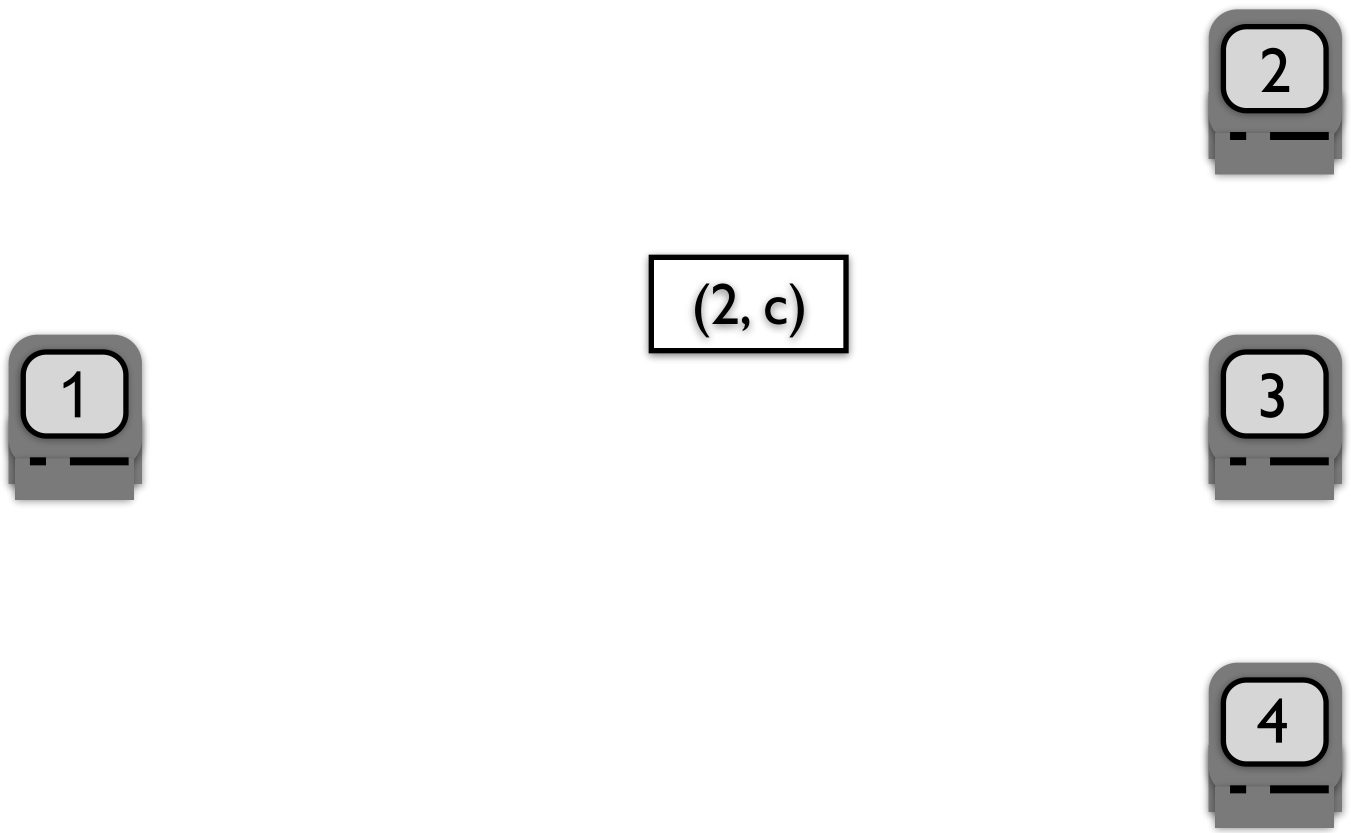

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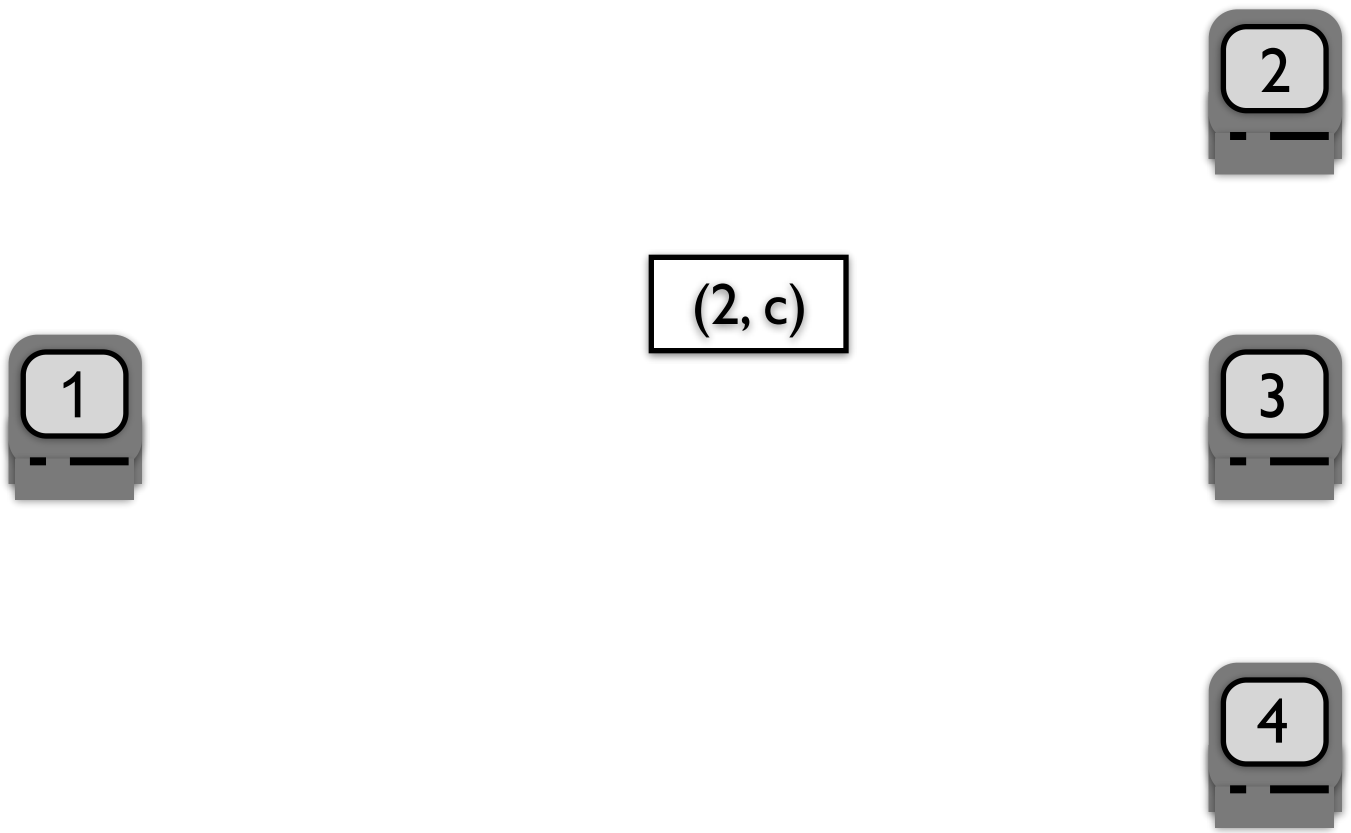

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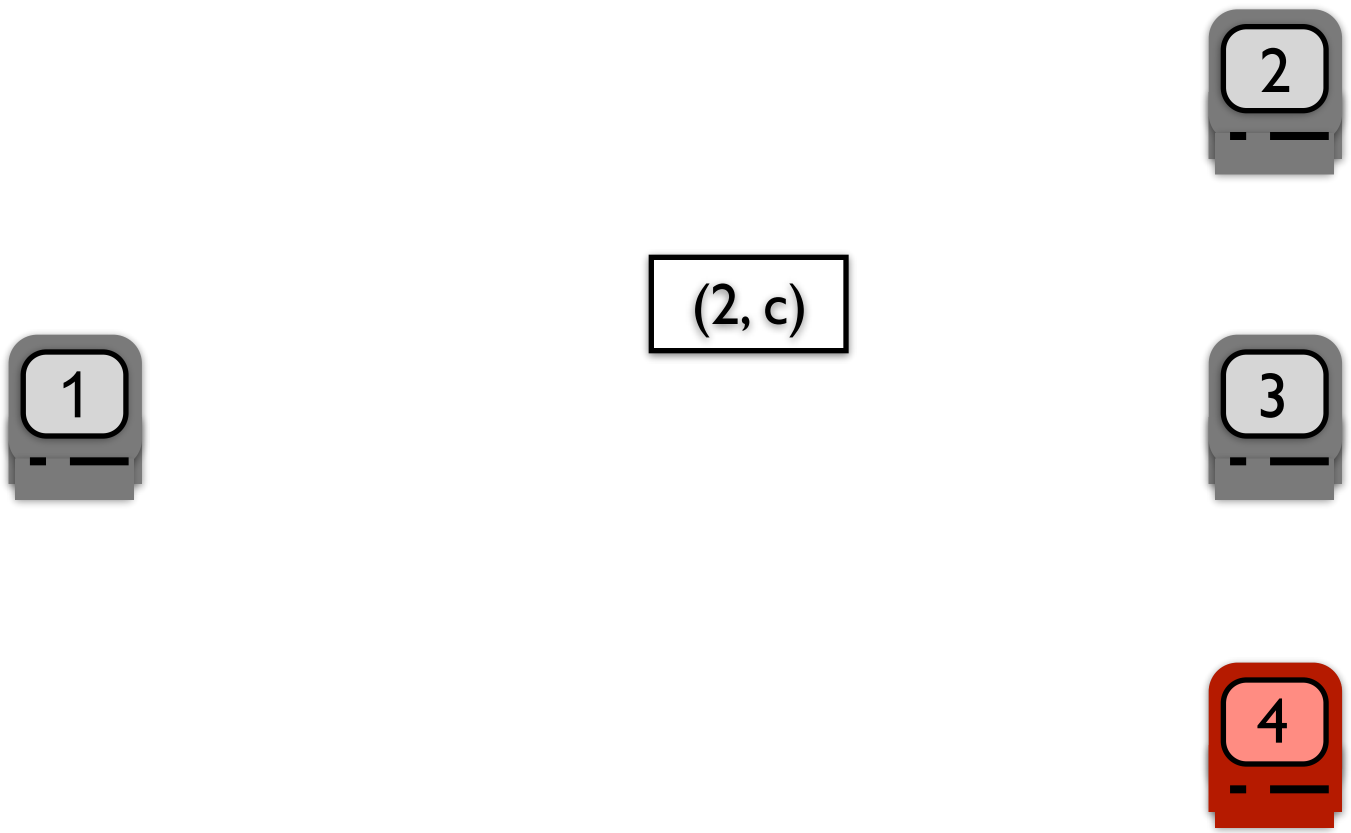

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Reliable Broadcast

The Equivocation Issue

Call it "broadcast" but…

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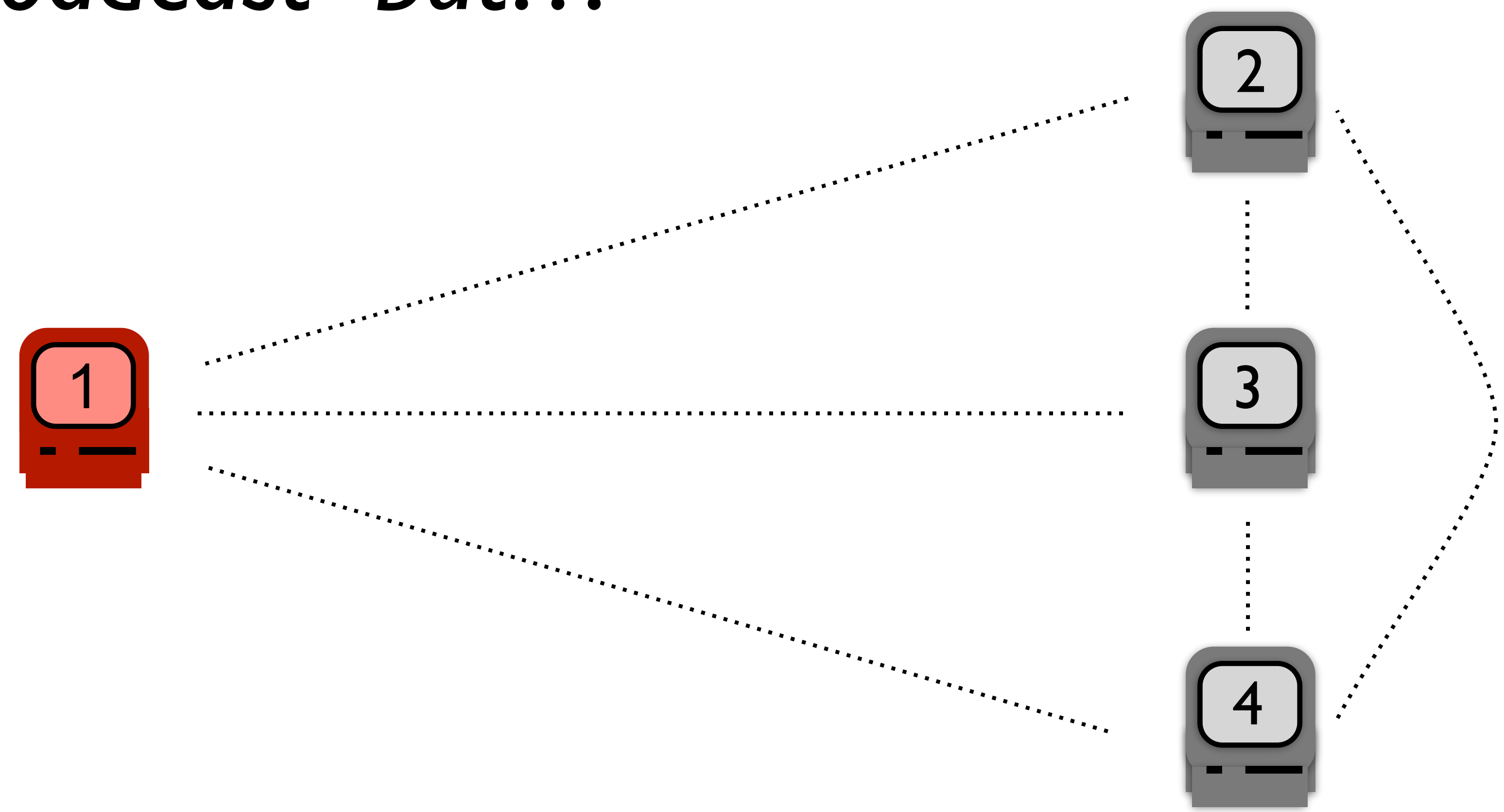

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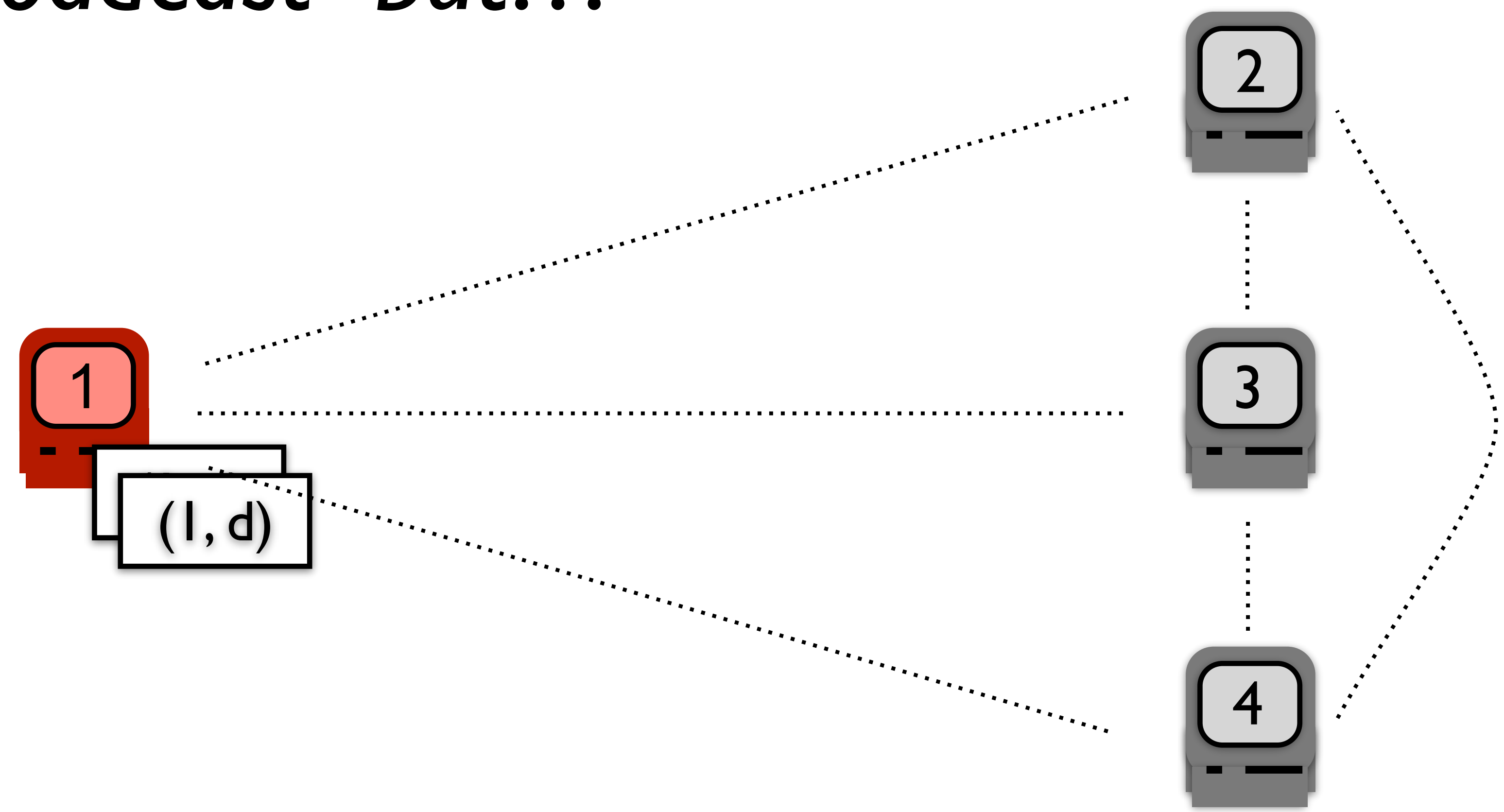

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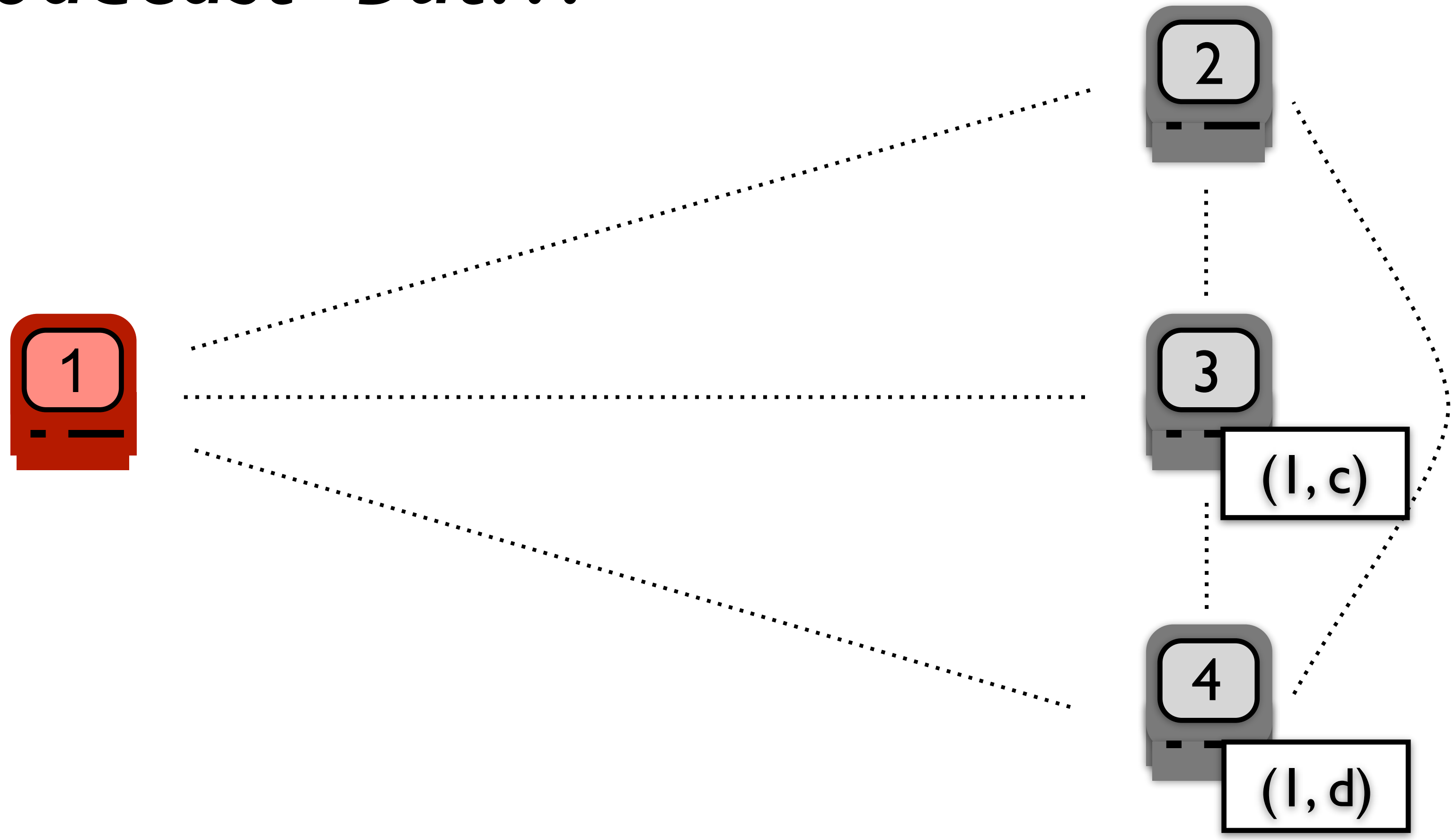

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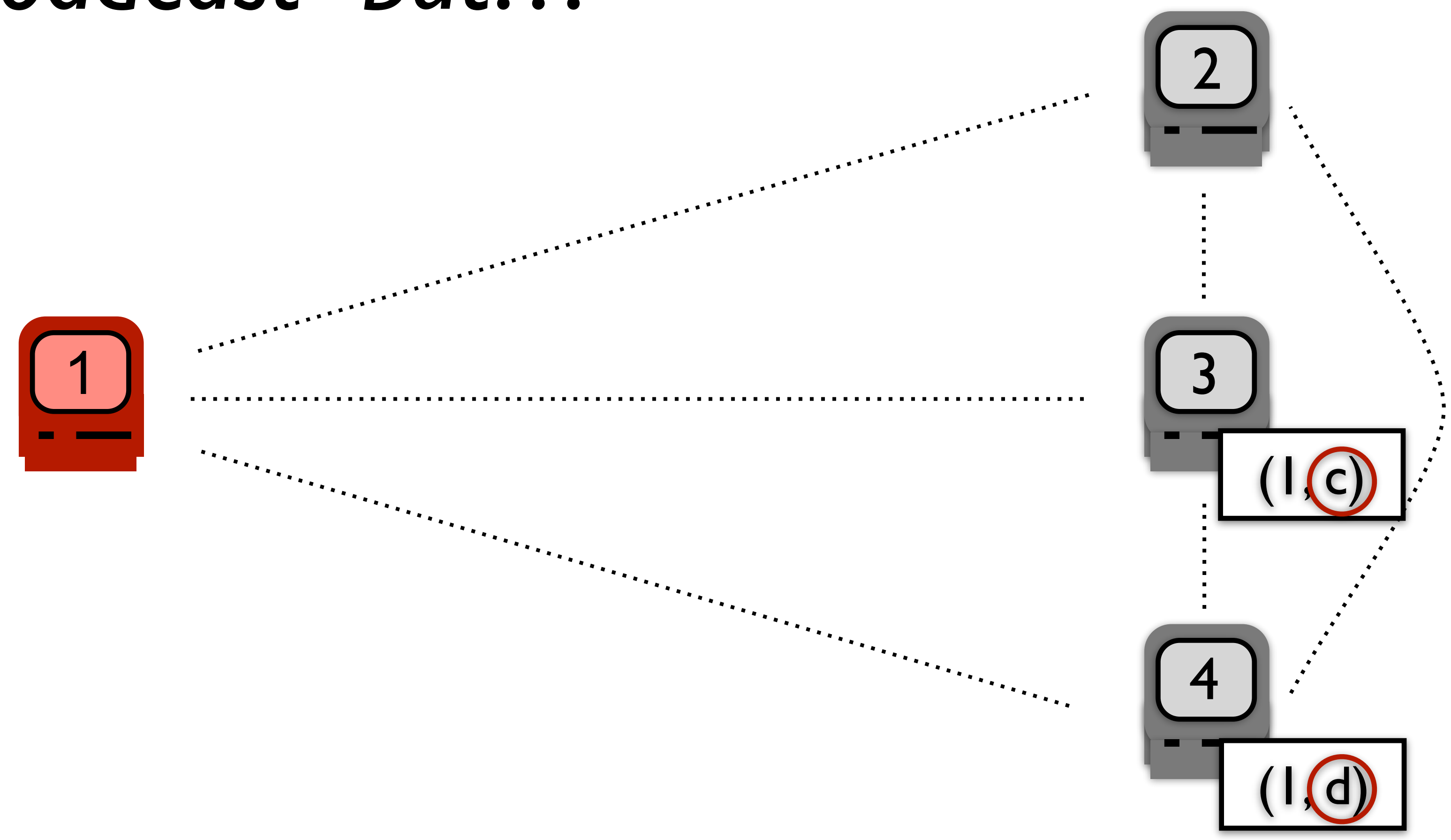

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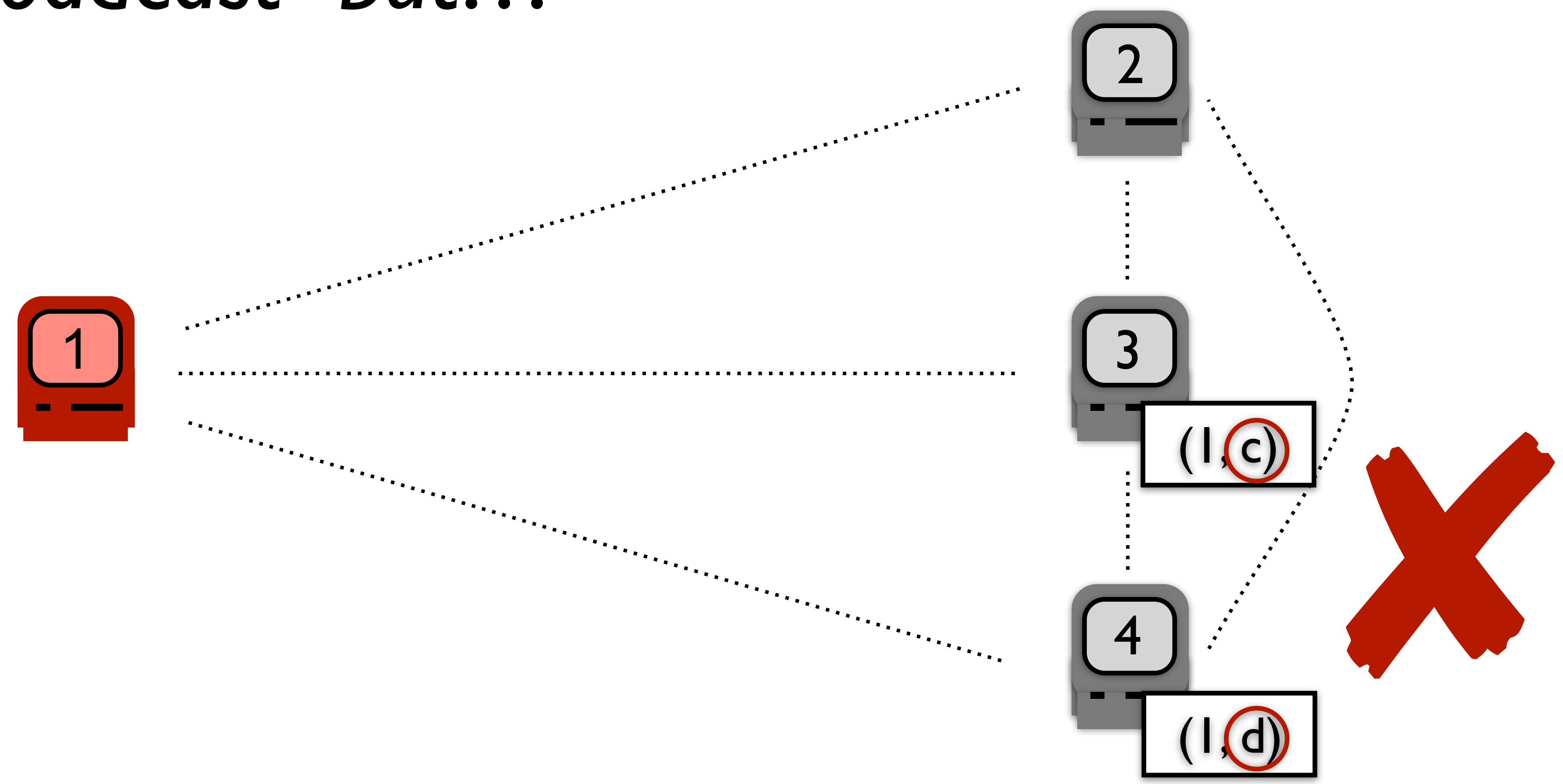

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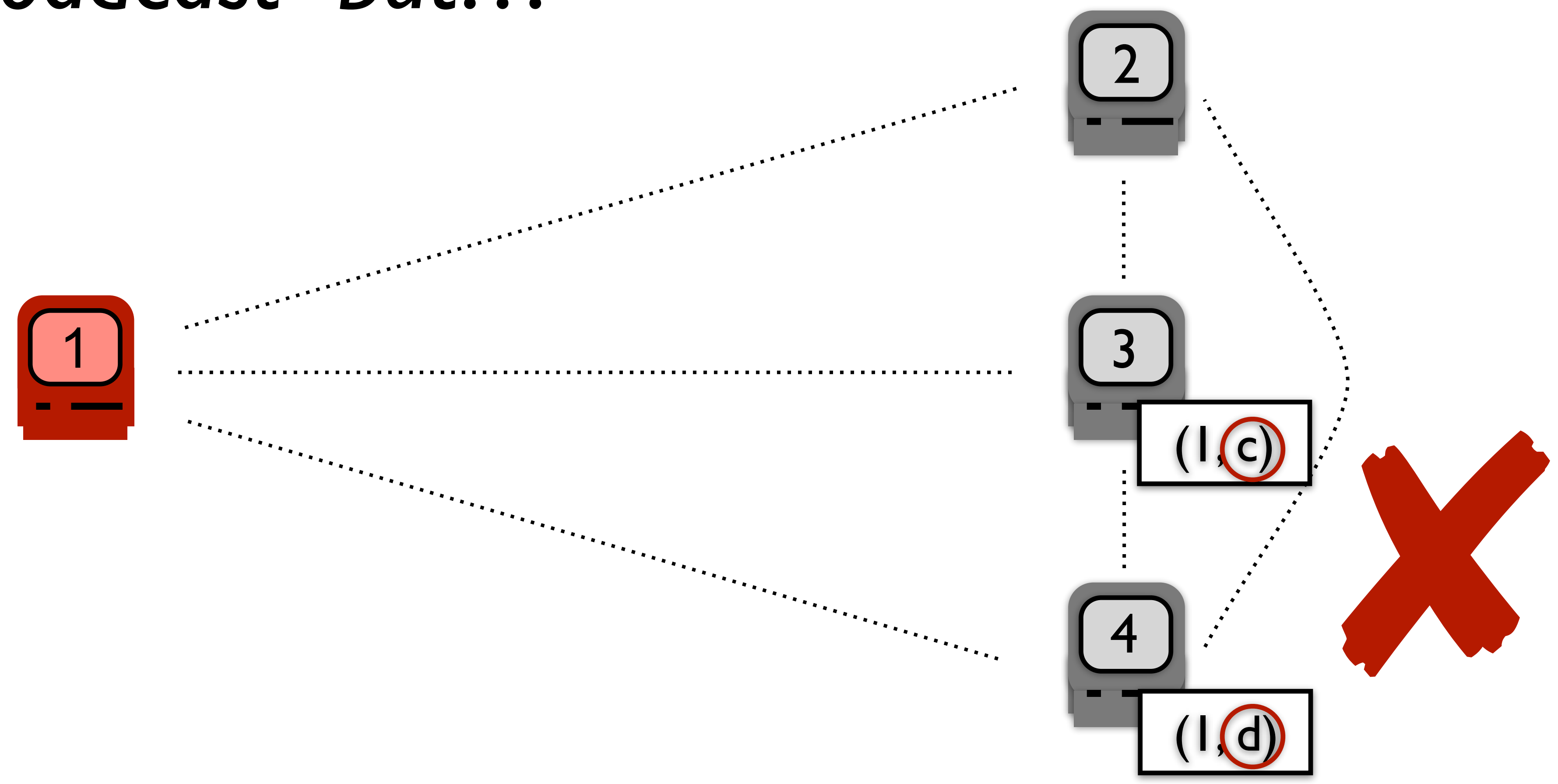

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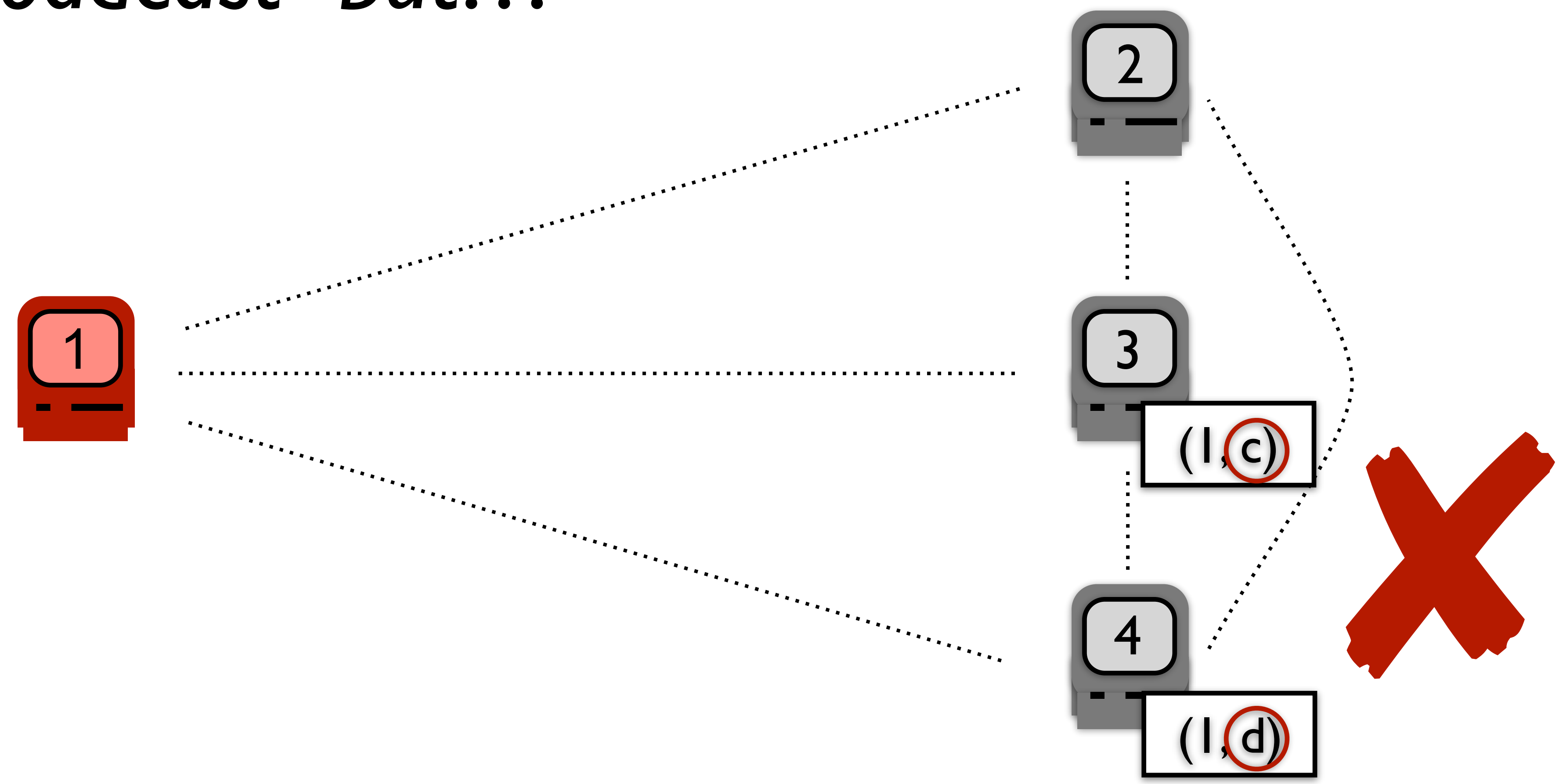

Using a primitive called Reliable Broadcast

Message contents are consistent

Full-Information Protocols

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Protocol = distributed algorithm

$s \leftarrow I_i$
for $r : 1 \rightarrow R$ **do**
    Send s via reliable broadcast
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Modeling Tasks

Tasks and Simplicial Complexes

Tasks modeled as simplicial complexes

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Solvability in terms of topological properties

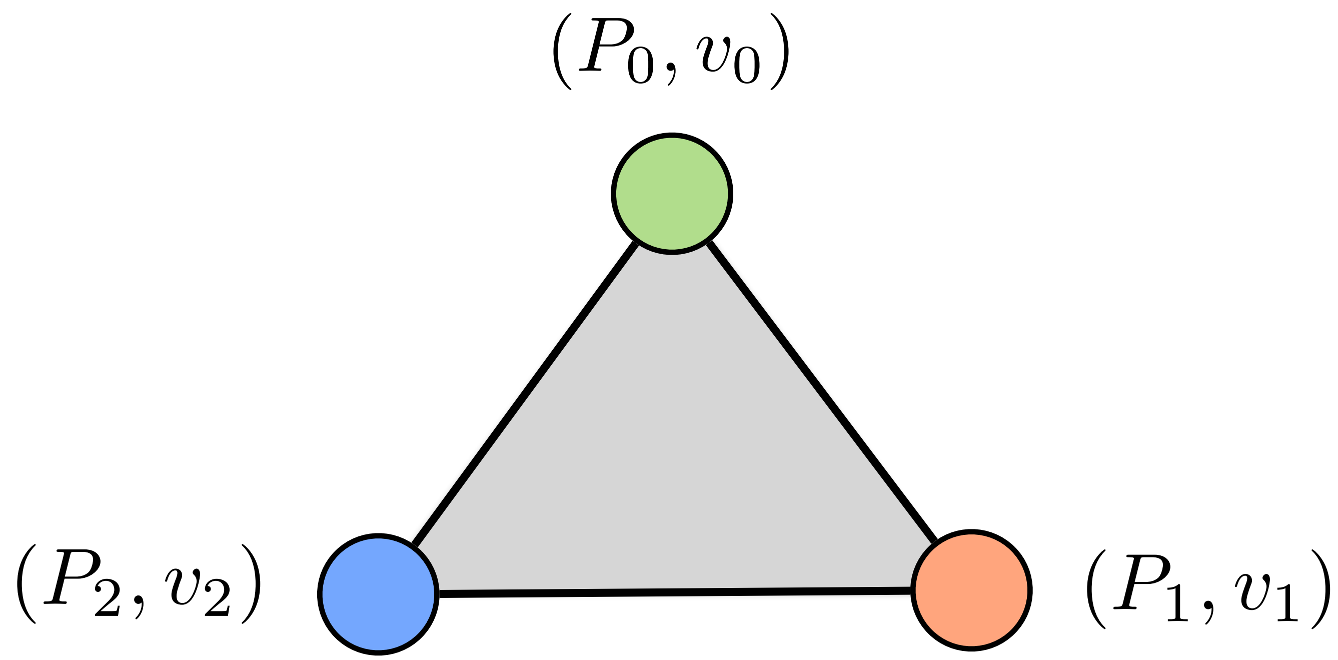
Tasks and Simplicial Complexes

Tasks modeled as simplicial complexes

Solvability in terms of topological properties

Let's start with crash failures

Many Simplexes → Simplicial Complex



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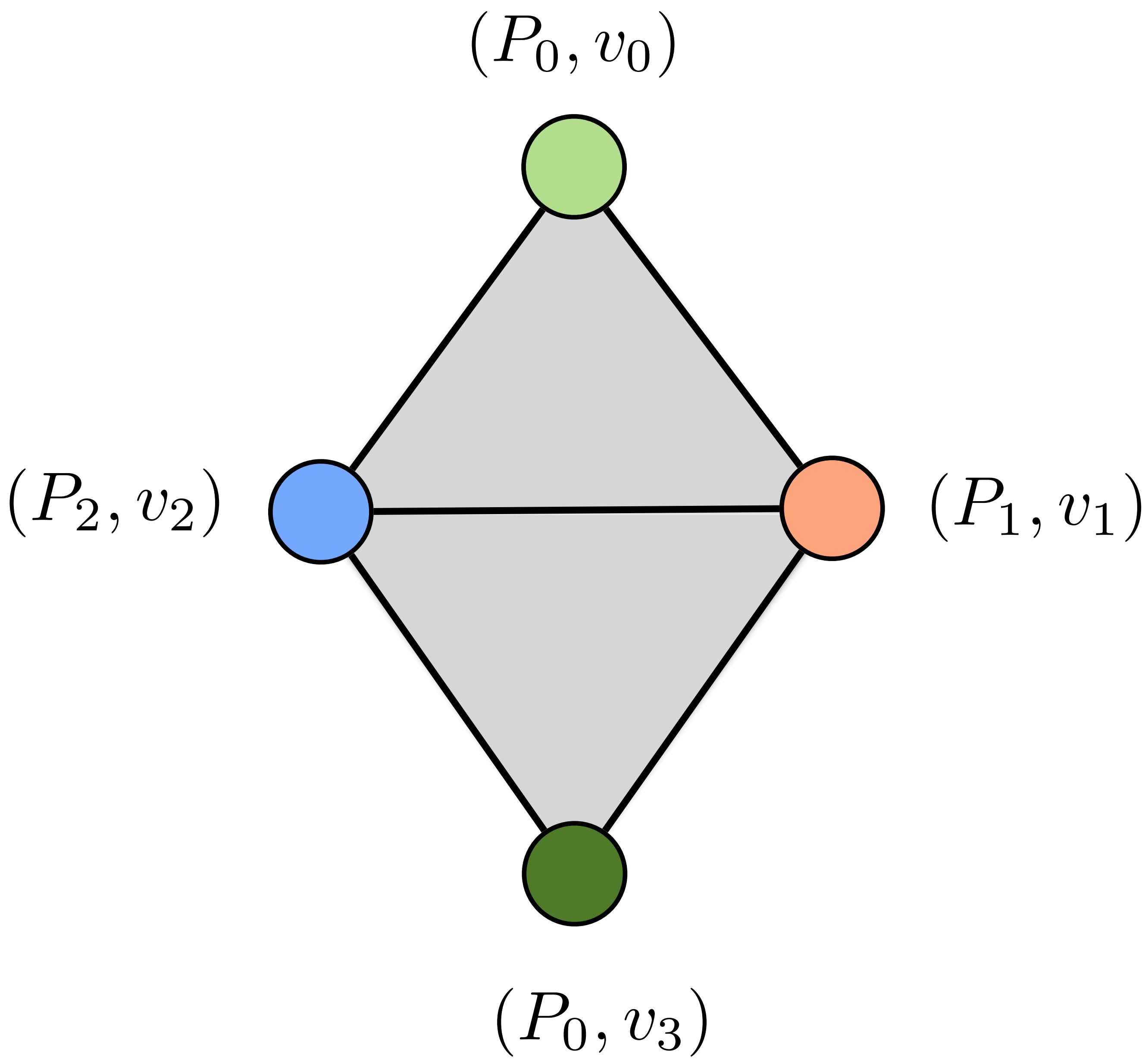

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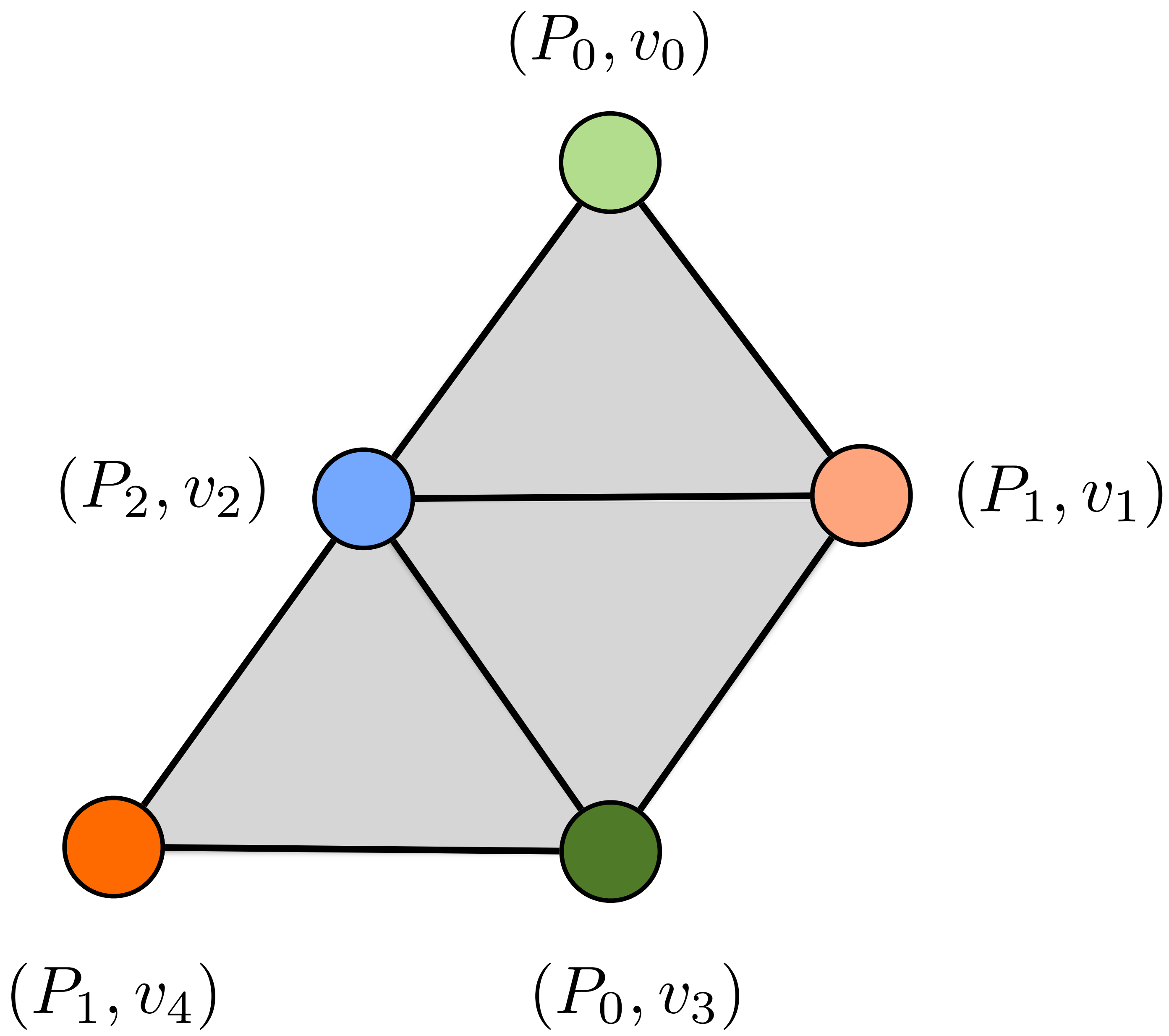

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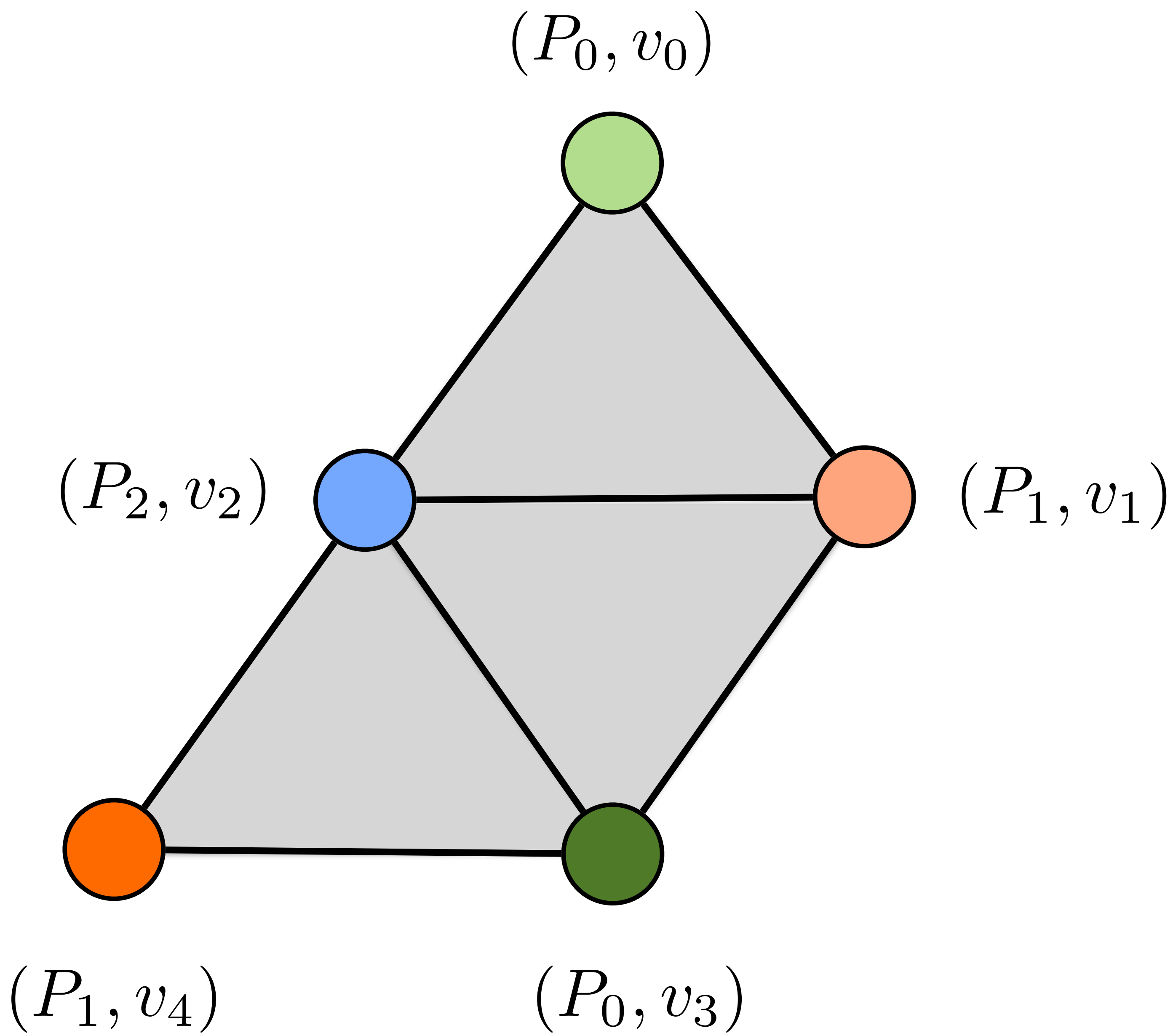

...

$\mathcal{I}$ (input complex)

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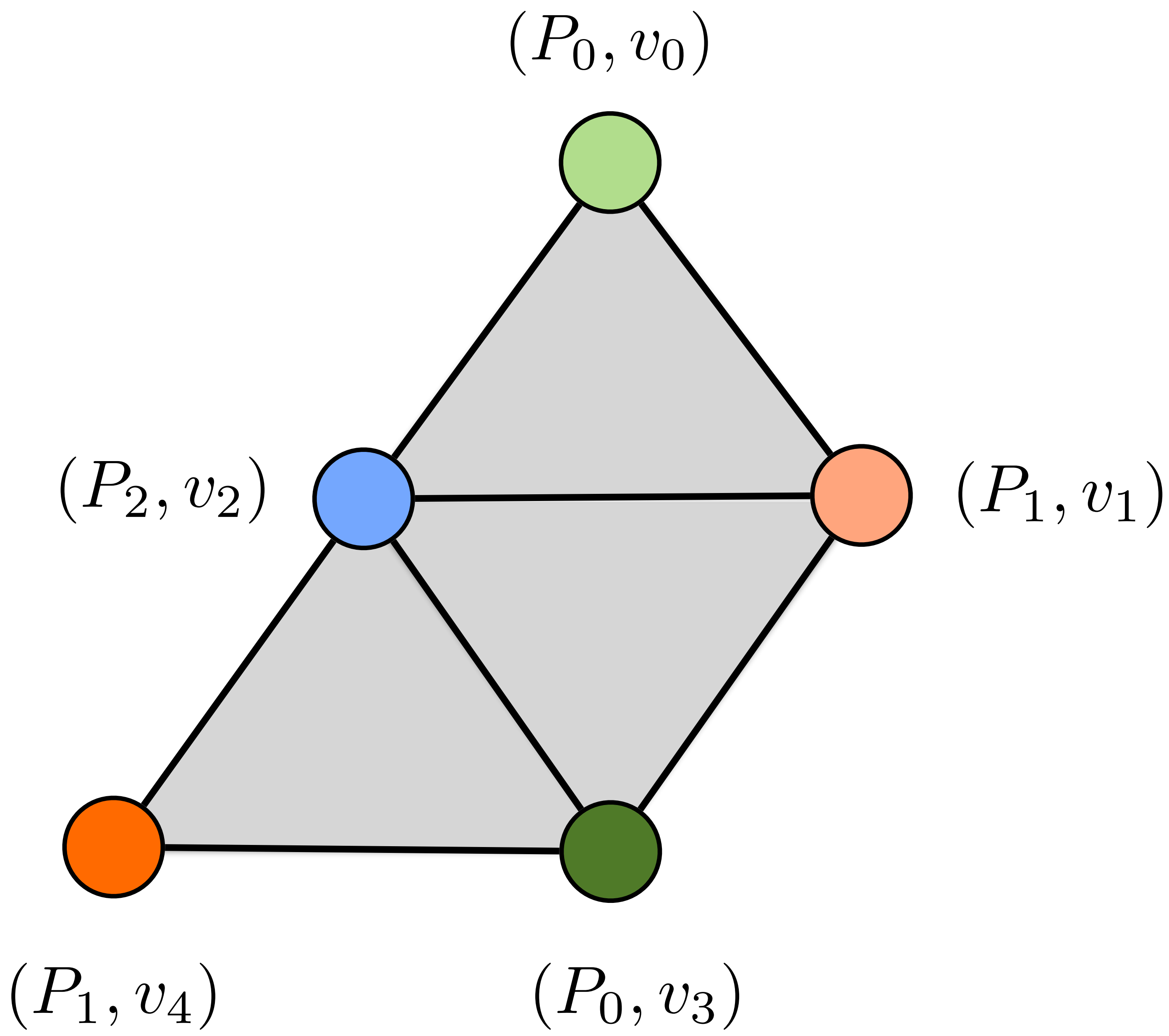

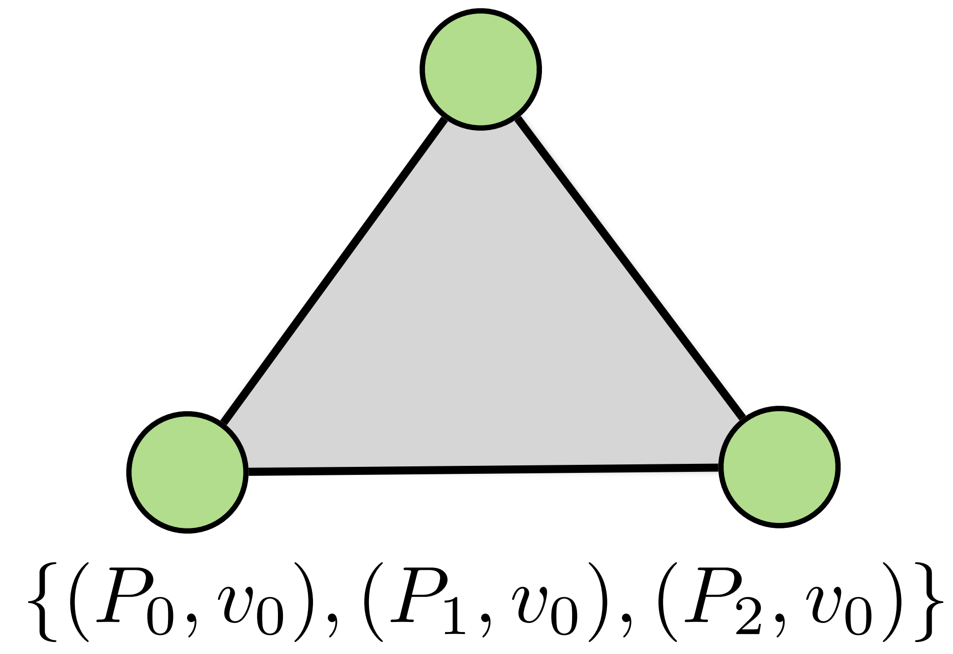

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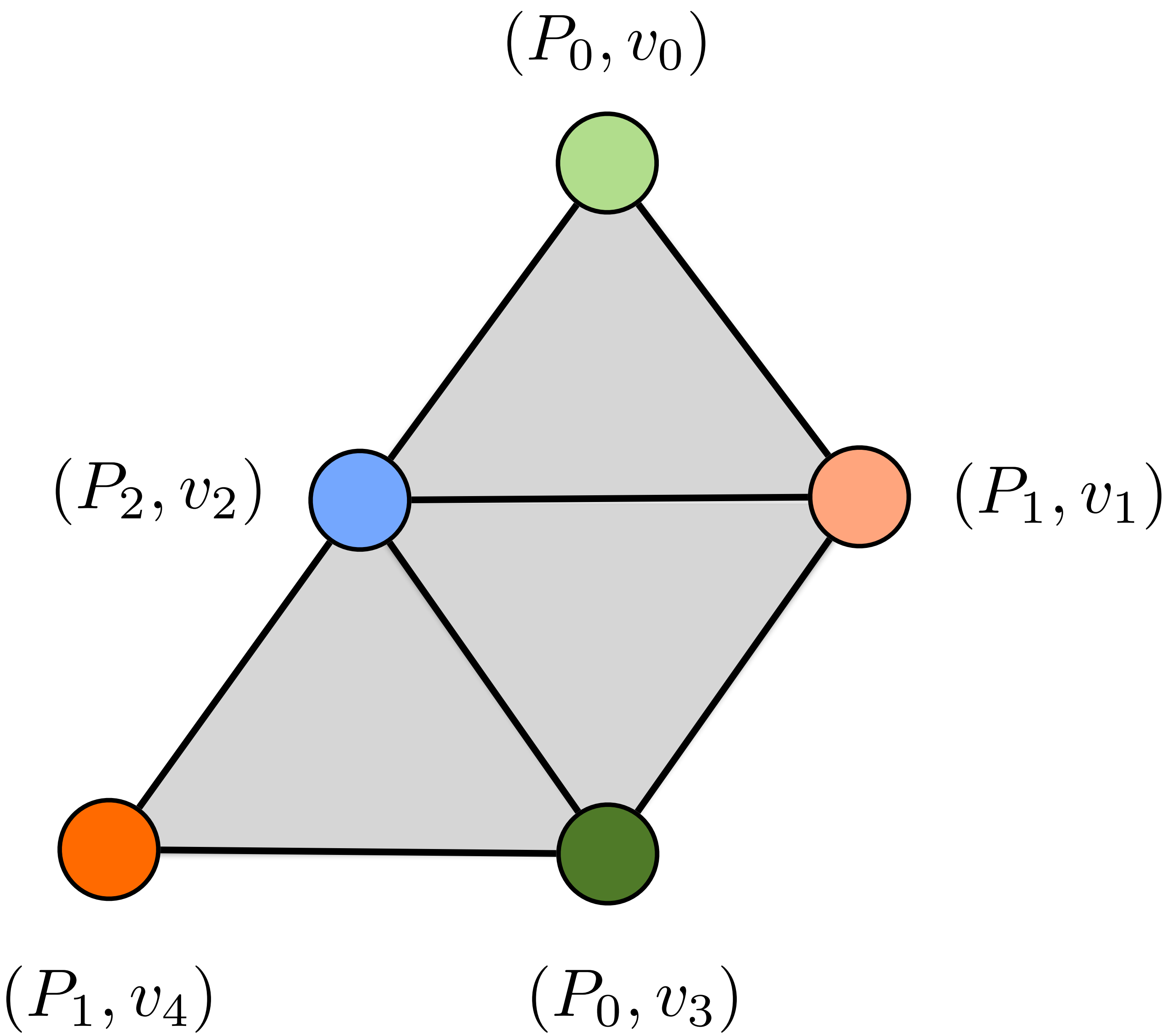

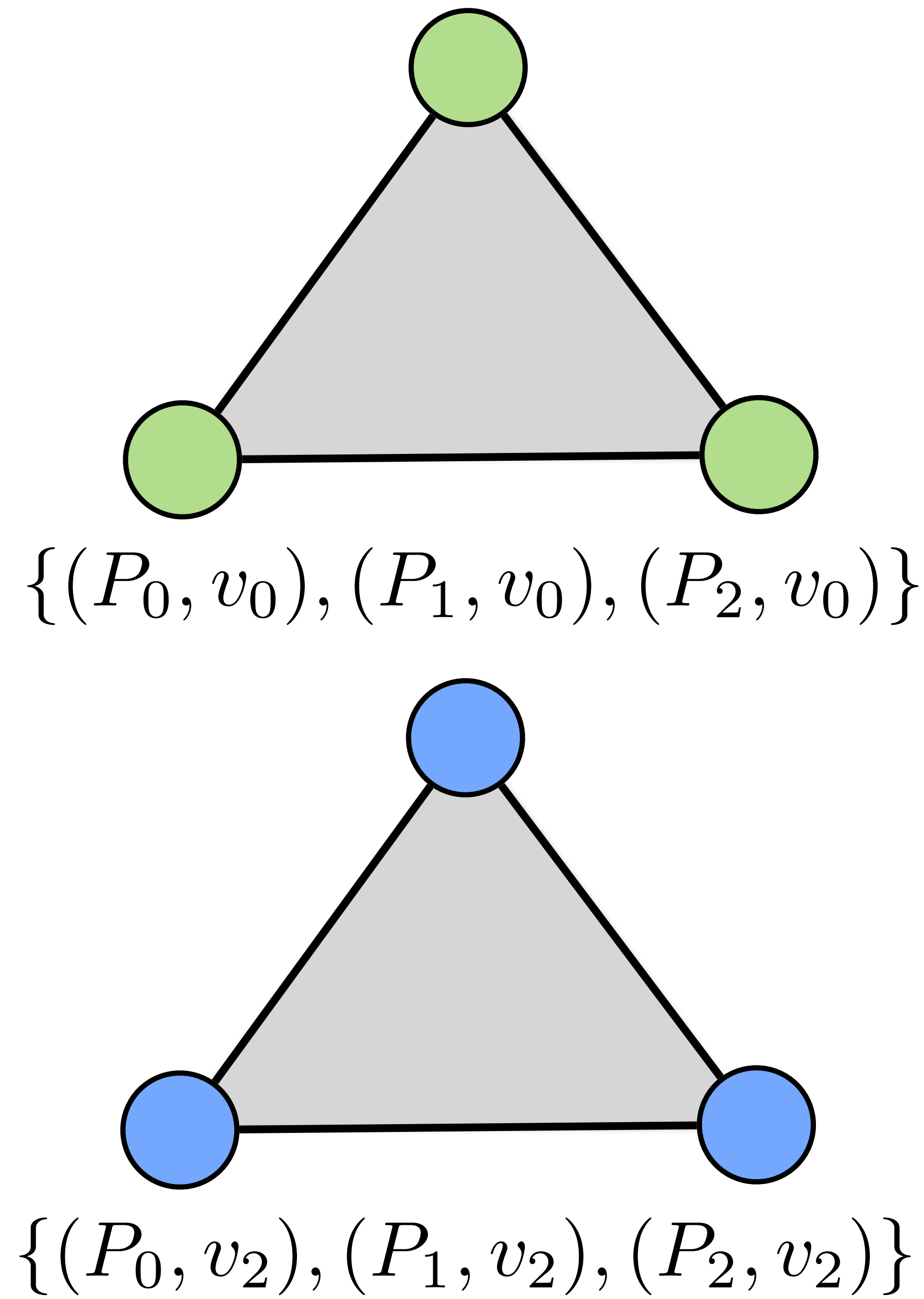

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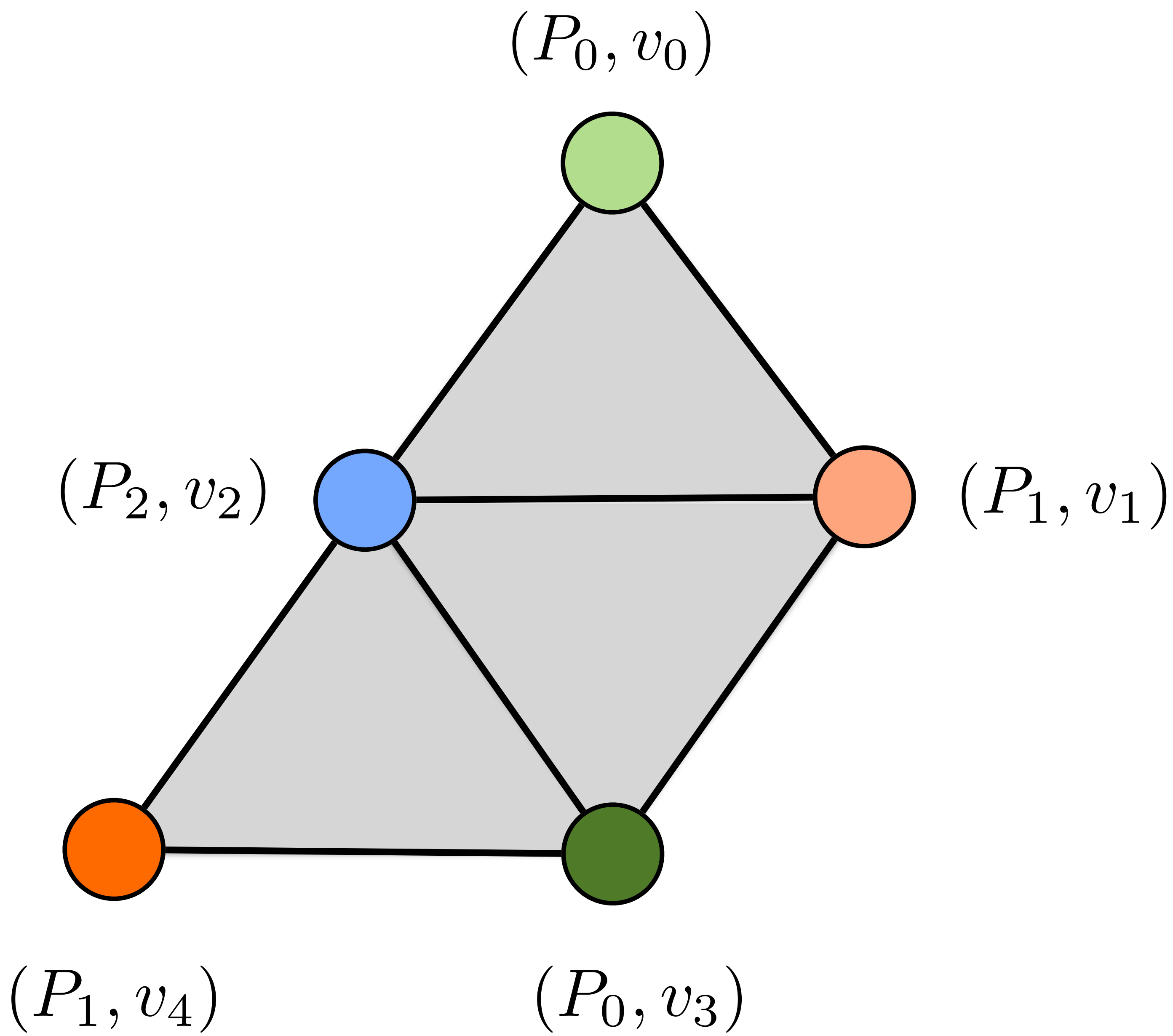

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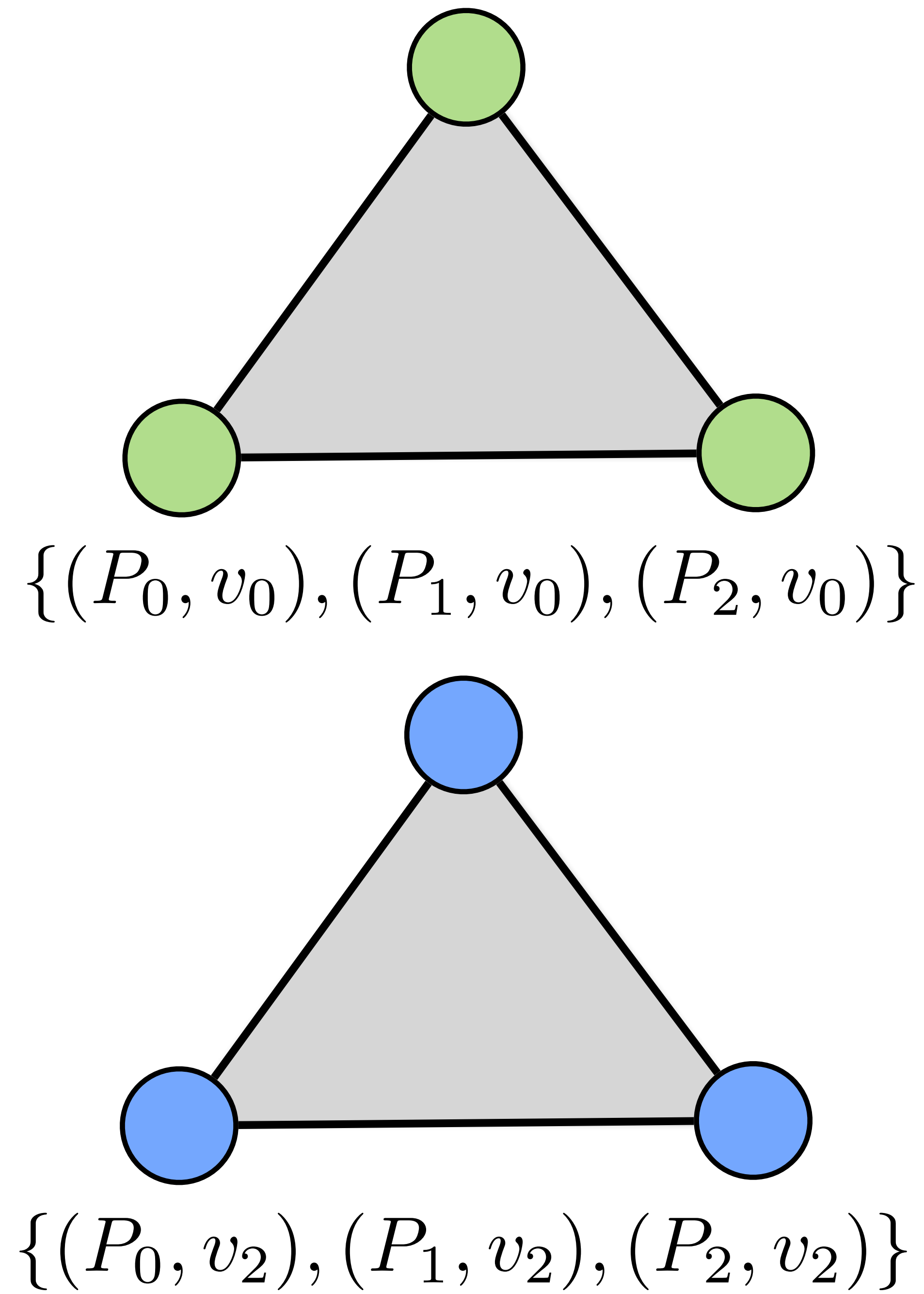

...

$\mathcal{O}$ (output complex)

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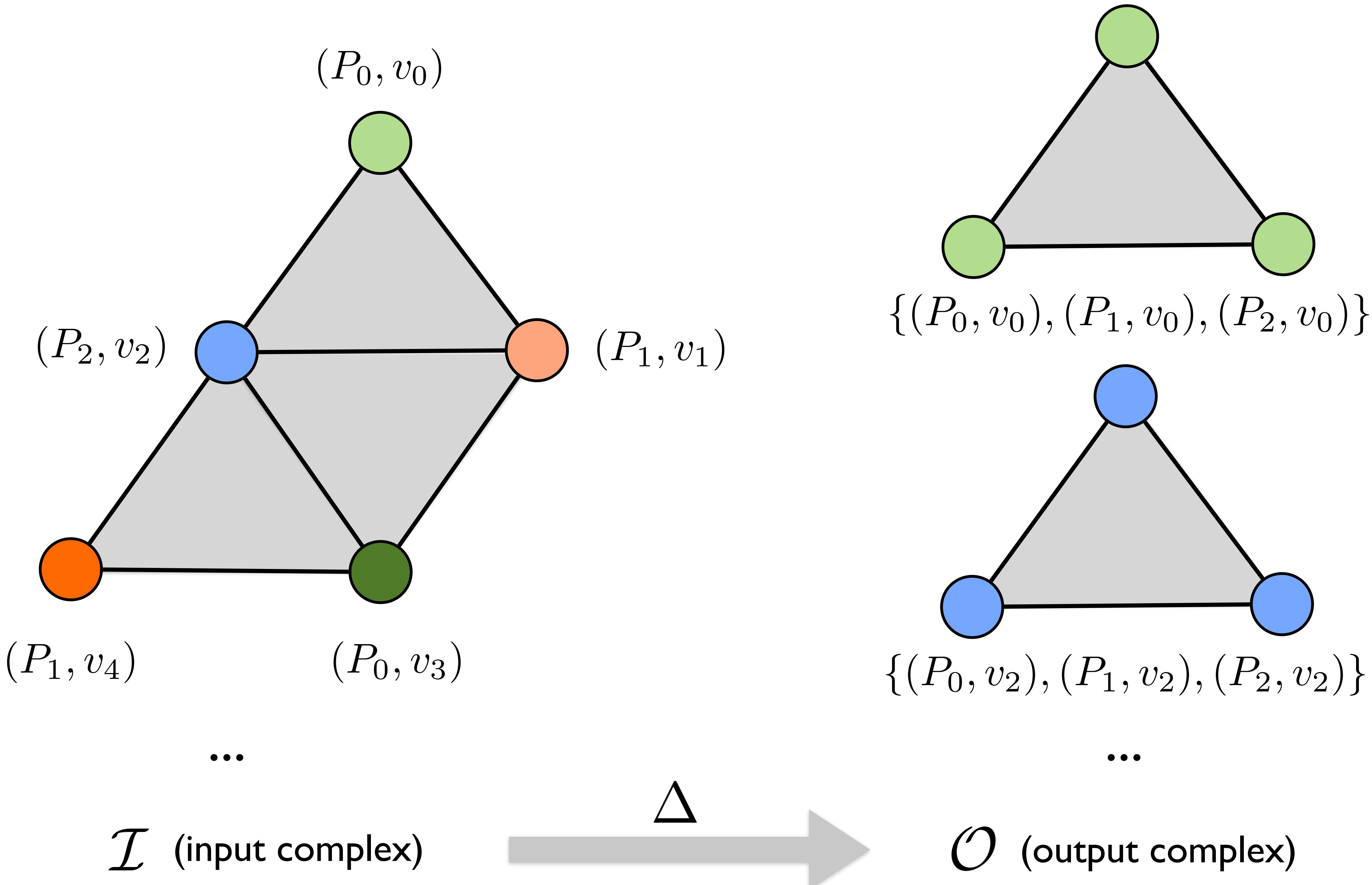
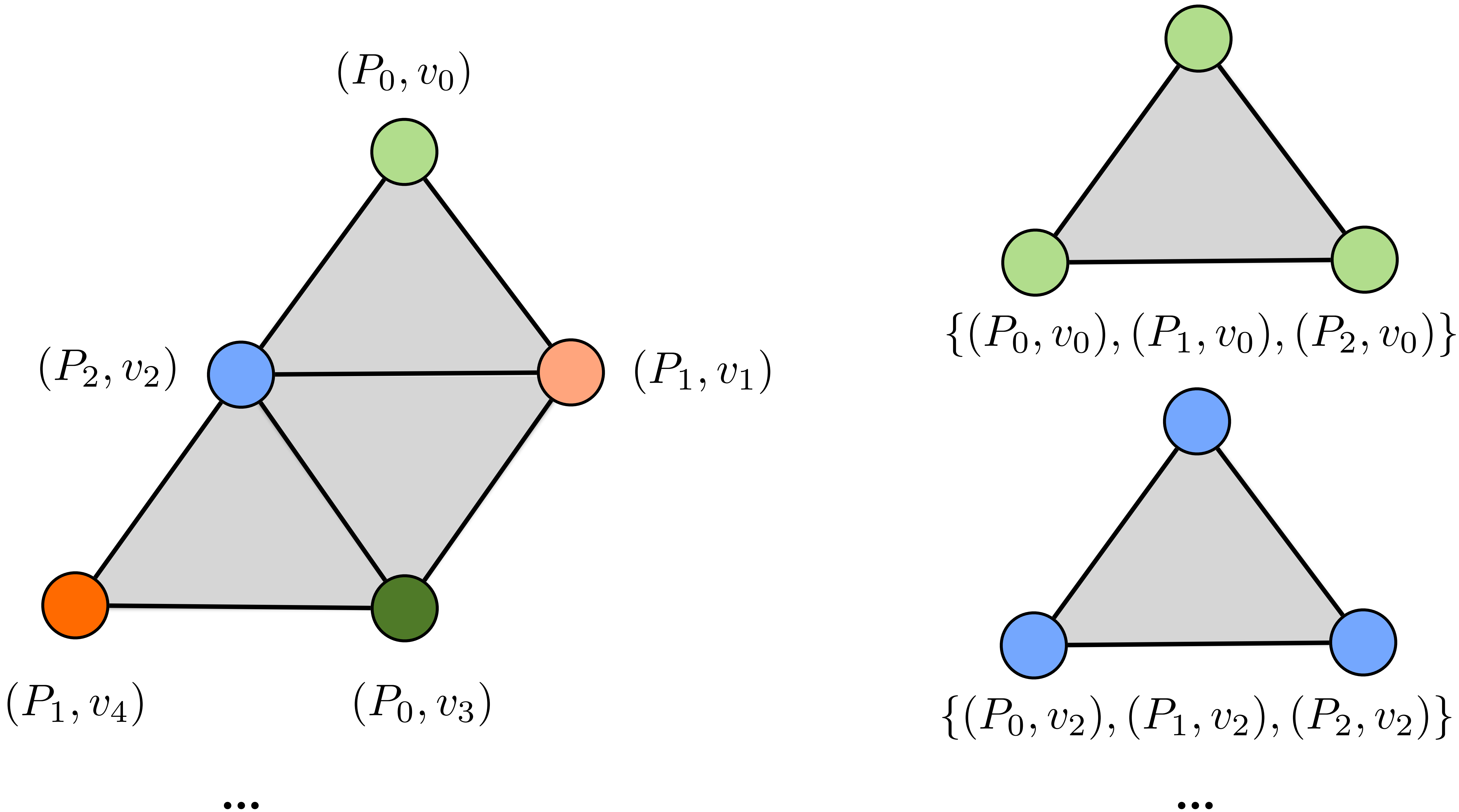


The map Δ



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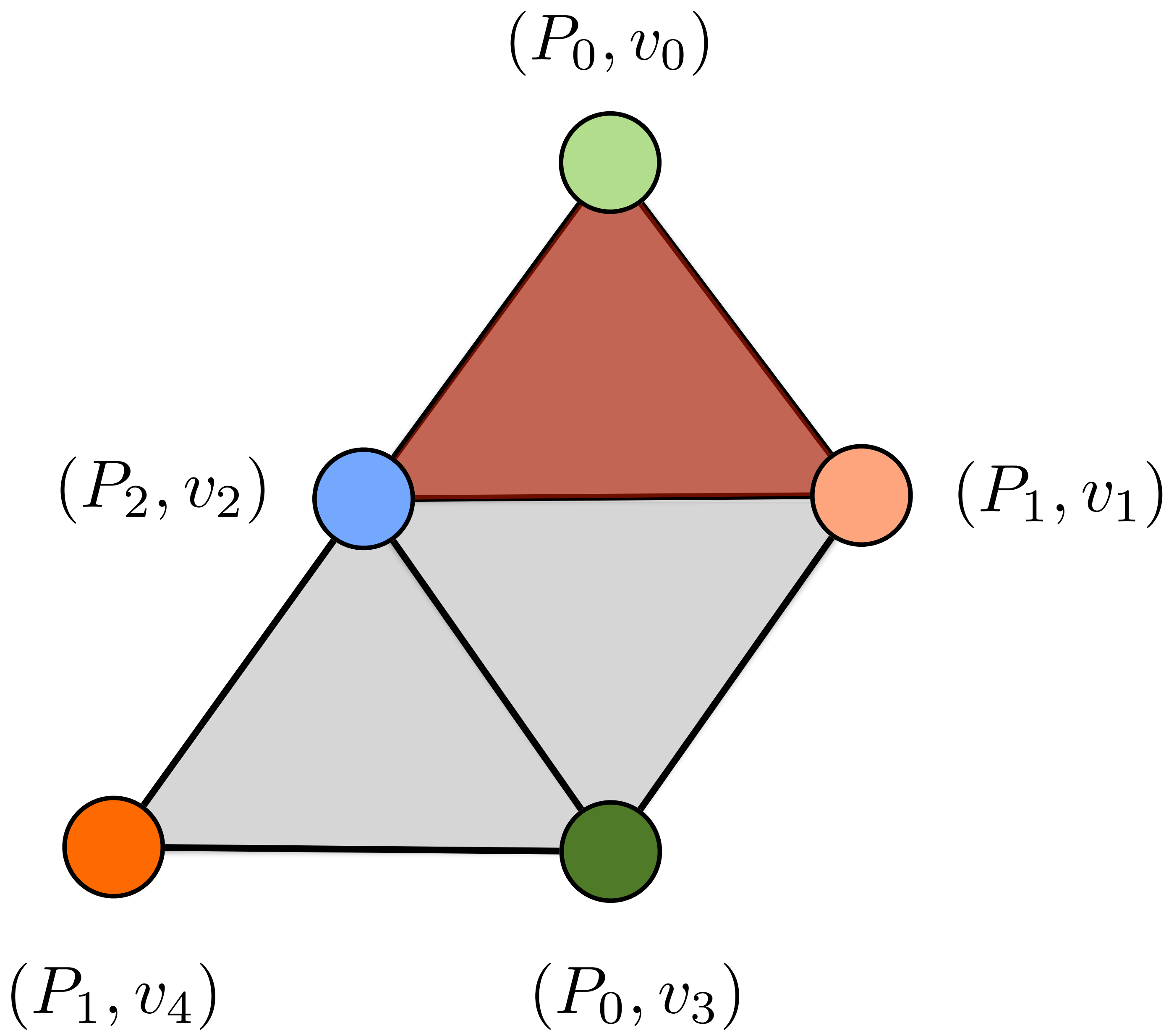

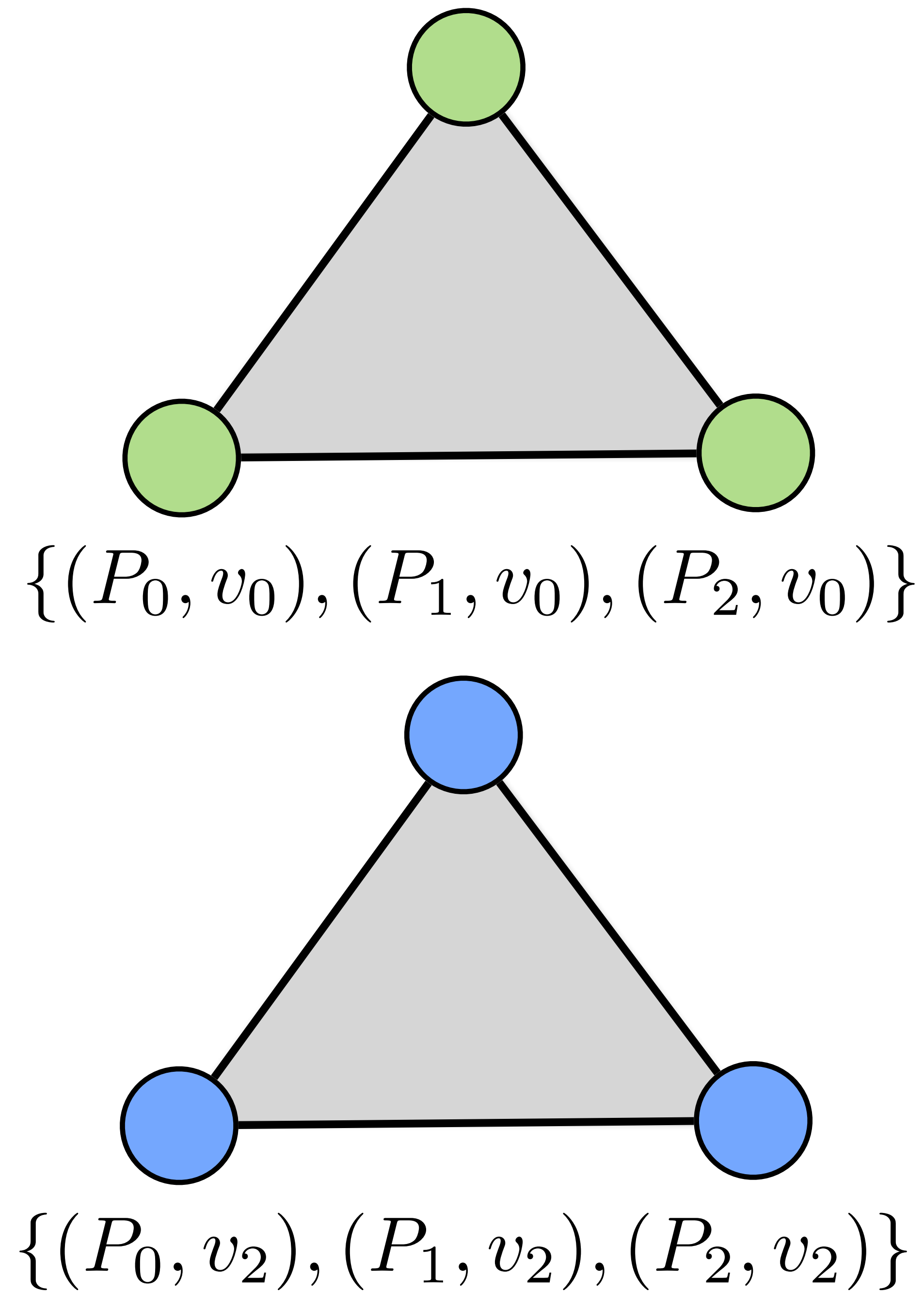

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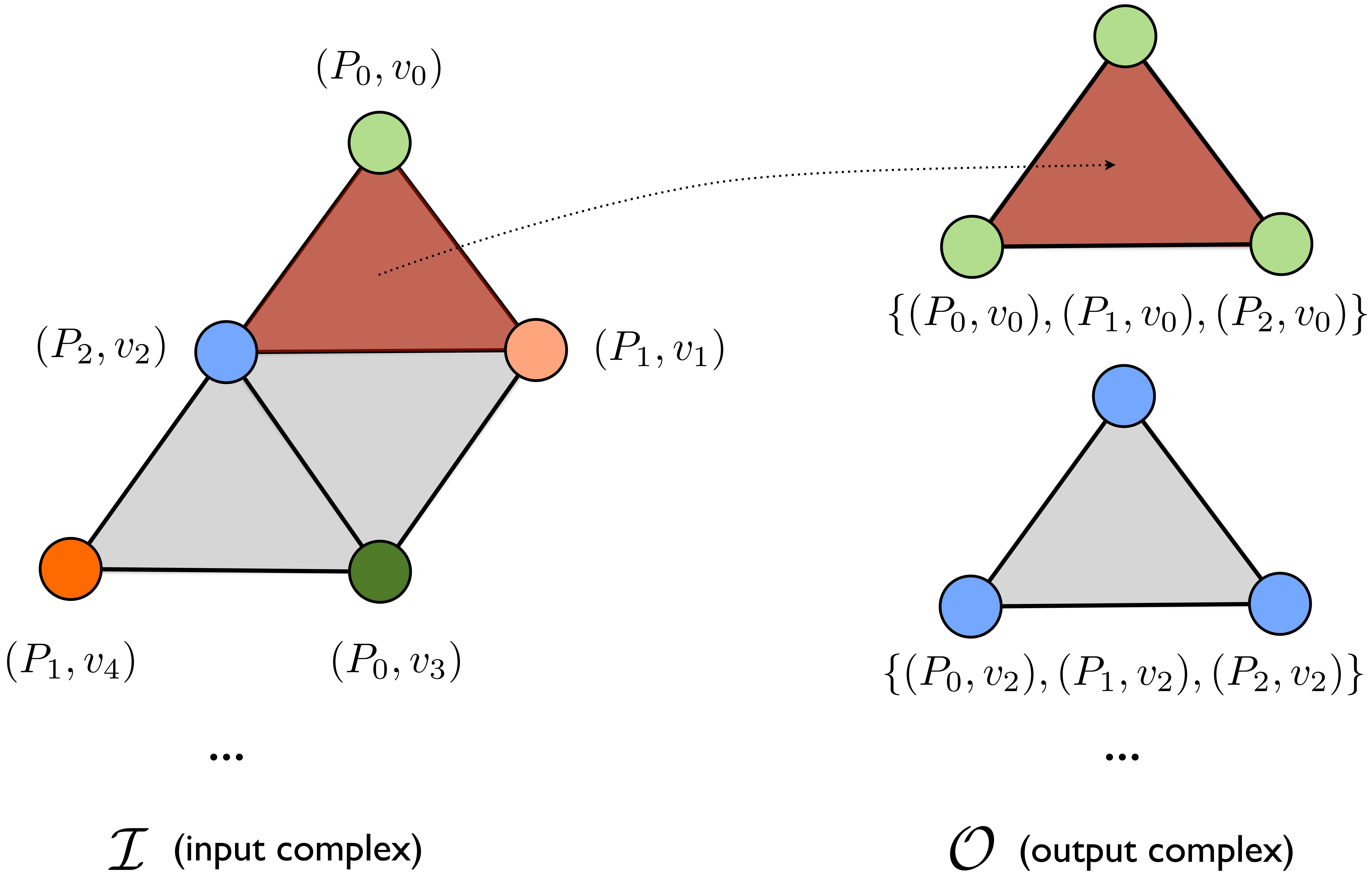

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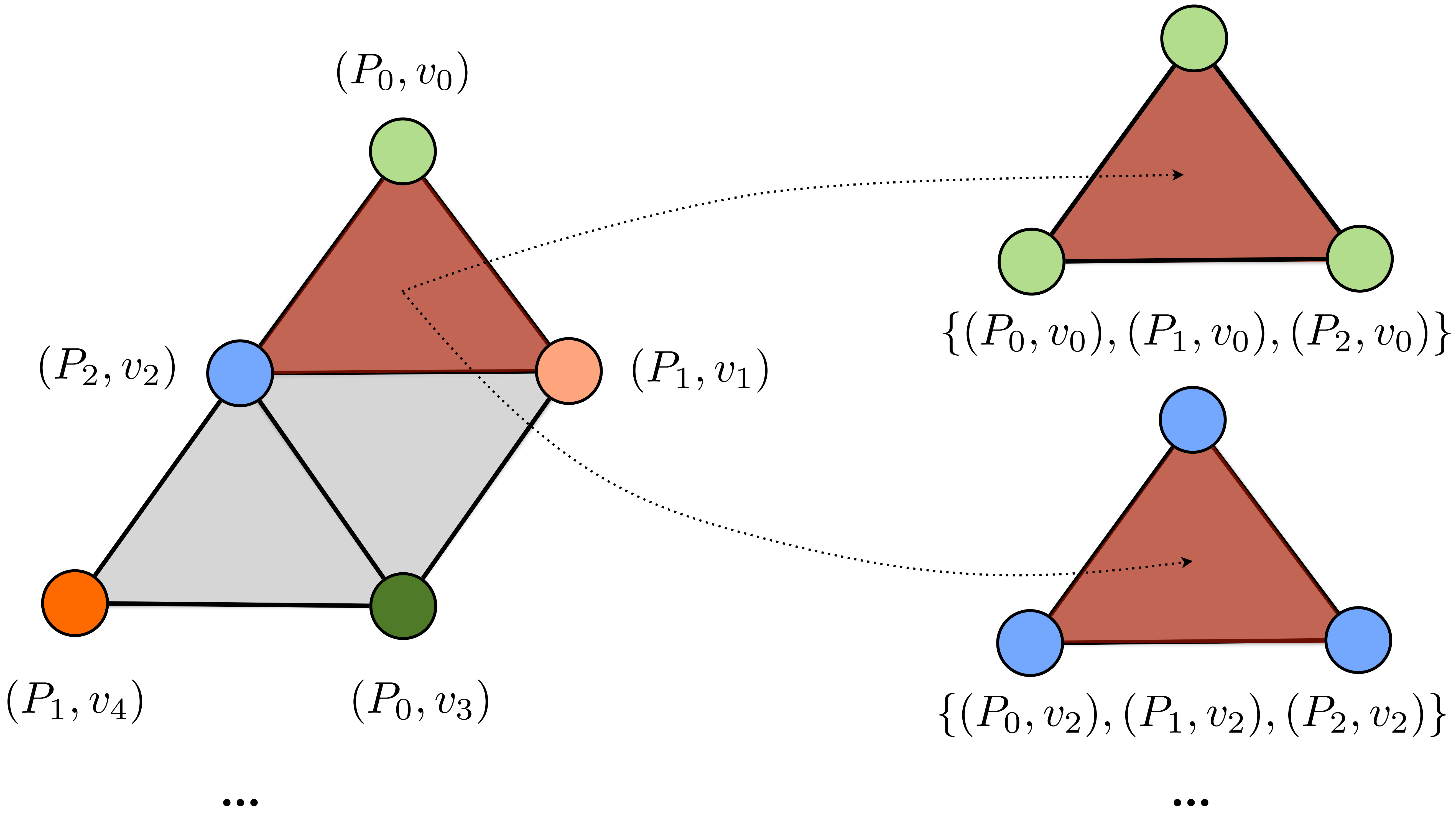

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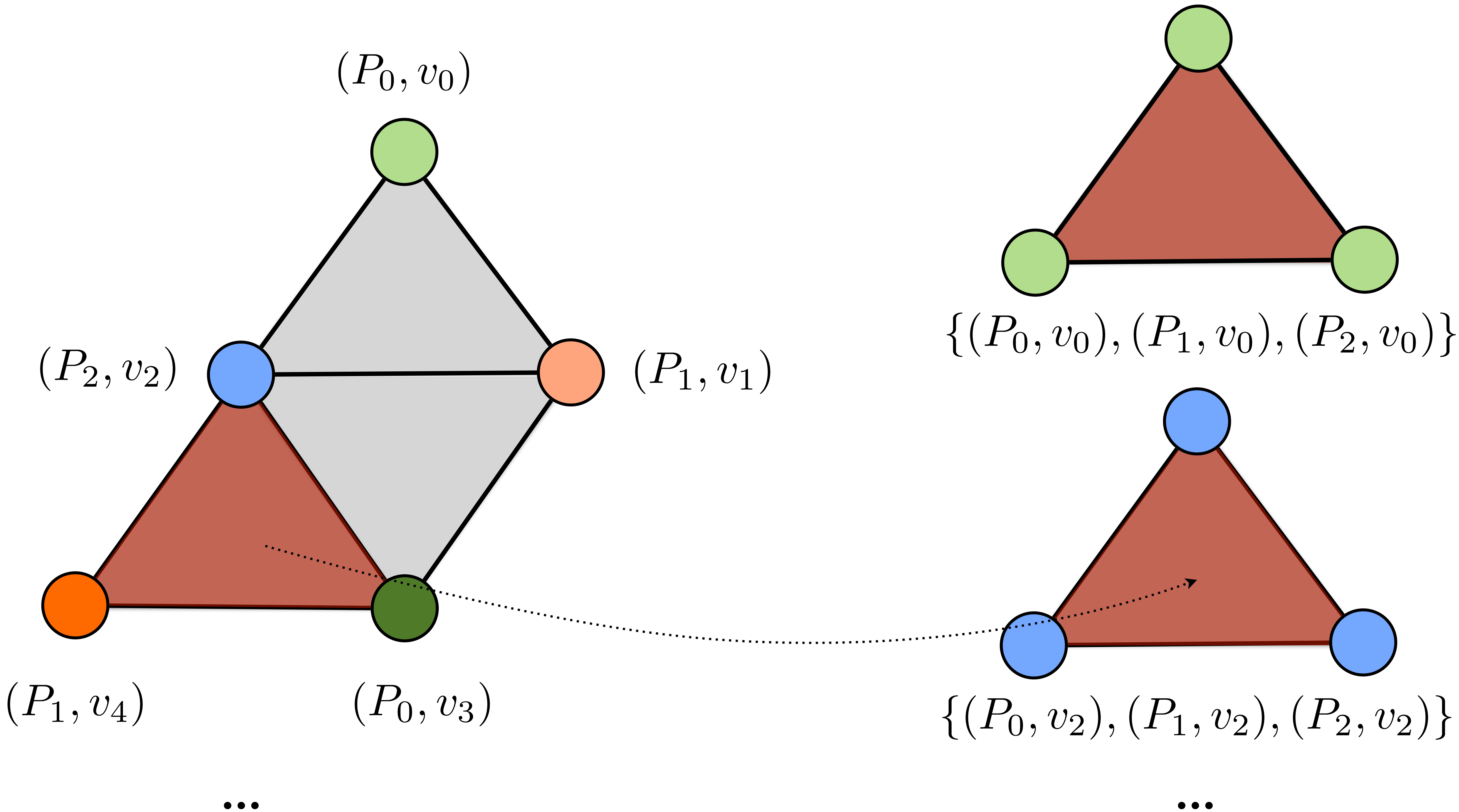

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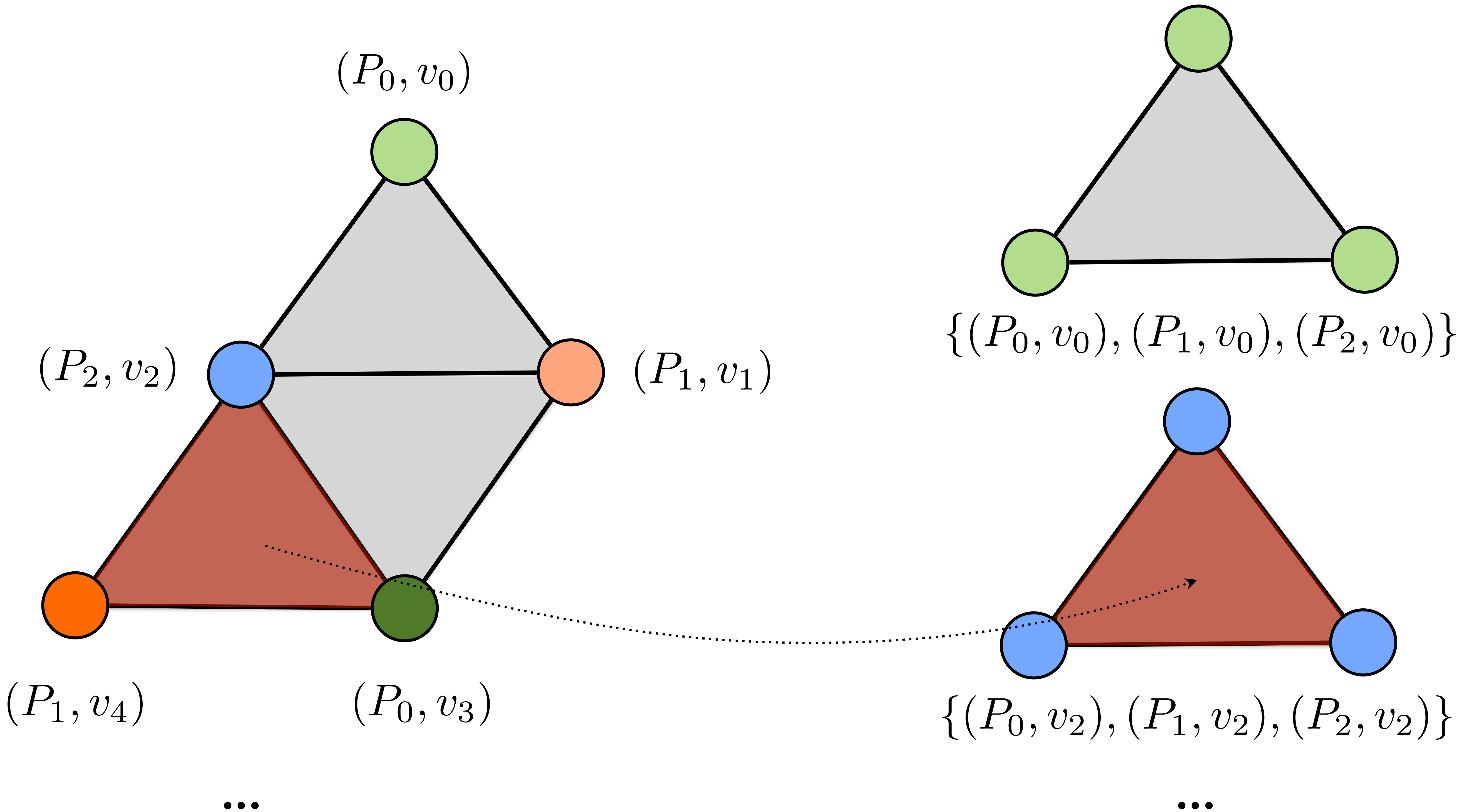

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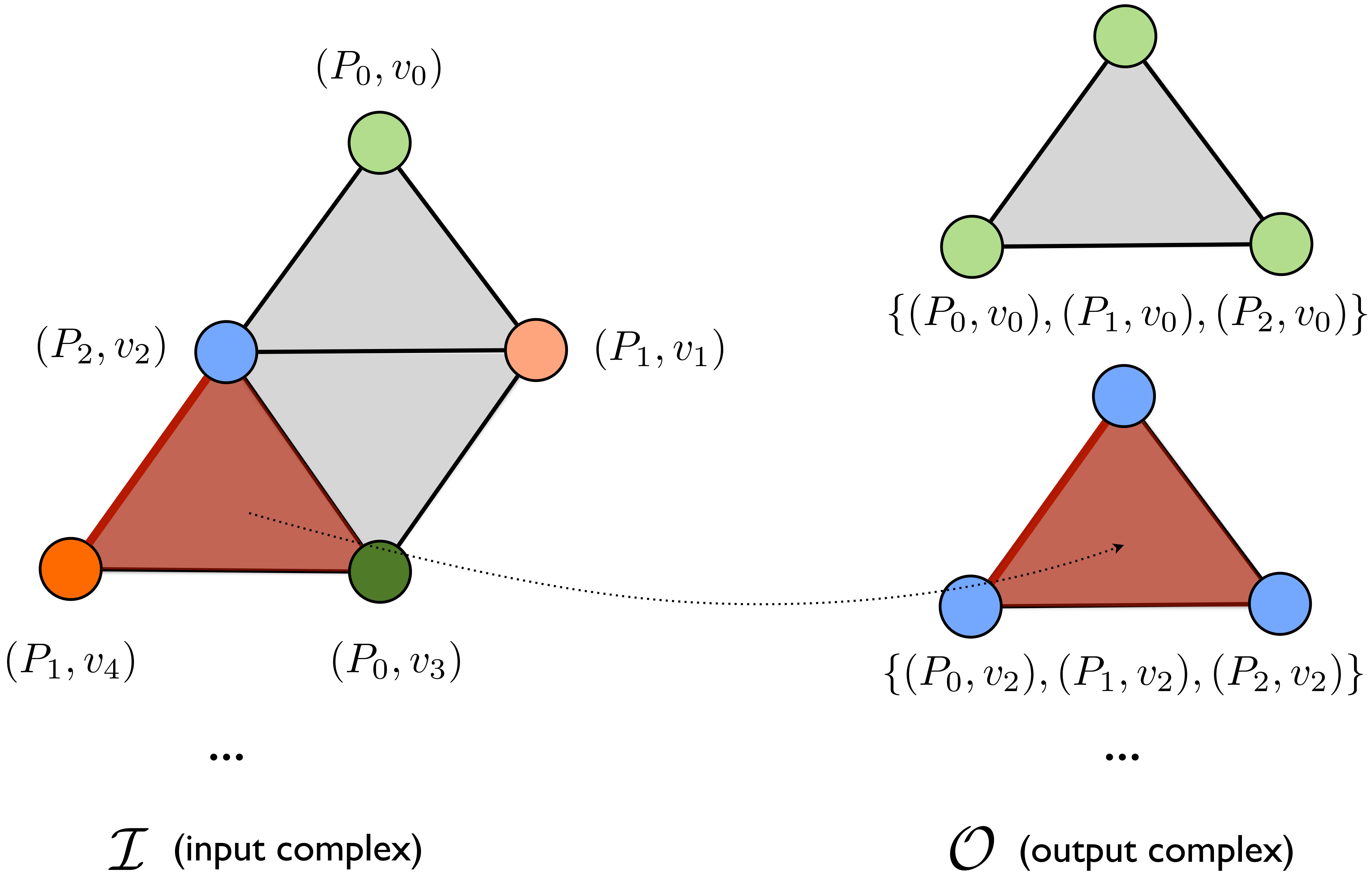

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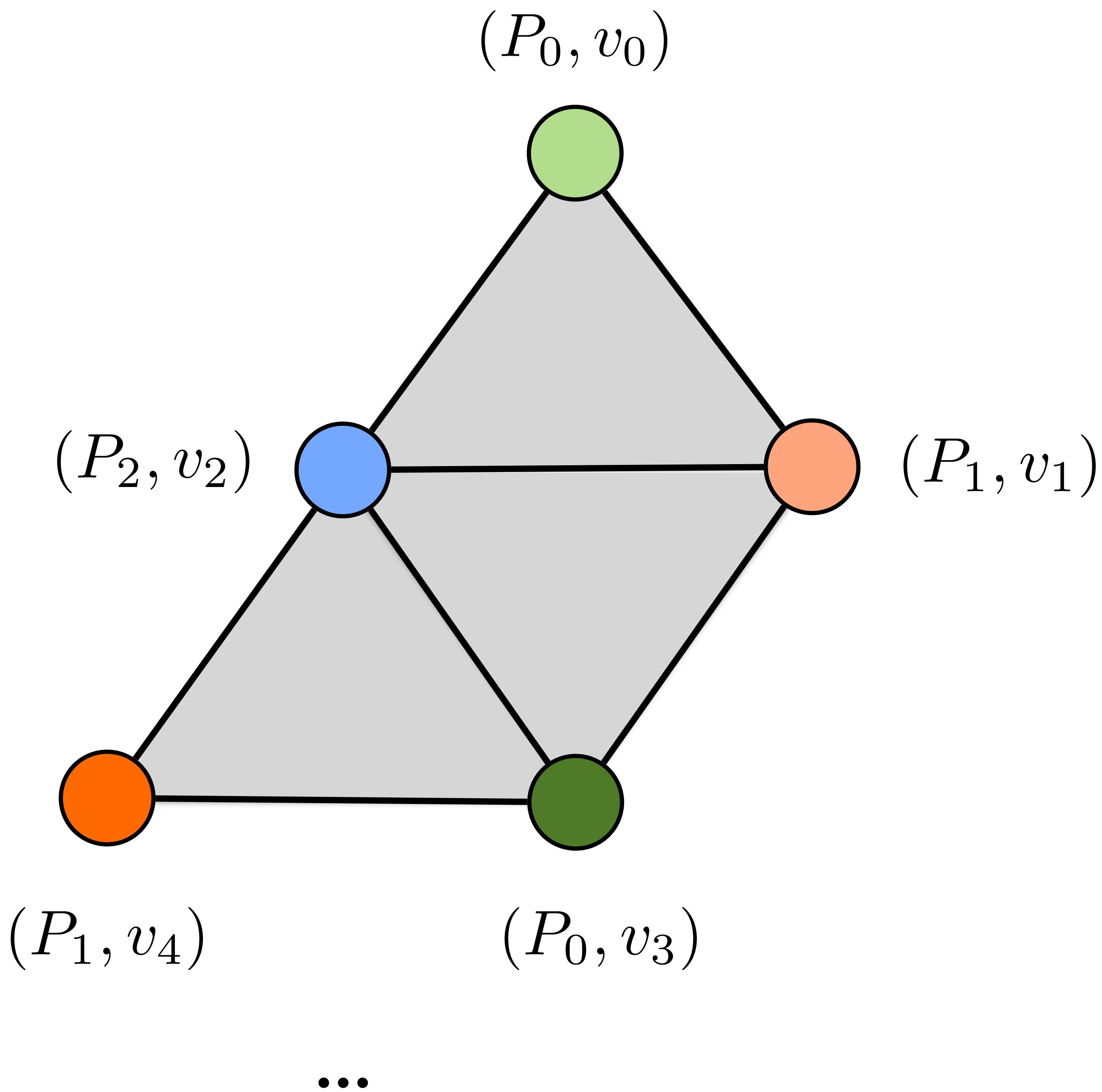

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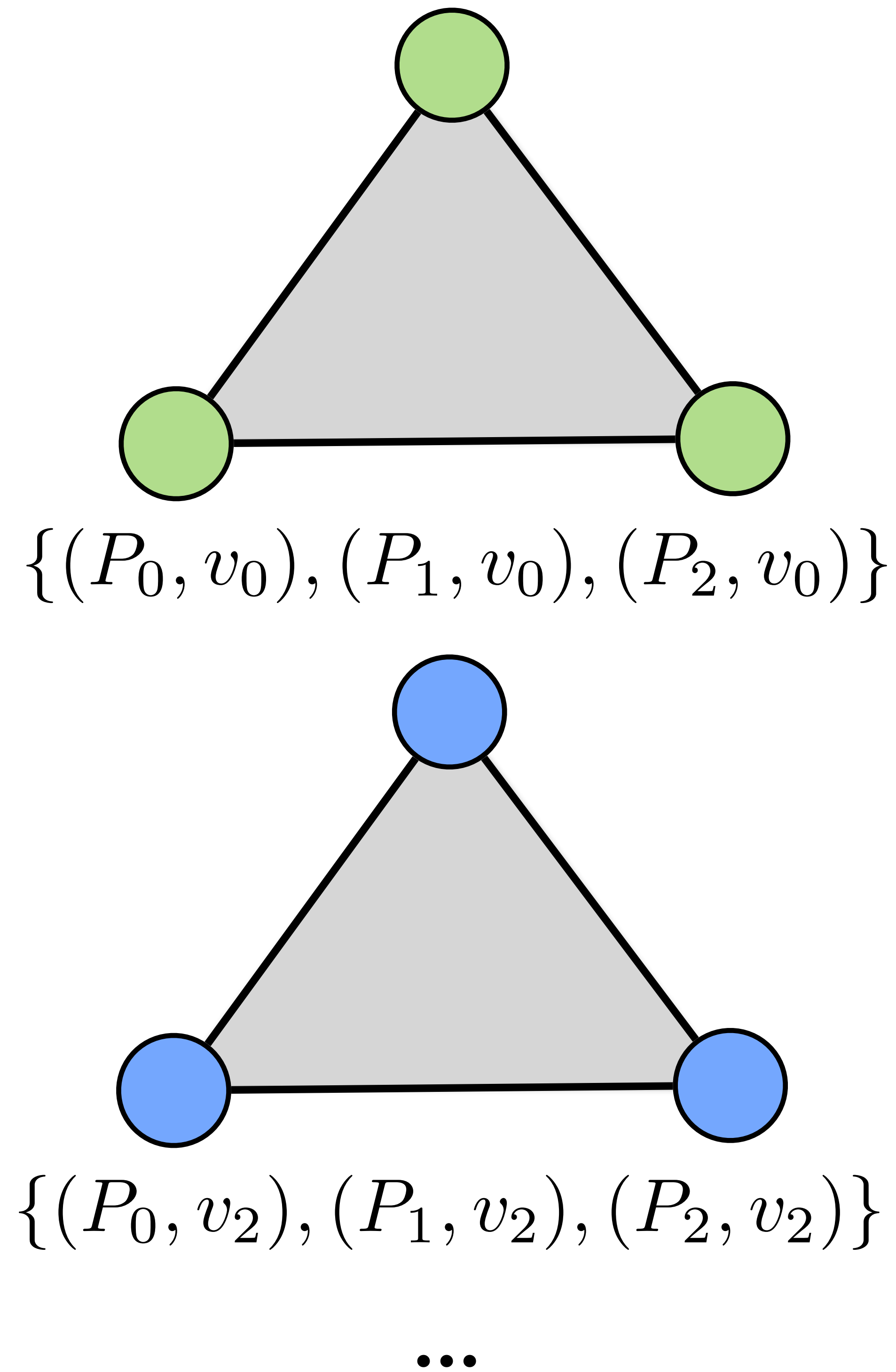

$\mathcal{O}$ (output complex)

Formal Specification

In the crash-failure model: $(\mathcal{I}, \mathcal{O}, \Delta)$

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$\tau \in \Delta(\sigma)$ if finishing with τ ...

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The Topological Structure of Asynchronous Computability

MAURICE HERLIHY

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AND

NIR SHAVIT

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Outline

1. Introduction

2. Asynchronous Byzantine Systems

3. Synchronous Byzantine Systems

4. Conclusion & Future Work

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Adversarial Model

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Byzantine processes

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up to t chosen by *adversary*

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Non-faulty processes output values...

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Δ constrains non-faulty processes *only*

$\mathcal{I}$ and $\mathcal{O}$ represent non-faulty processes *only*

What about Equivocation?

What about Equivocation?

Different information to different processes?

What about Equivocation?

Different information to different processes? ✘

What about Equivocation?

Different information to different processes? ✗

Reliable Broadcast!

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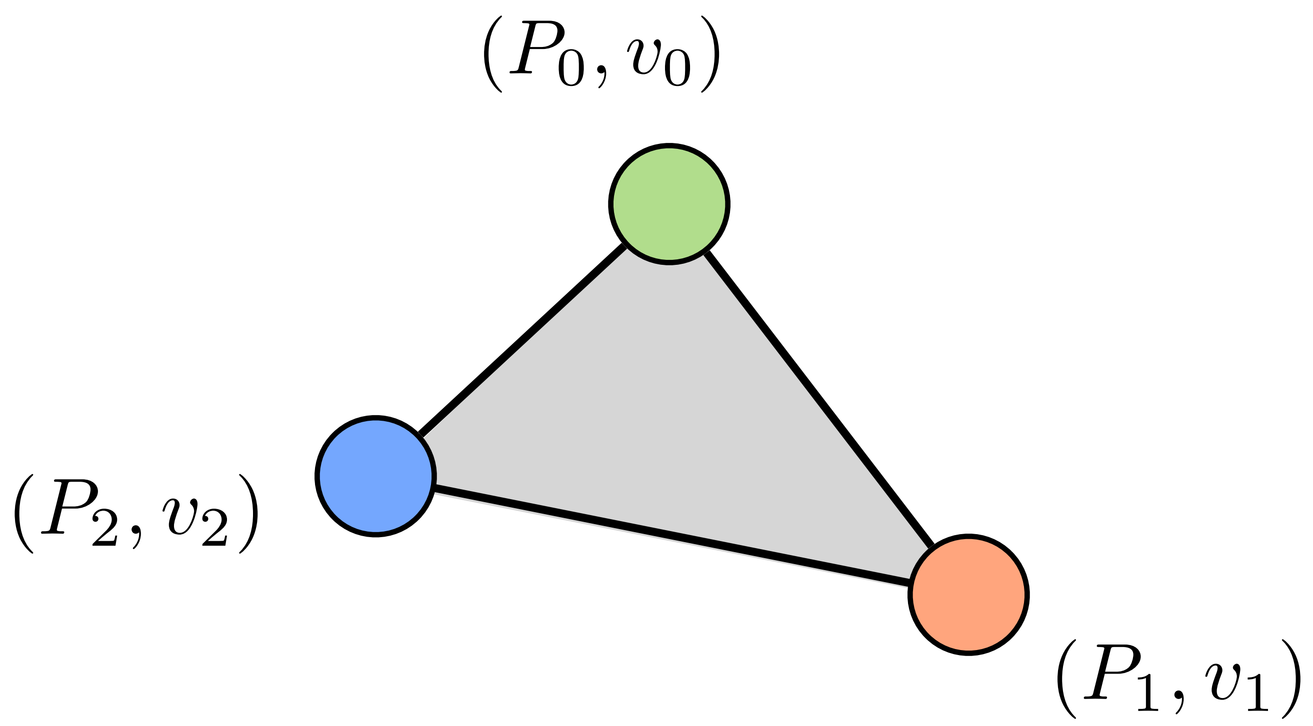

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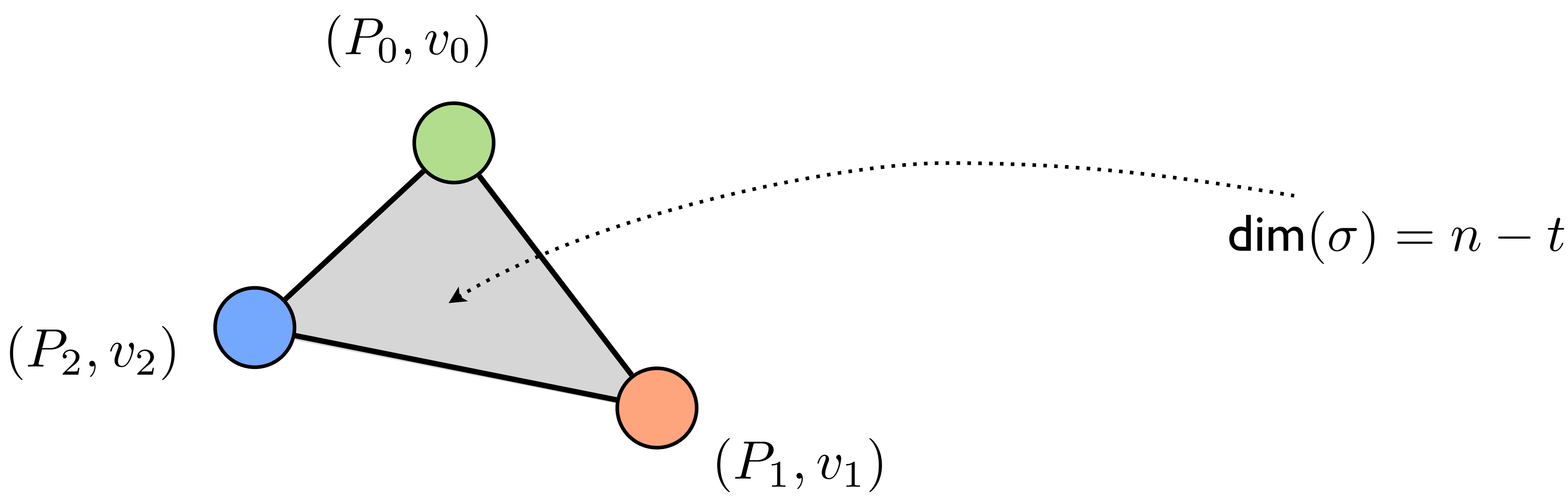

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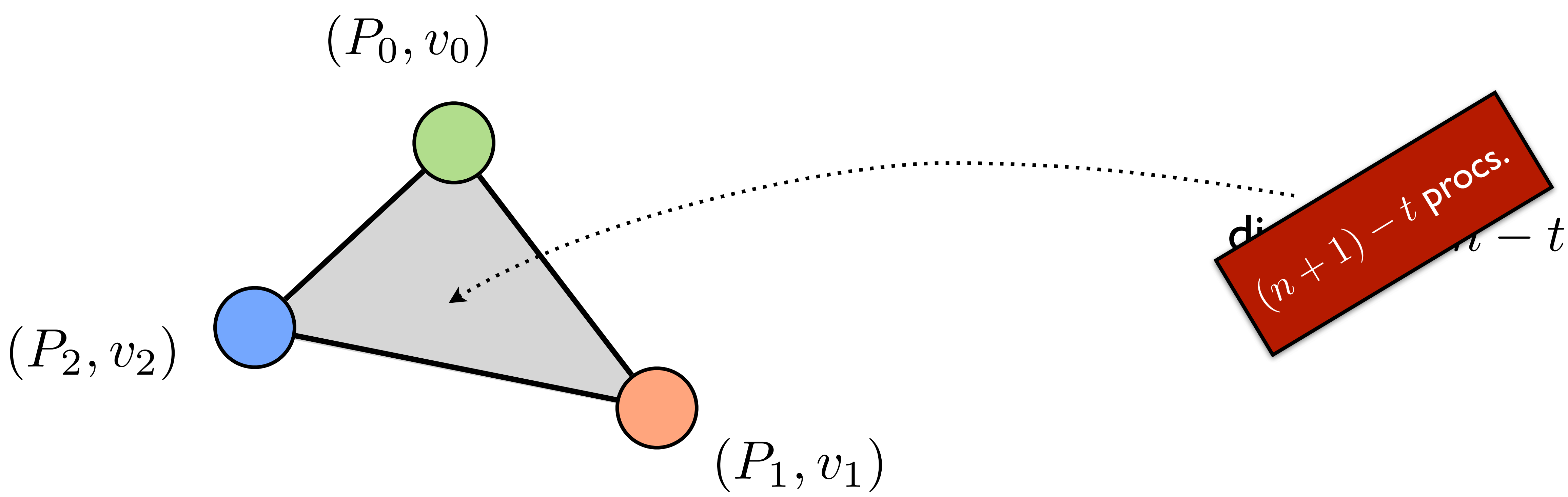

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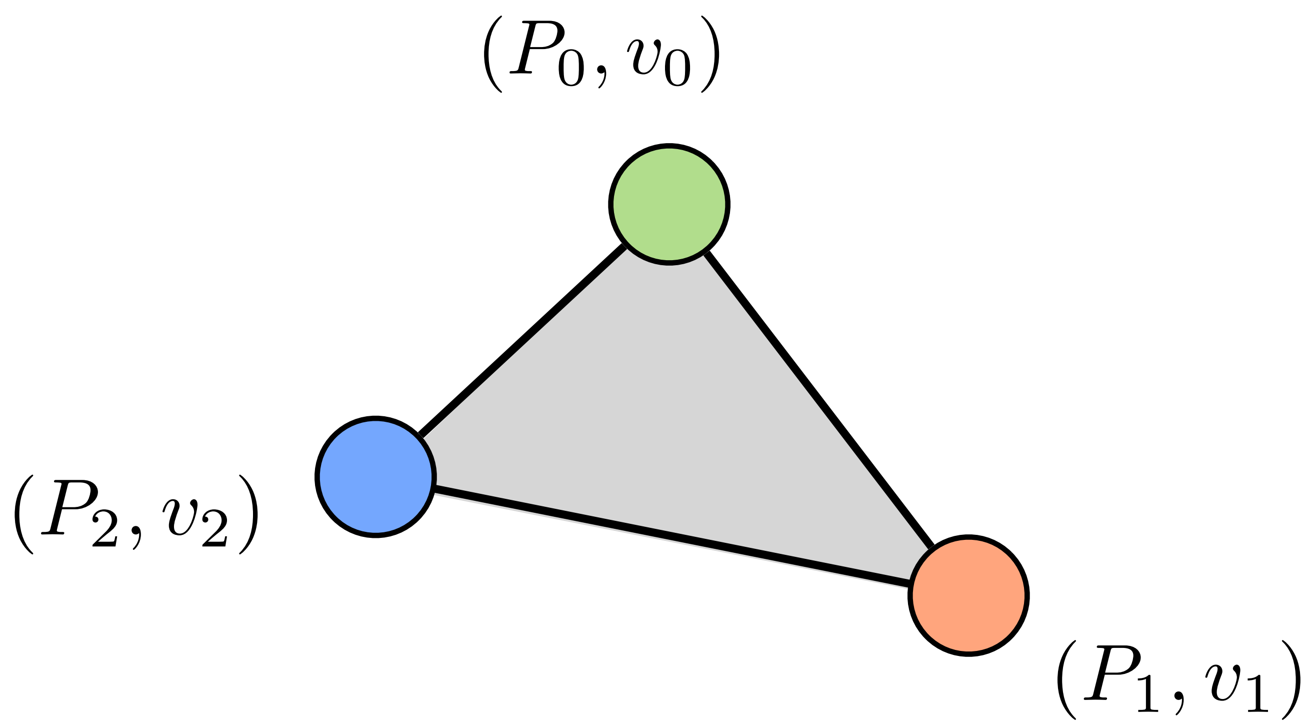

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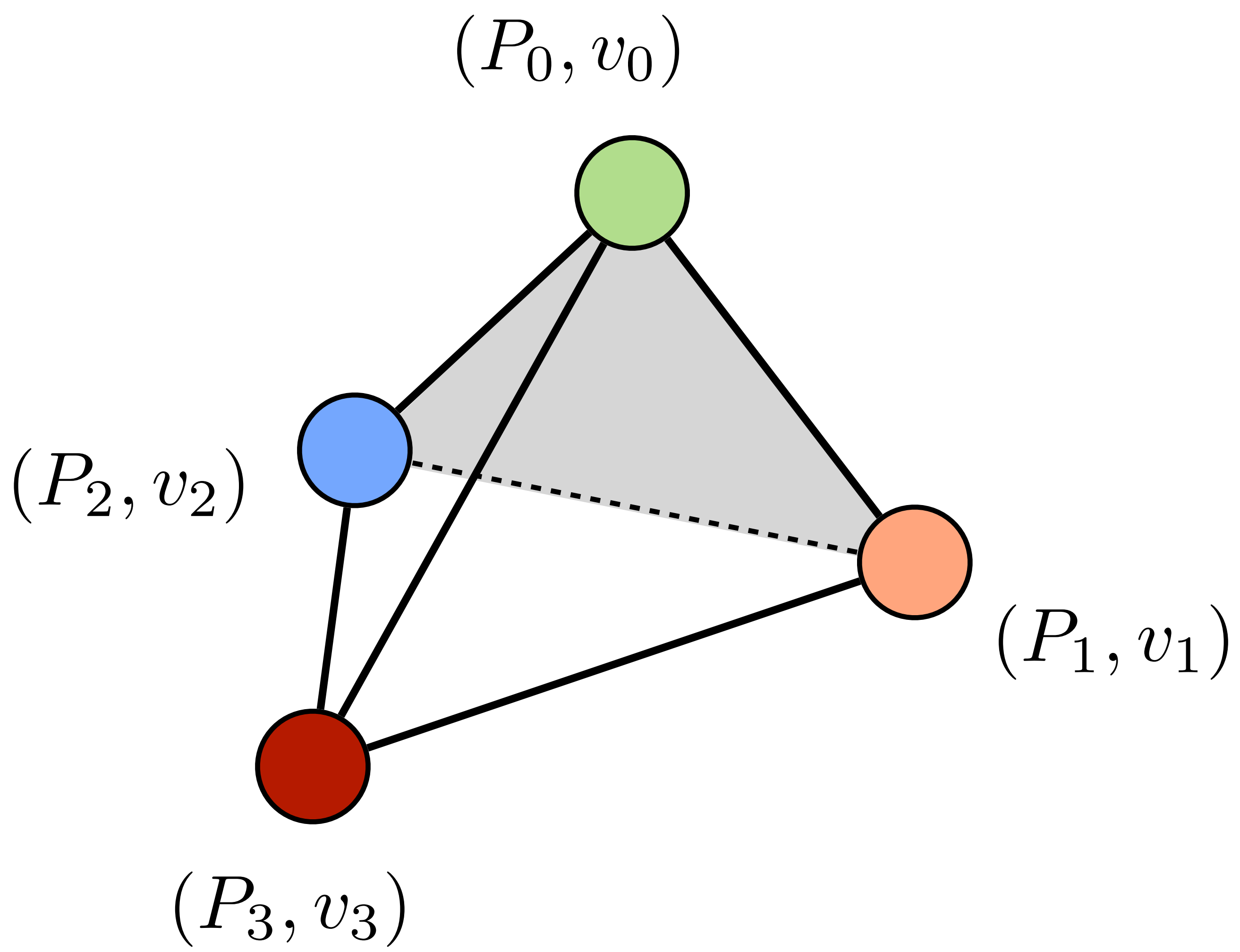

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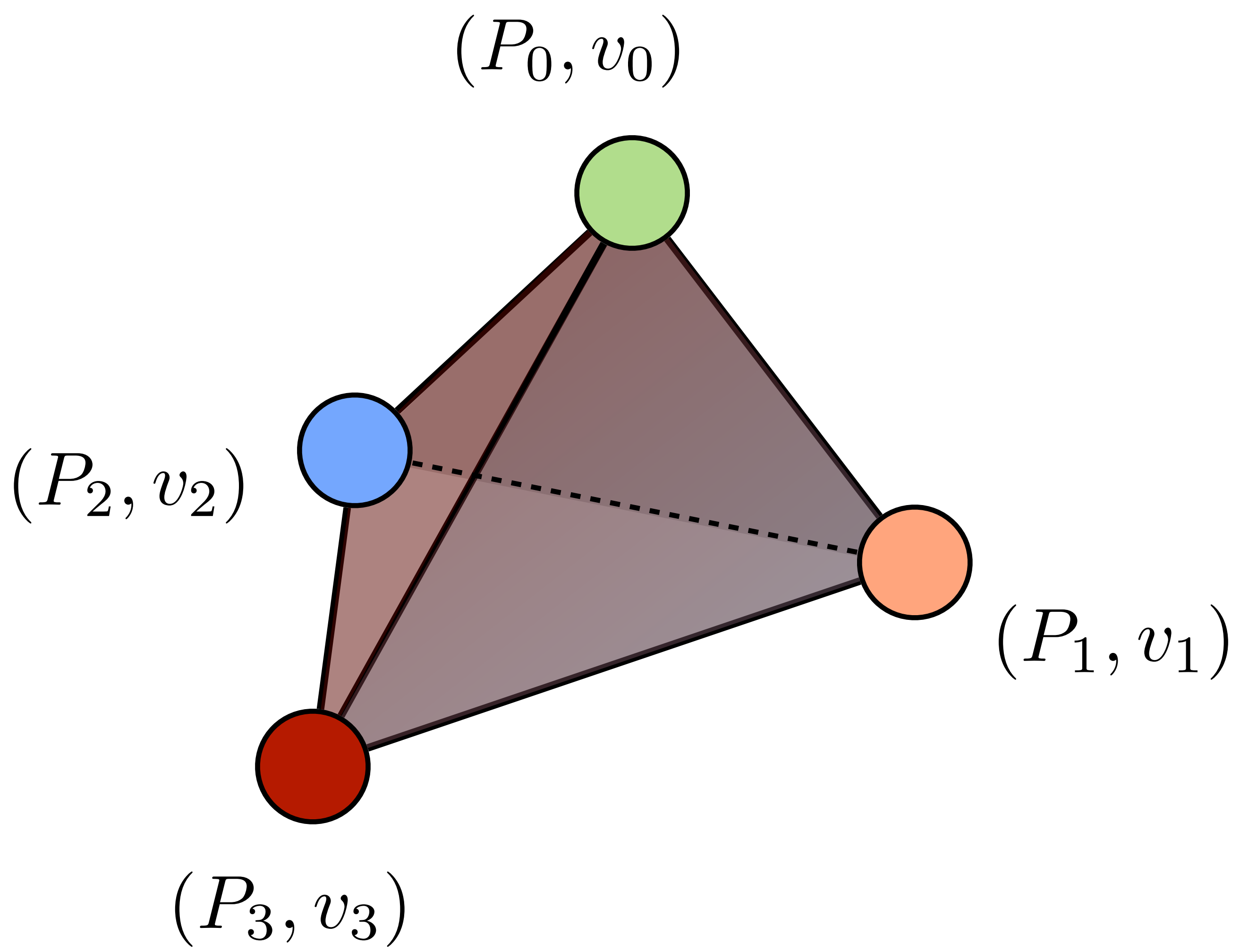

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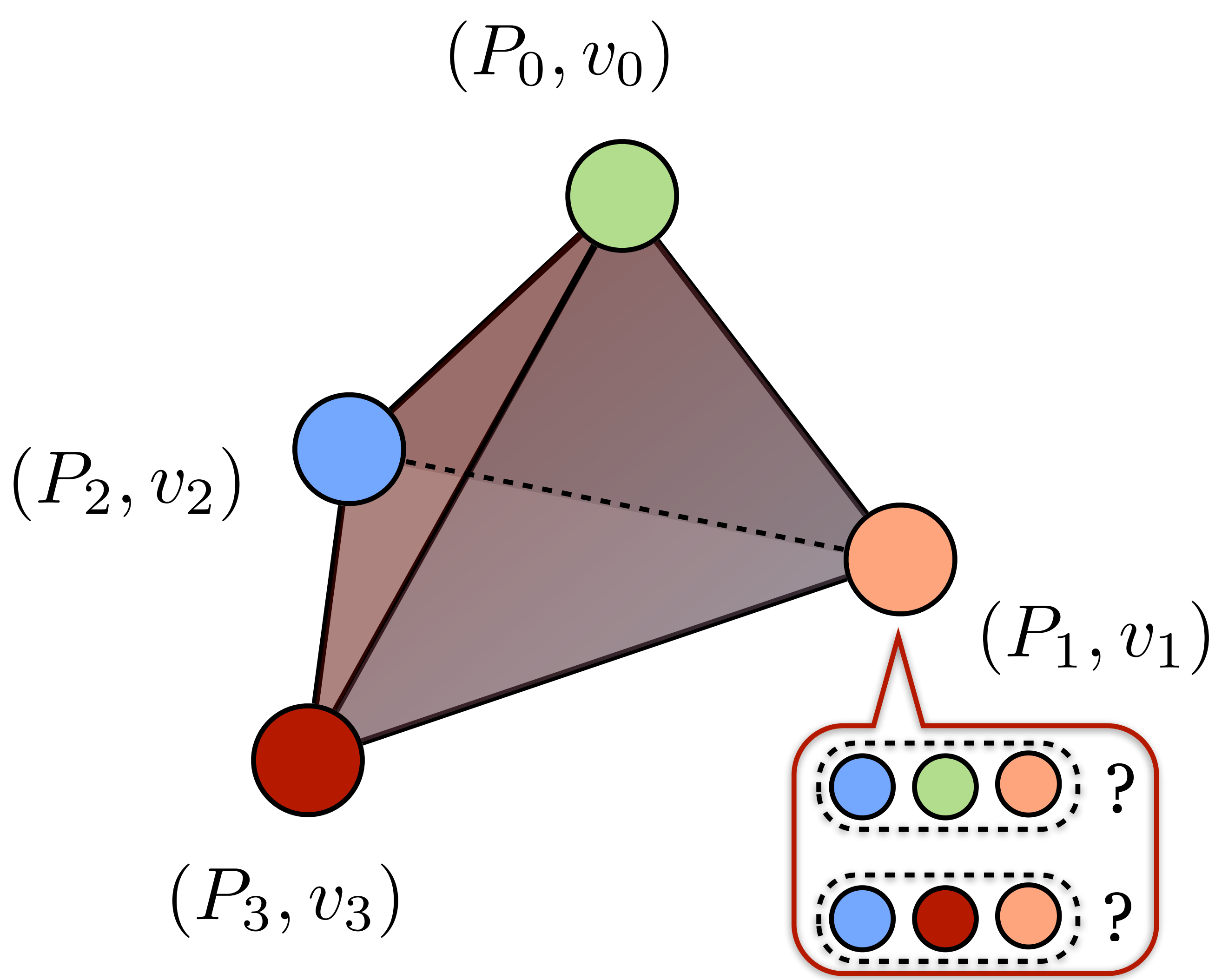

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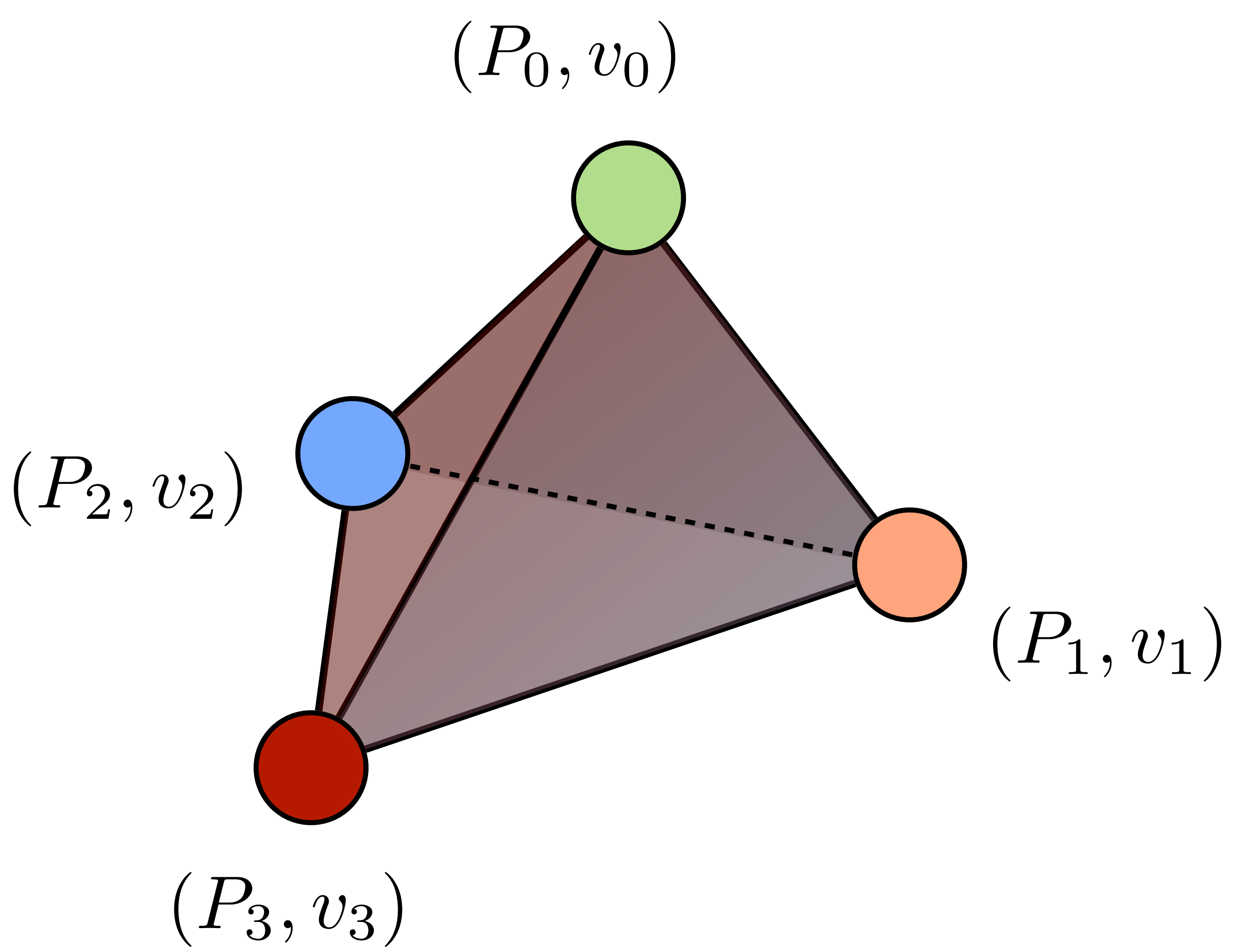

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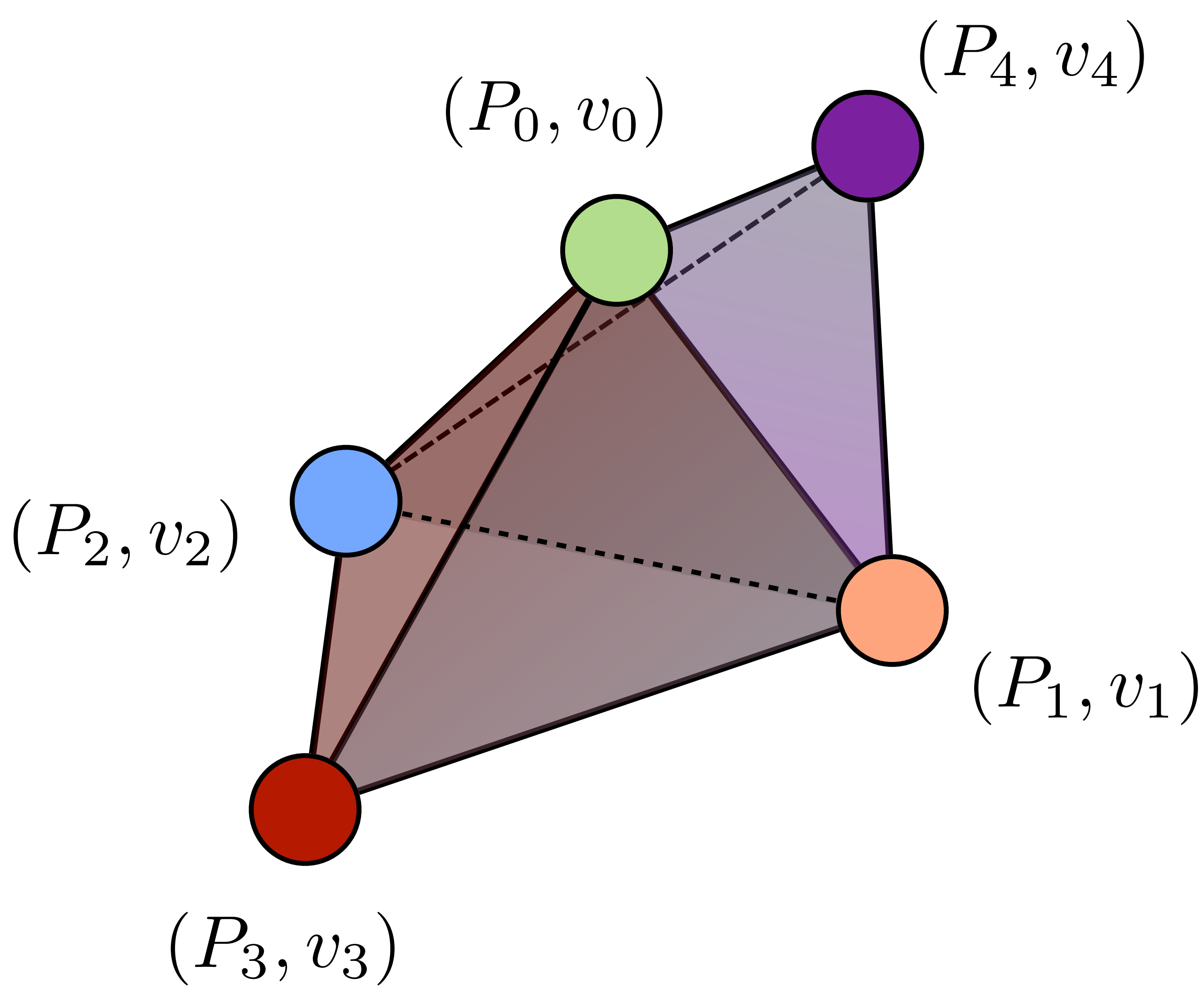

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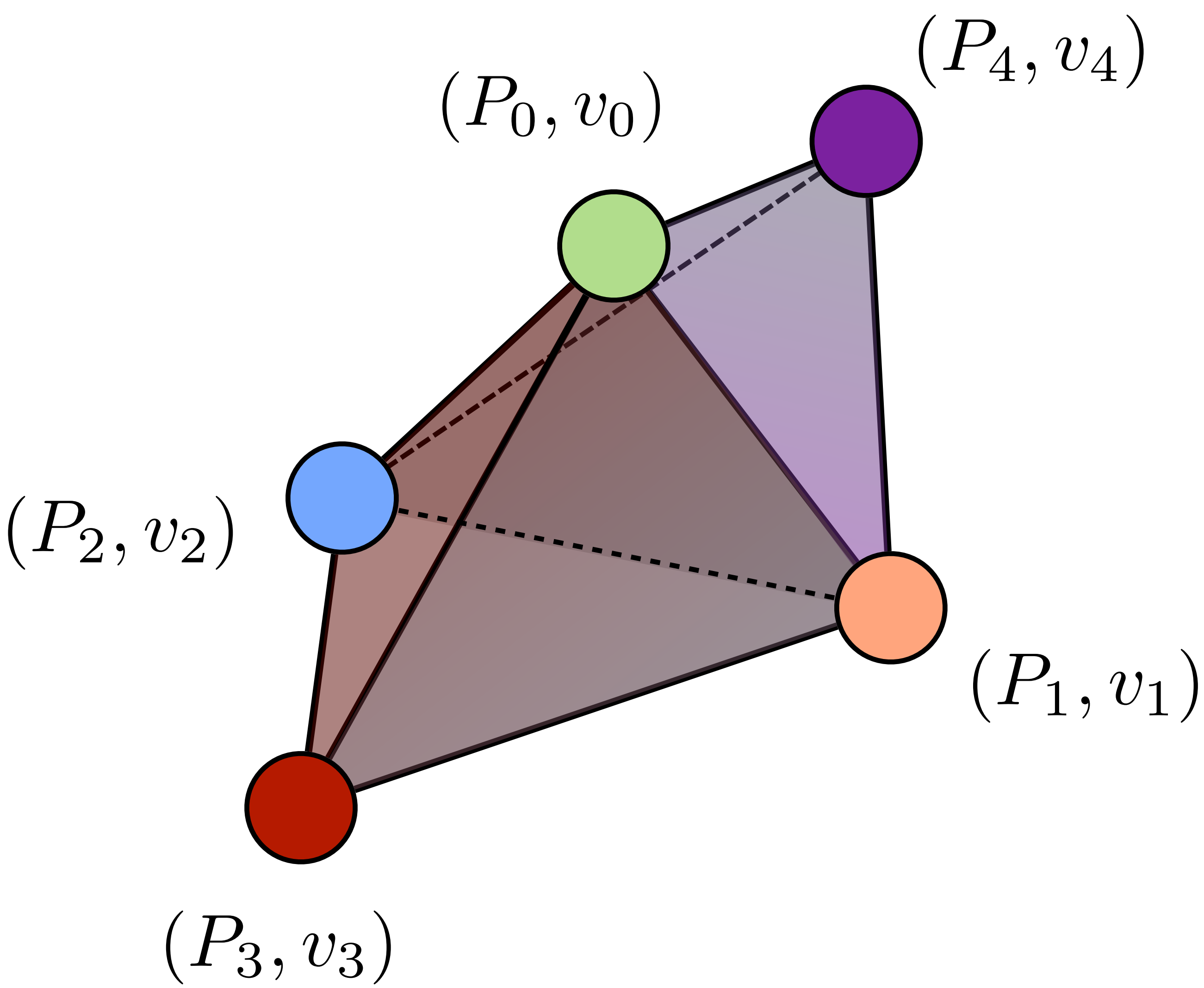

Byzantine processes pick fake inputs...

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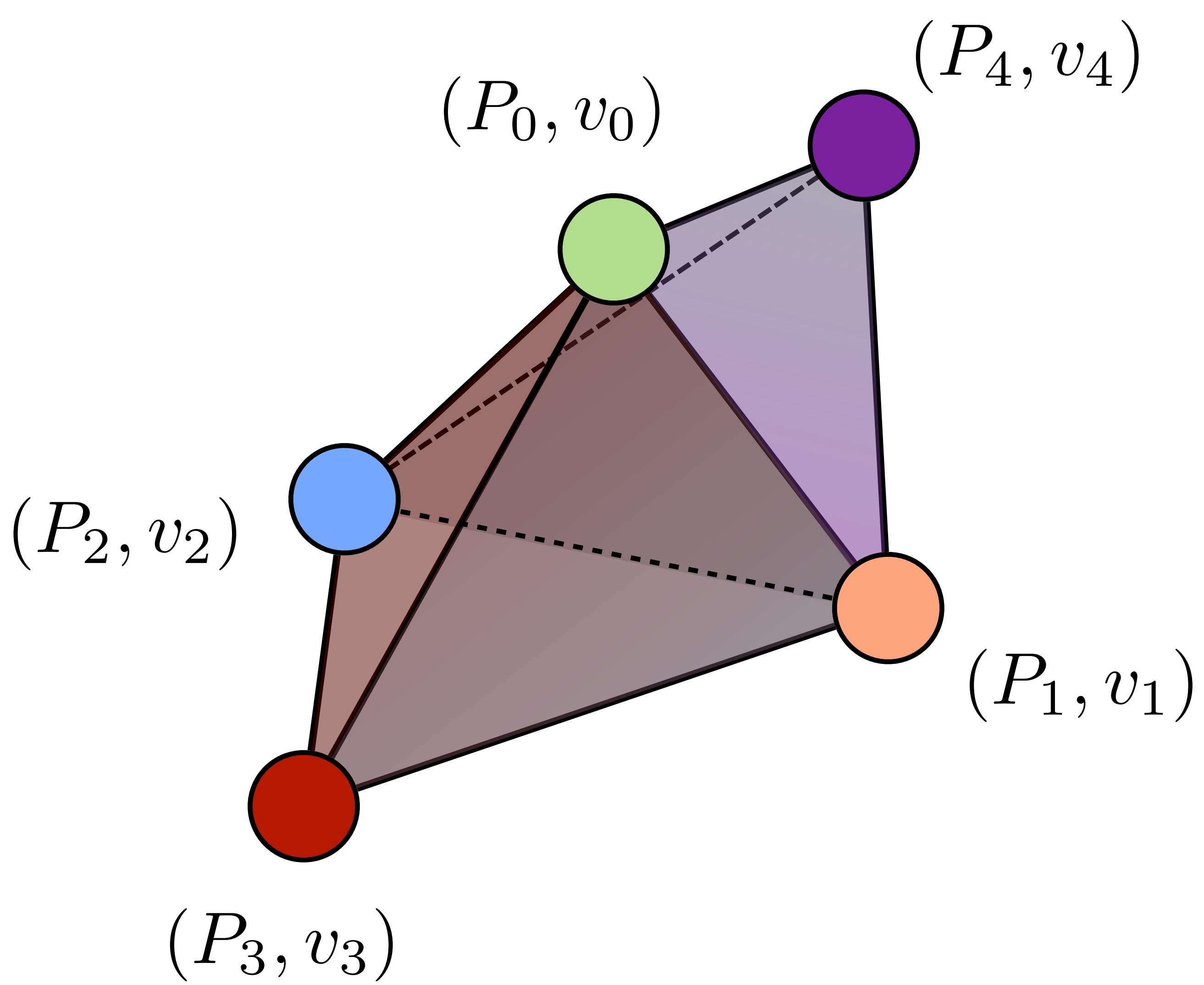

Byzantine processes
pick fake inputs…

… yet behave "correctly"

Formal Specification

In the Byzantine-failure model: $(\mathcal{I}, \mathcal{O}, \Delta)$

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When are these Byzantine tasks solvable?

Reduction to Crash-Failure Tasks

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Theorem: (Equivalence Theorem)

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In [STOC14], we show what is the dual task,

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In [STOC14], we show what is the dual task,

and how the equivalence holds

*Algorithmic methods**

used to prove theorem

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used to prove theorem

** simulations, reductions*

[STOC14]

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$\Updownarrow$

protocol $\Leftrightarrow \exists$ continuous $f : |\tilde{\mathcal{I}}| \to |\tilde{\mathcal{O}}|$ carried by $\tilde{\Delta}$

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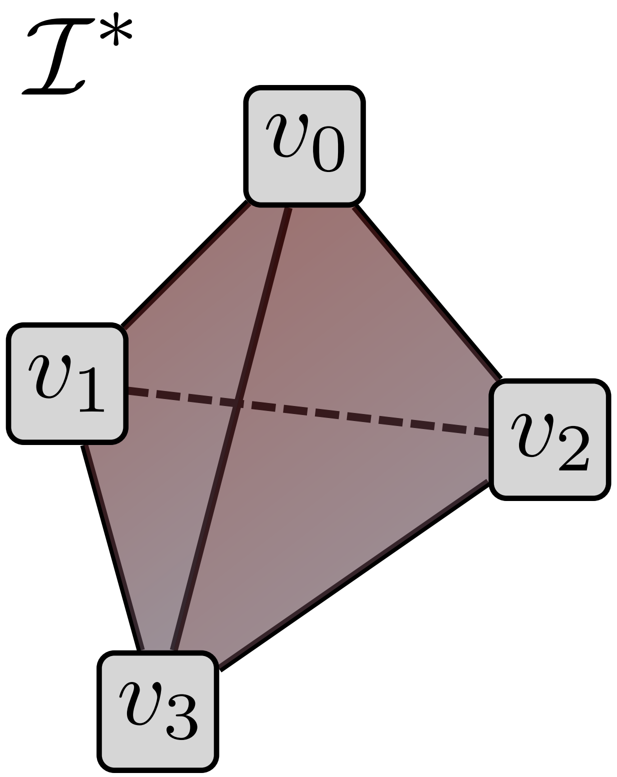

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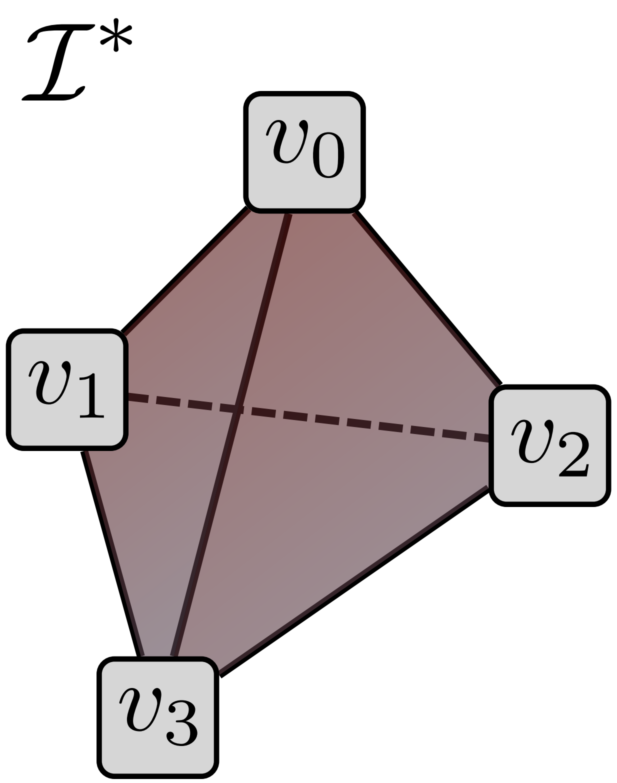

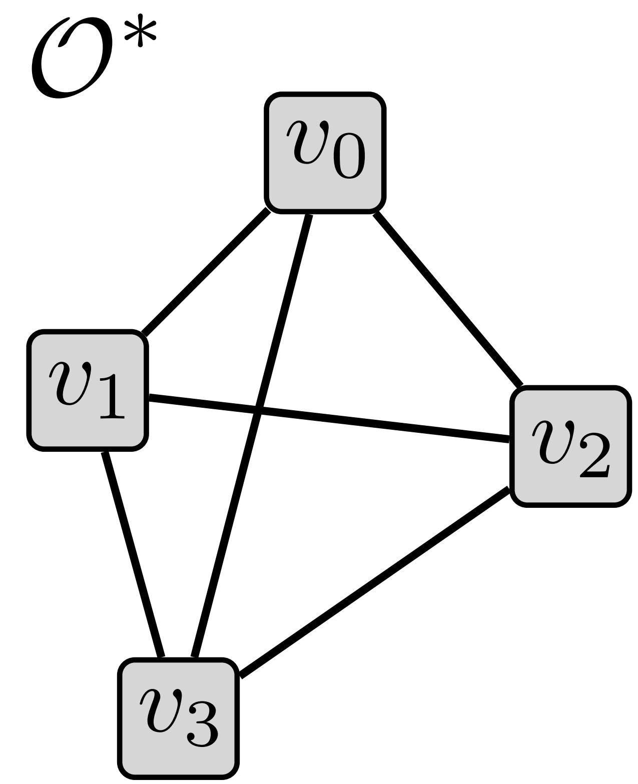

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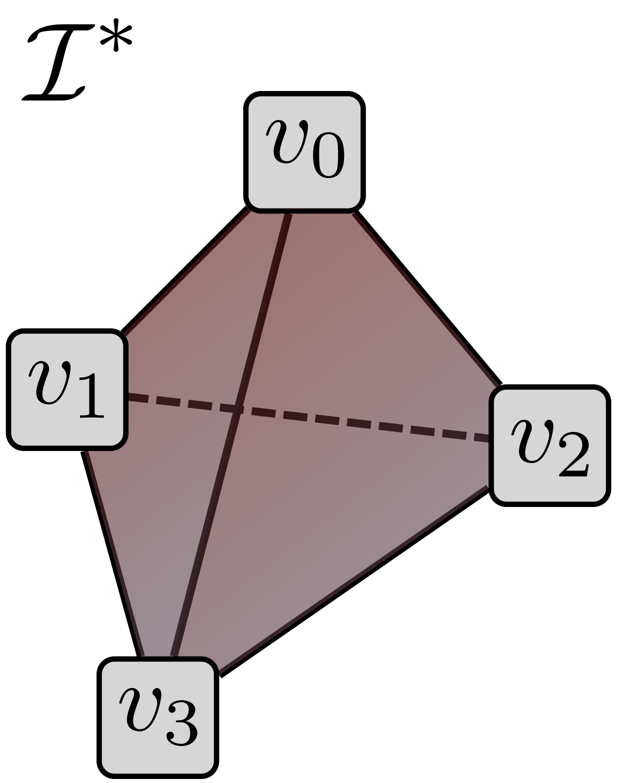
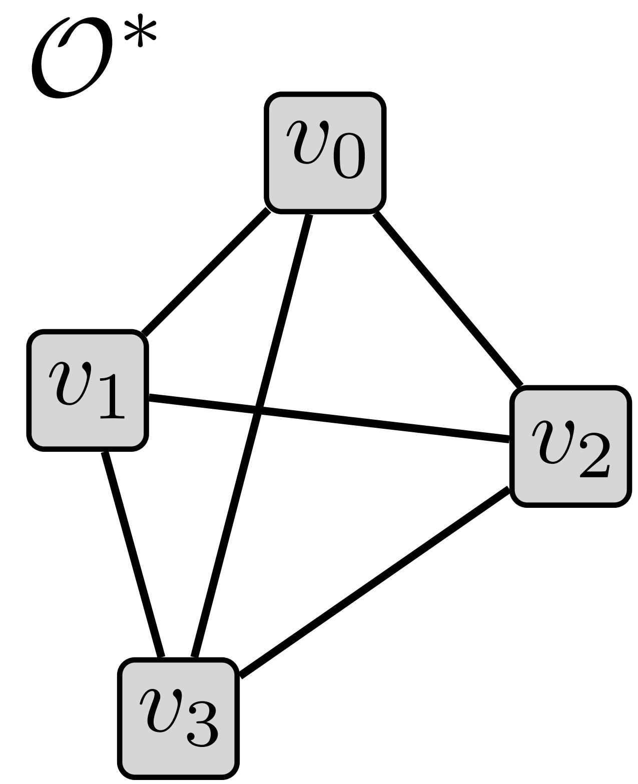


$\Delta^* = \text{skel}^{k-1}$ 1



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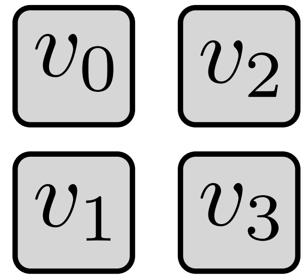
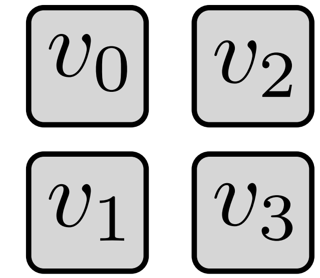
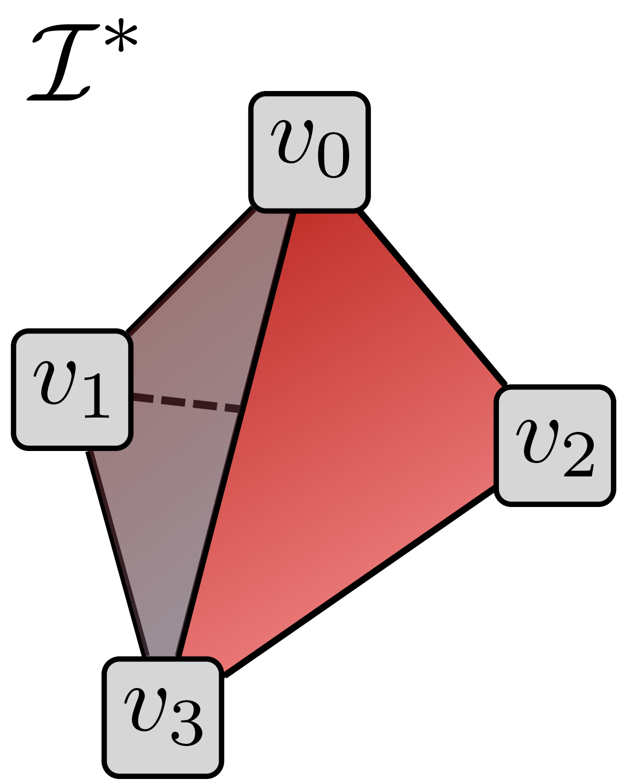


Finish with $\leq k$ of $(k = 2)$





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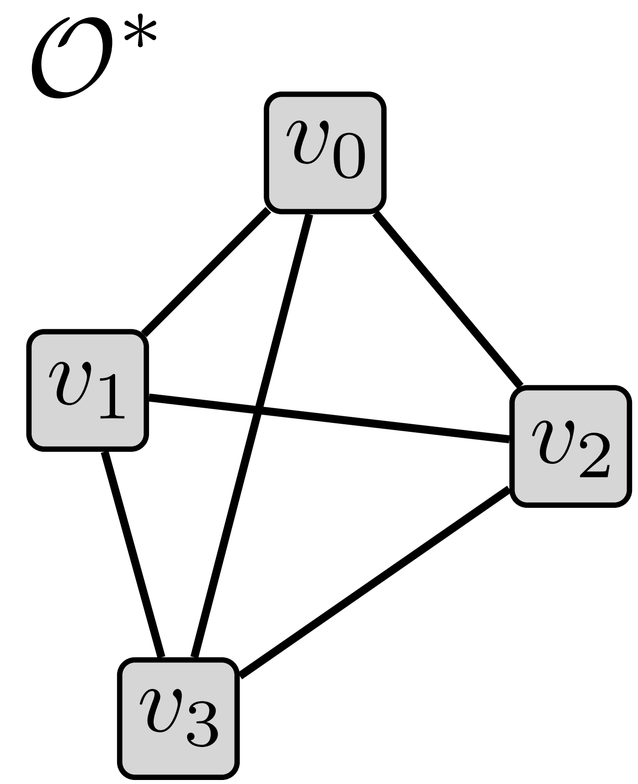

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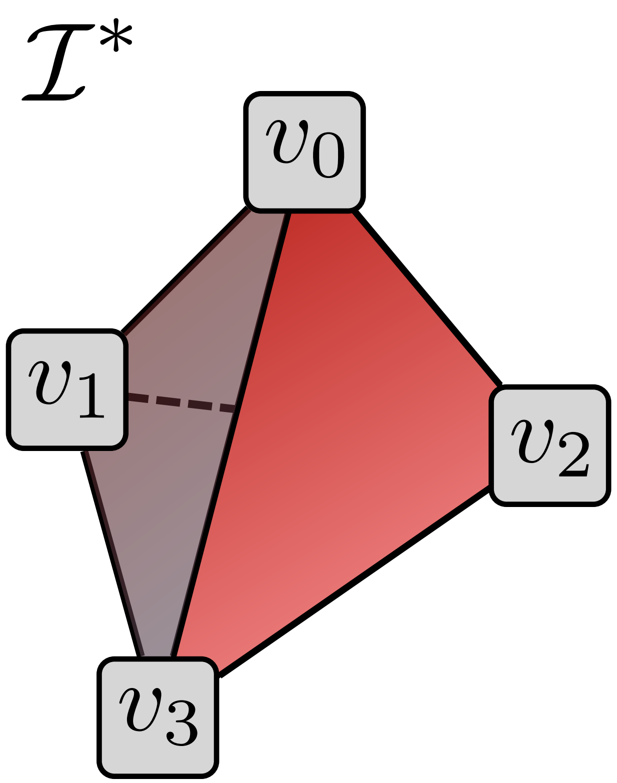

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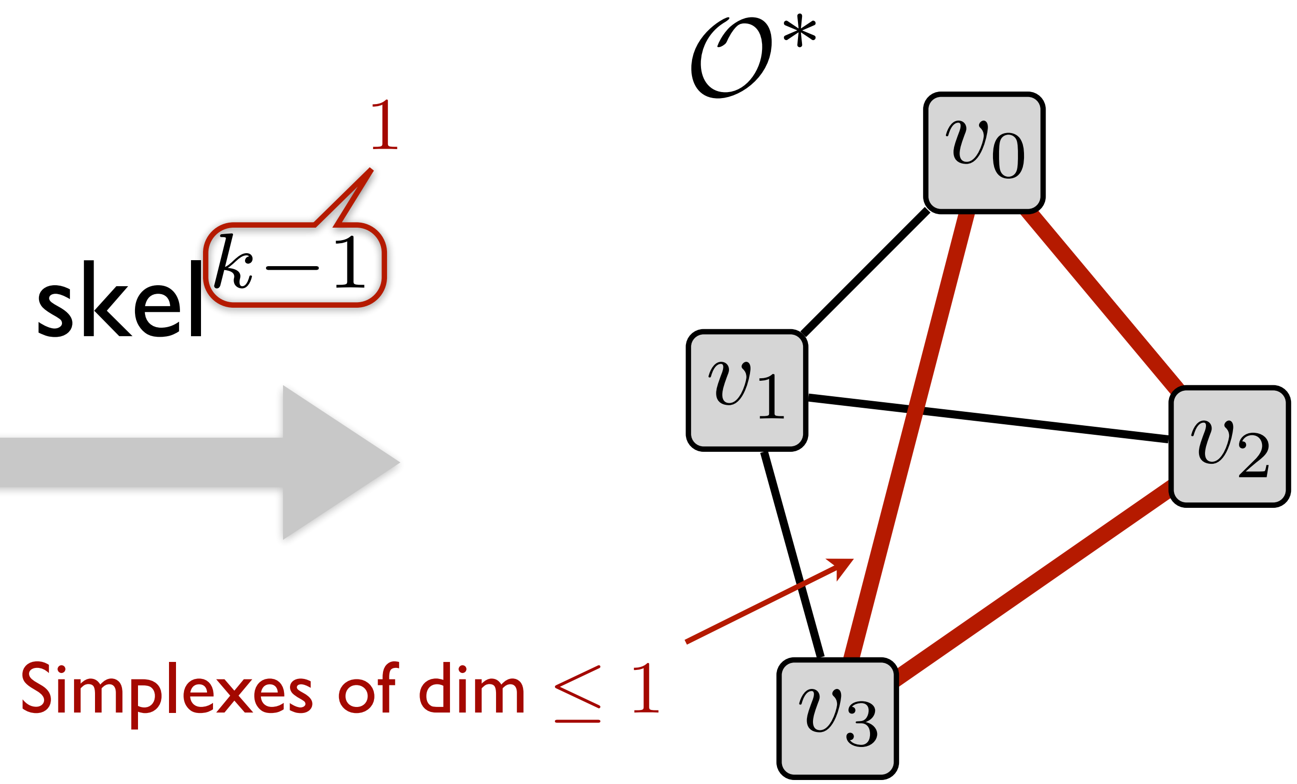

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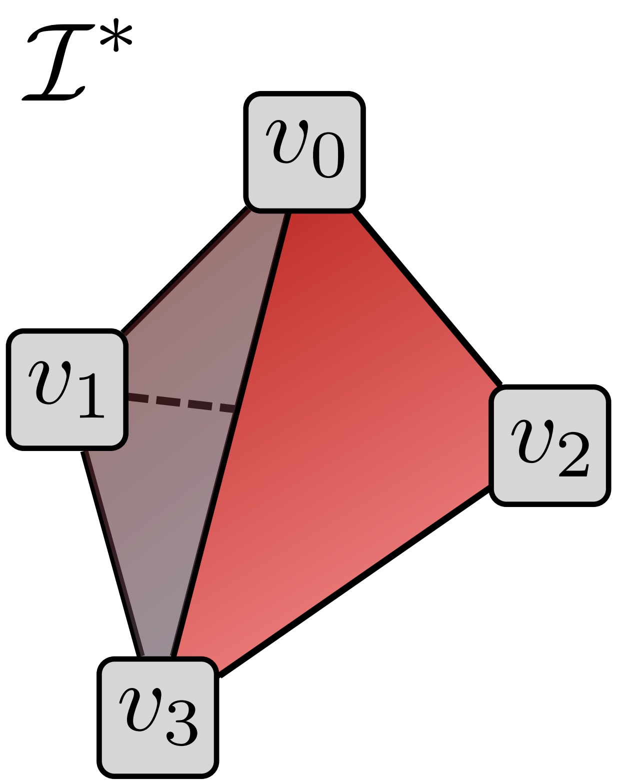

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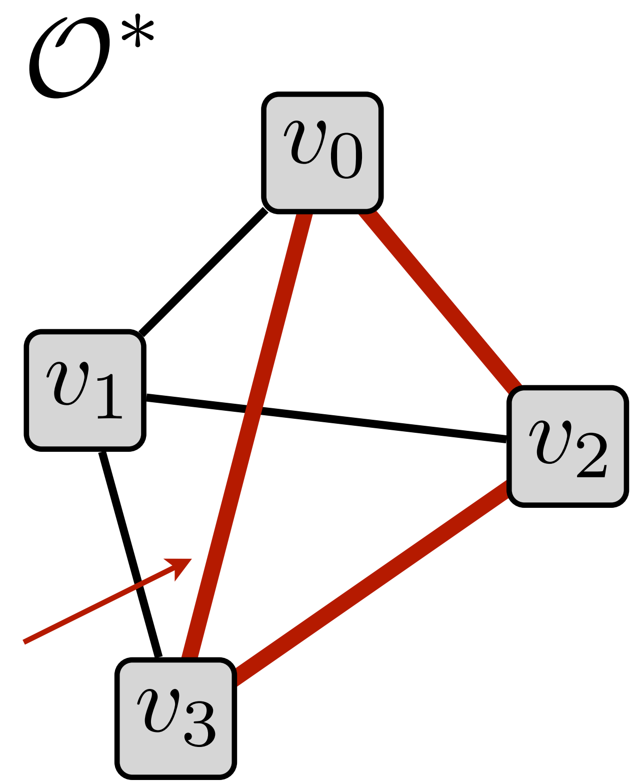

Strict k-set agreement

Theorems for Colorless Tasks

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Theorem:

The strict $(t+1)$-set agreement $(\mathcal{I}^*, \mathcal{O}^*, \text{skel}^t)$ has a t-resilient Byzantine asynchronous protocol *iff*

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$$n + 1 > t(\text{dim}(\mathcal{I}^*) + 2) \text{ or } \text{dim}(\mathcal{I}^*) \leq t.$$

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(application of our Equivalence Theorem)

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* Exchange values via Reliable Broadcast, and pick the 'smallest' one

Proof Sketch (contd.)

2 Run the Byzantine barycentric agreement protocol N times, landing on a simplex in $\text{Bary}^N \text{skel}^t(\mathcal{I}^*)$.

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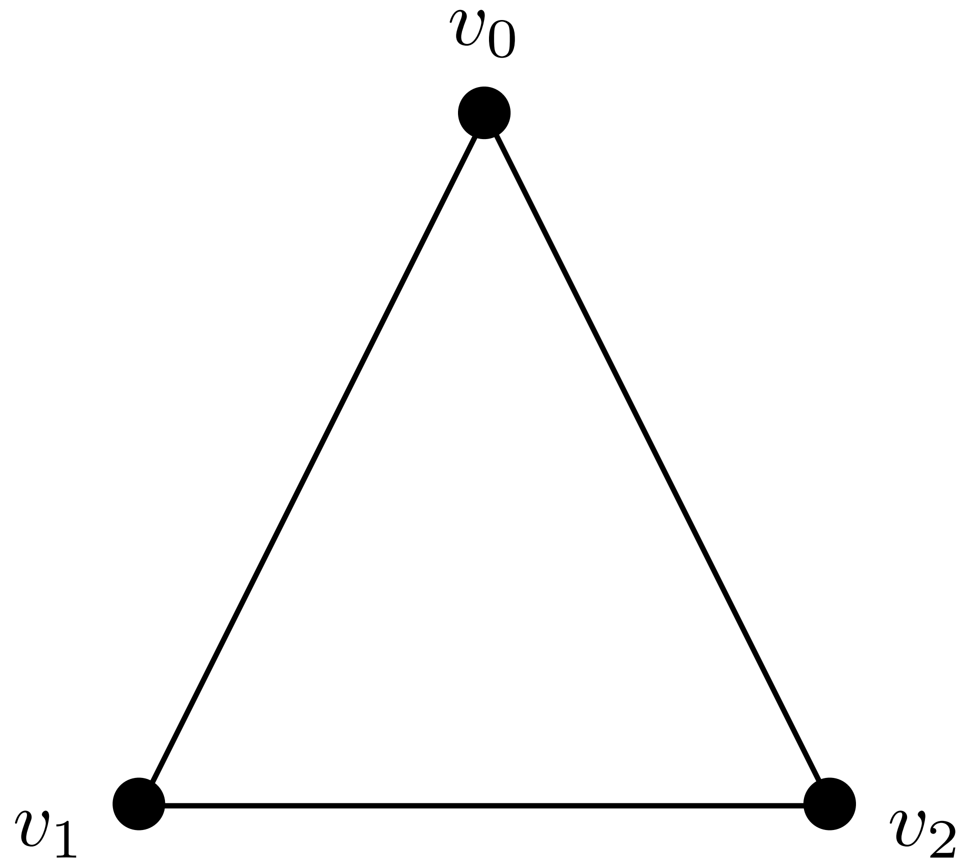

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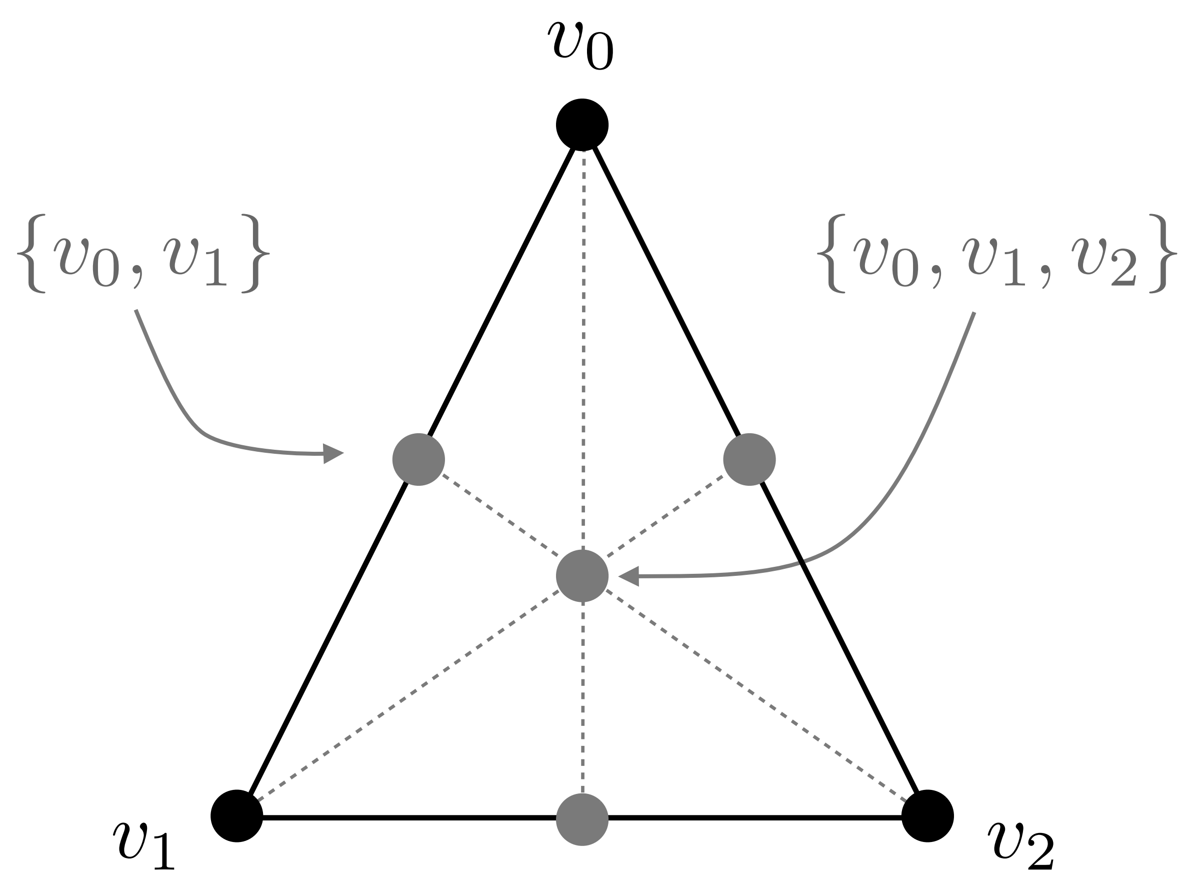

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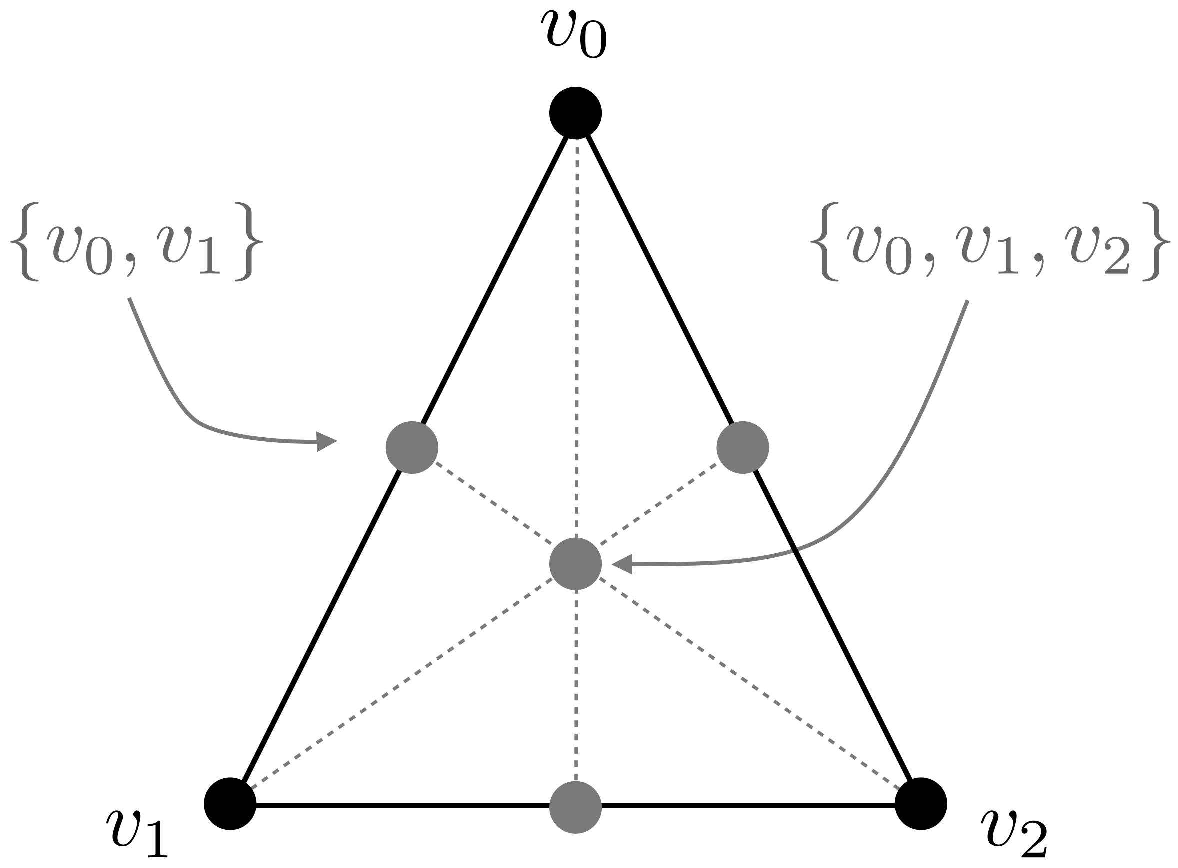
Barycentric Agreement task



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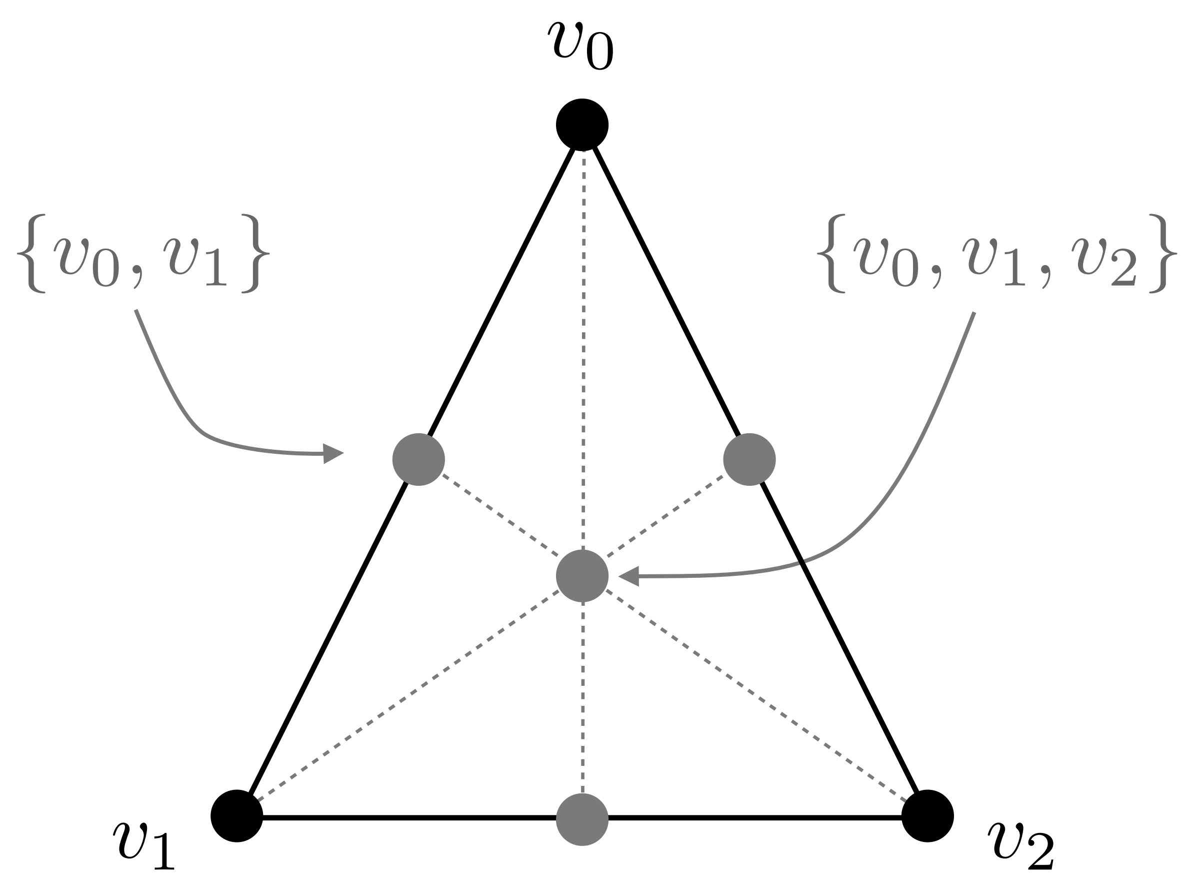
Barycentric Agreement task

Agree on vertices of a single simplex of Bary σ



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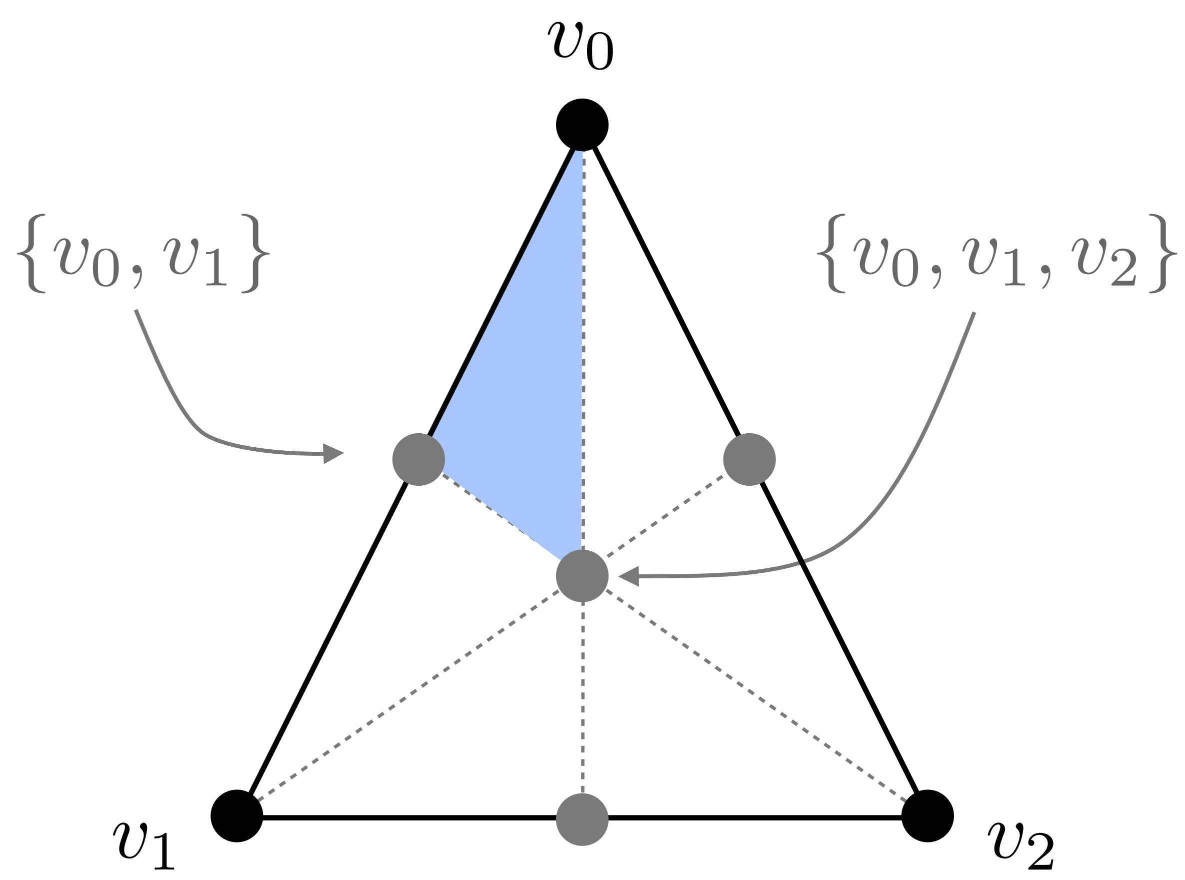
Barycentric Agreement task

Agree on vertices of a single simplex of Bary σ

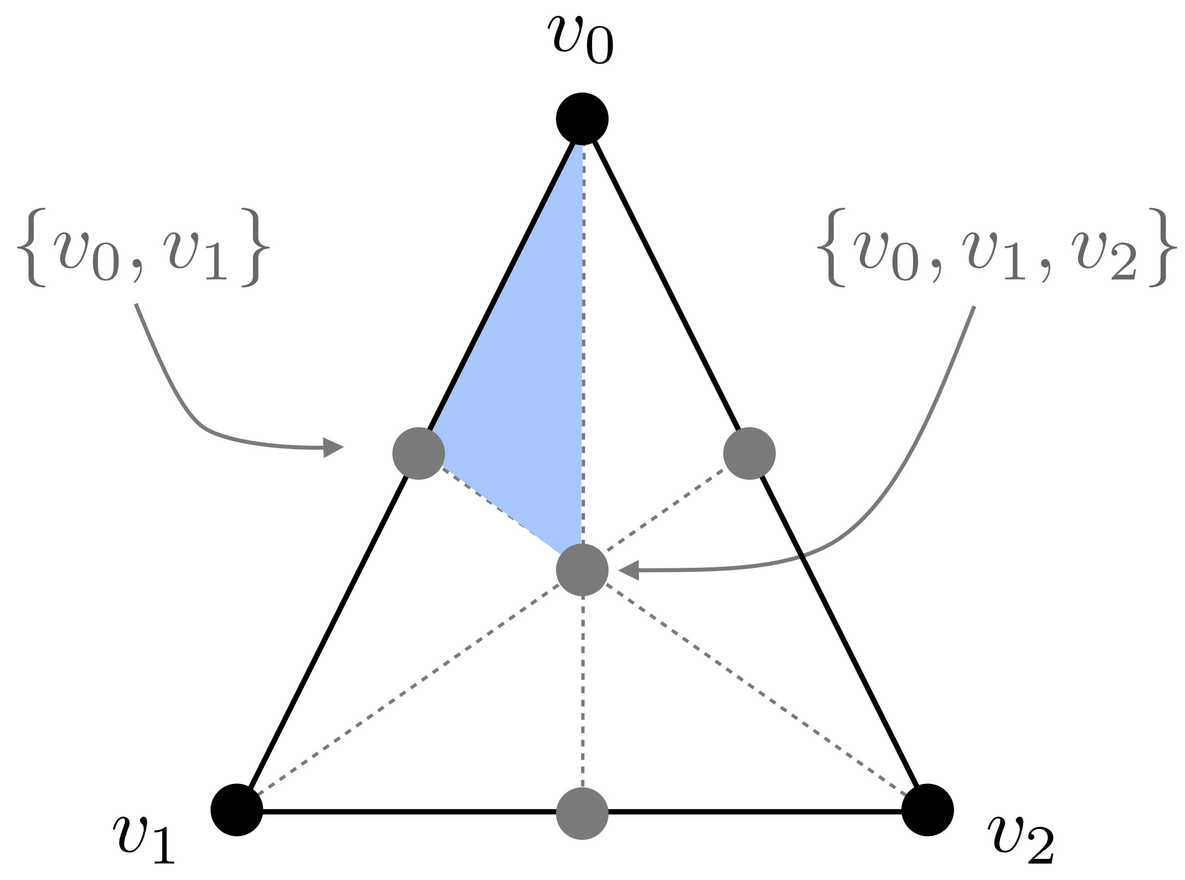


Proof Sketch (contd.)

2 Run the Byzantine barycentric agreement protocol N times, landing on a simplex in $\text{Bary}^N \text{skel}^t(\mathcal{I}^*)$.

Barycentric Agreement task

Agree on vertices of a single simplex of Bary σ



(protocol based on the ϵ-multidimensional agreement!) [STOC 13]

Proof Sketch (contd.)

Proof Sketch (contd.)

By the *Simplicial Approximation Theorem*, $f : |\,\mathsf{skel}^t(\mathcal{I}^*)| \to |\mathcal{O}^*|$ has a *simplicial approximation*

Proof Sketch (contd.)

hypothesis

By the *Simplicial Approximation Theorem*, $f : |\,\text{skel}^t(\mathcal{I}^*)| \to |\mathcal{O}^*|$ has a *simplicial approximation*

Proof Sketch (contd.)

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By the *Simplicial Approximation Theorem,* $f : |\,\mathsf{skel}^t(\mathcal{I}^*)| \to |\mathcal{O}^*|$ has a *simplicial approximation*

$$\mathsf{Bary}^N\,\mathsf{skel}^t(\mathcal{I}^*) \to \mathcal{O}^* \text{ for some } N > 0.$$

Proof Sketch (contd.)

hypothesis

By the *Simplicial Approximation Theorem*, $f : |\,\text{skel}^t(\mathcal{I}^*)| \rightarrow |\mathcal{O}^*|$ has a *simplicial approximation*

$$\text{Bary}^N \text{skel}^t(\mathcal{I}^*) \rightarrow \mathcal{O}^* \text{ for some } N > 0.$$

fine-grain the input, so we can "approximate" f by a *simplicial* map

Proof Sketch (contd.)

hypothesis

By the *Simplicial Approximation Theorem*, $f : |\,\text{skel}^t(\mathcal{I}^*)| \to |\mathcal{O}^*|$ has a *simplicial approximation*

$$\text{Bary}^N \text{skel}^t(\mathcal{I}^*) \to \mathcal{O}^* \text{ for some } N > 0.$$

fine-grain the input, so we can "approximate" f by a *simplicial* map

We then...

3 Apply $\phi : \text{Bary}^N \text{skel}^t(\mathcal{I}^*) \to \mathcal{O}^*$ to choose vertices in $\mathcal{O}^*$.

Proof Sketch (contd.)

hypothesis

By the *Simplicial Approximation Theorem*, $f : |\,\text{skel}^t(\mathcal{I}^*)| \to |\mathcal{O}^*|$ has a *simplicial approximation*

$$\text{Bary}^N \text{skel}^t(\mathcal{I}^*) \to \mathcal{O}^* \text{for some } N > 0.$$

fine-grain the input, so we can "approximate" f by a *simplicial* map

We then...

3 Apply $\phi : \text{Bary}^N \text{ skel}^t(\mathcal{I}^*) \to \mathcal{O}^*$ to choose vertices in $\mathcal{O}^*$.

(because it's a simplicial approximation, choosing outputs based on the approximation is consistent with choosing outputs based on f)

Outline

1. Introduction
2. Asynchronous Byzantine Systems
3. Synchronous Byzantine Systems
4. Conclusion & Future Work

Outline

1. Introduction
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Outline

1. Introduction
2. Asynchronous Byzantine Systems
3. Synchronous Byzantine Systems *quick overview*
4. Conclusion & Future Work

Protocol Complex

Protocol Complex

Before

Protocol Complex

Before

Tasks modeled as simplicial complexes

Protocol Complex

Before

Tasks modeled as simplicial complexes

Now

Protocol Complex

Before

Tasks modeled as simplicial complexes

Now

Executions also modeled as simplicial complexes

Protocol Complex

Before

Tasks modeled as simplicial complexes

Now

Executions also modeled as simplicial complexes

Global state evolving throughout the rounds

Protocol Complex

Before

Tasks modeled as simplicial complexes

Now

Executions also modeled as simplicial complexes

Global state evolving throughout the rounds

Protocol Complex

Protocol Complexes

Protocol Complexes

Consensus **task**

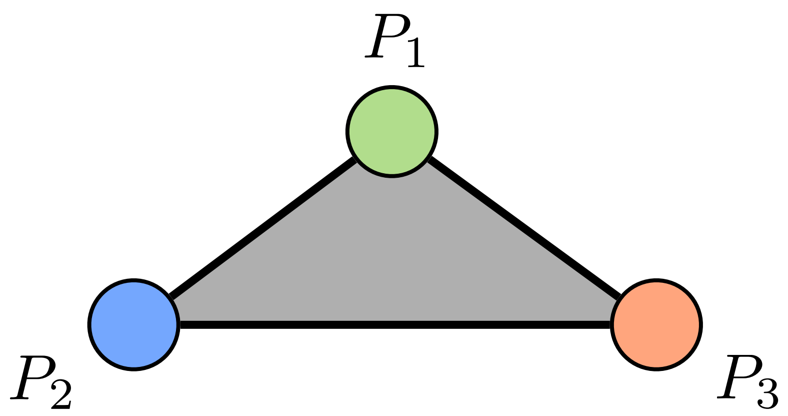
Protocol Complexes

Consensus **task** <u>t = 1</u>

Protocol Complexes

Consensus task t = 1

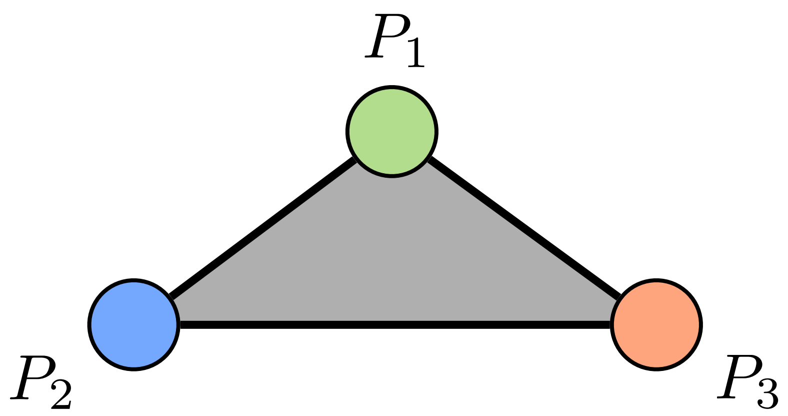
input



Protocol Complexes

Consensus task t = 1

input

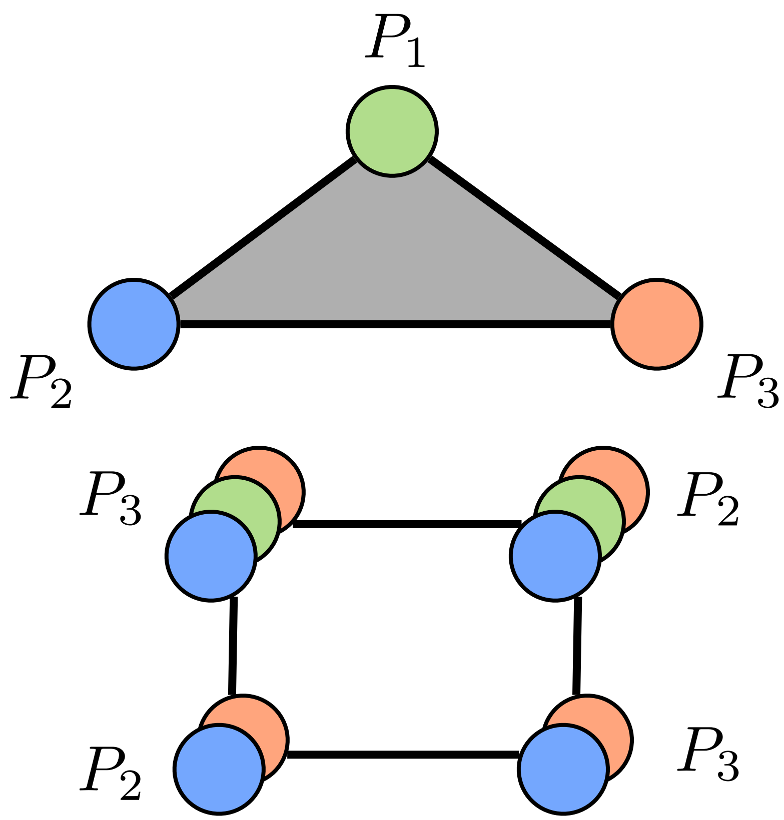


round 1:
P_1 fails

Protocol Complexes

Consensus task t = 1

input



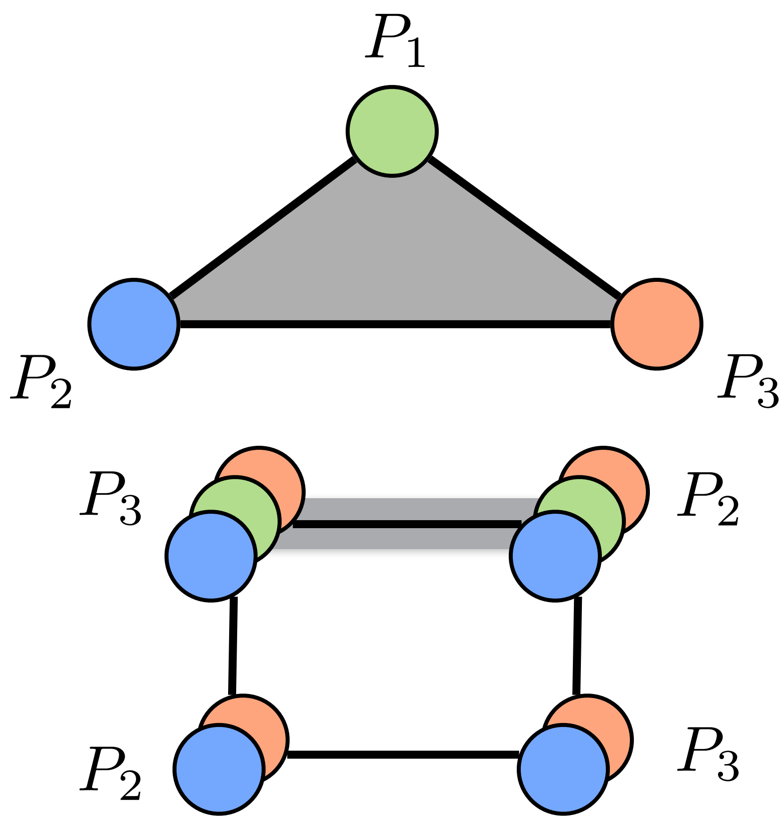
round 1:
P_1 fails

Protocol Complexes

Consensus task t = 1

input

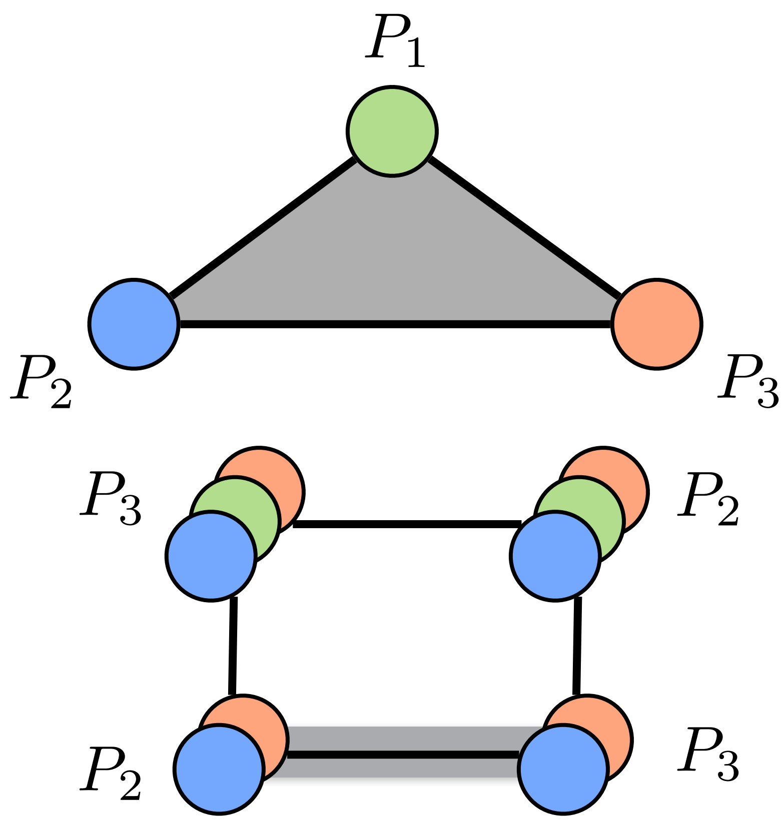
round 1:
P_1 fails



Protocol Complexes

Consensus task t = 1

input

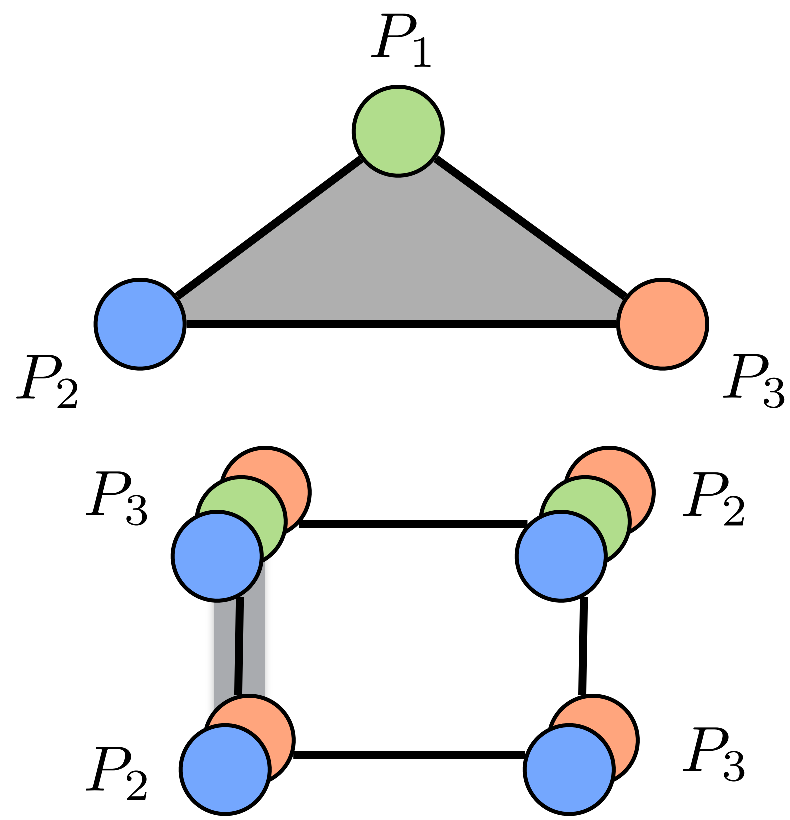


round 1: P_1 fails

Protocol Complexes

Consensus task t = 1

input

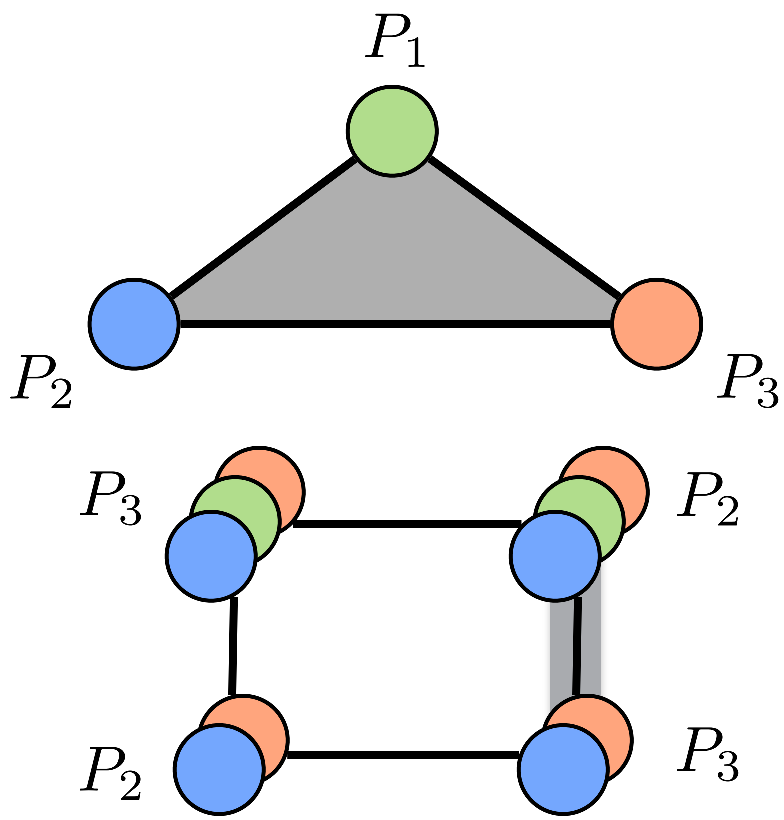


round 1:
P_1 fails

Protocol Complexes

Consensus task t = 1

input



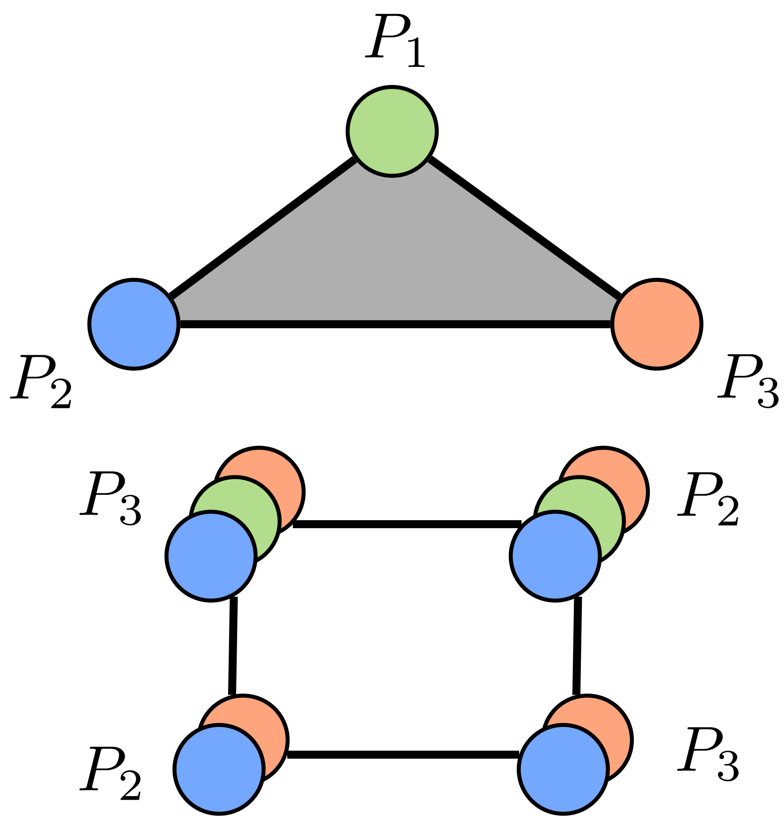
round 1:
P_1 fails

Protocol Complexes

Consensus task t = 1

input

round 1:
P_1 fails

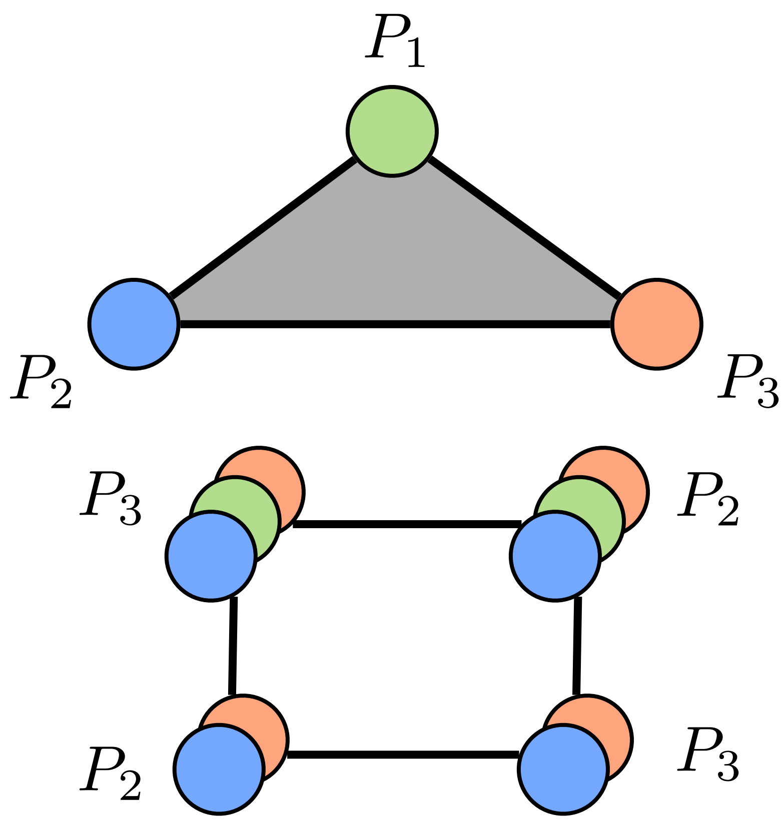


Protocol Complexes

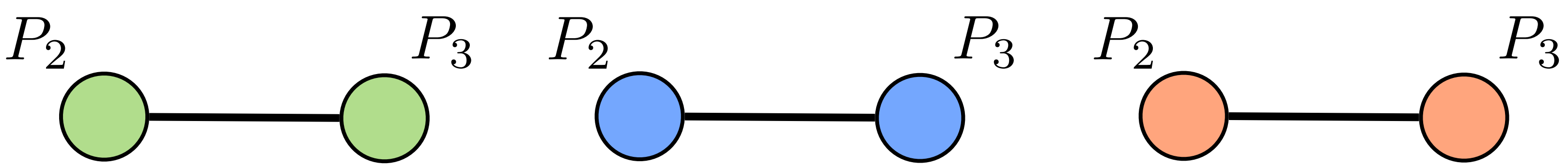
Consensus task t = 1

input

round 1: P_1 fails



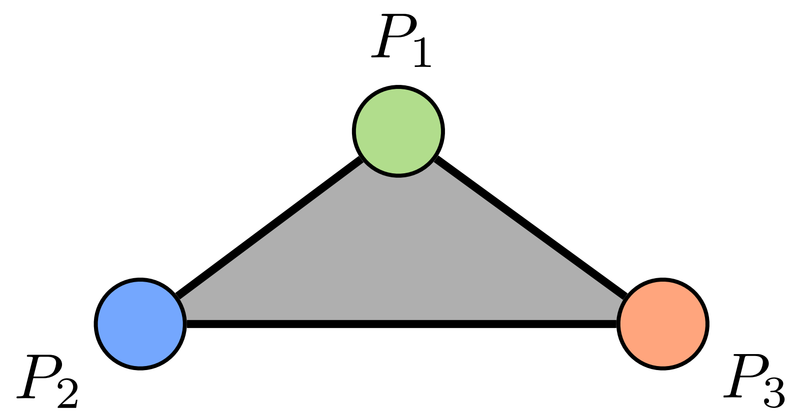
output



Protocol Complexes

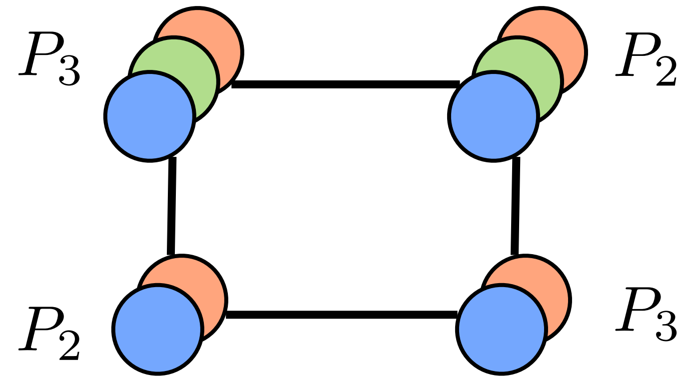
Consensus task t = 1

input

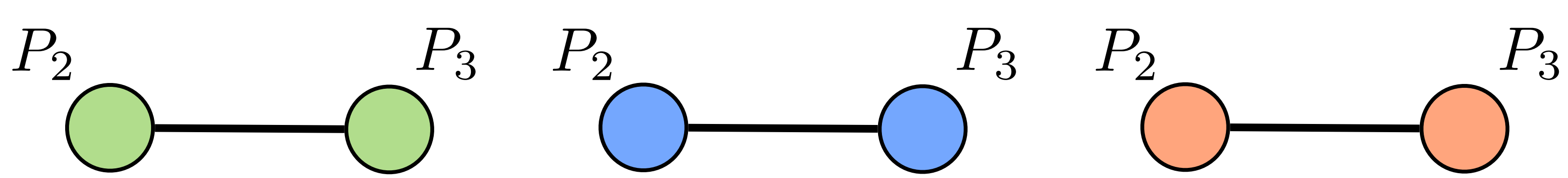


connected

round 1:
P_1 fails



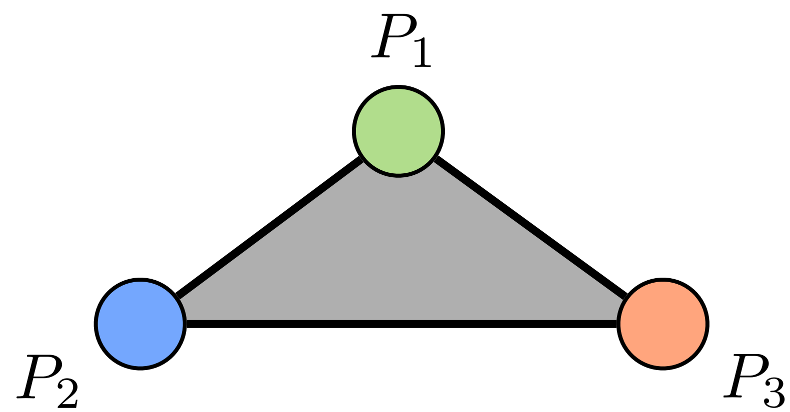
output



Protocol Complexes

Consensus task t = 1

input



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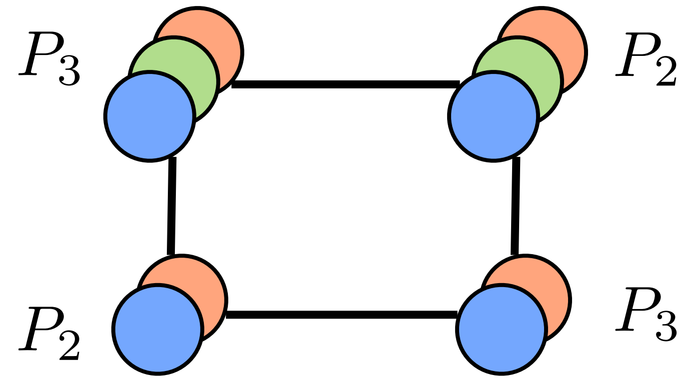
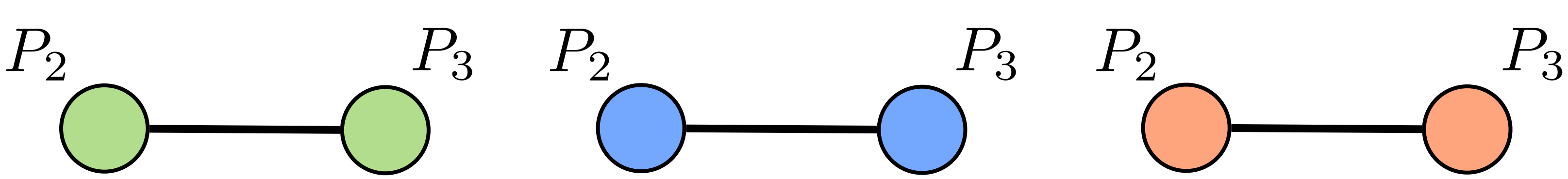


connected

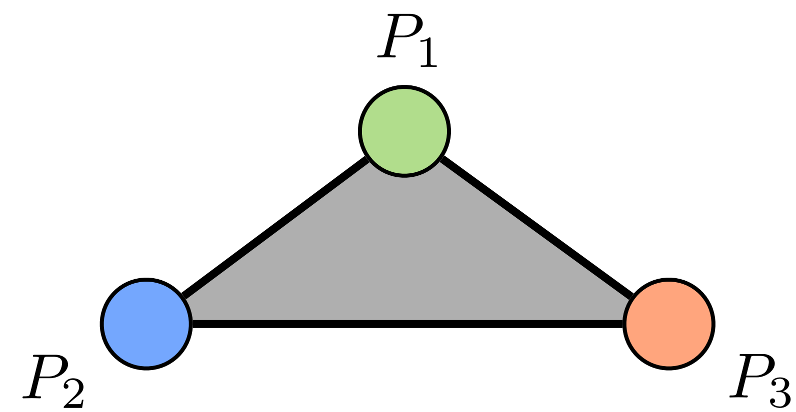
output



Protocol Complexes

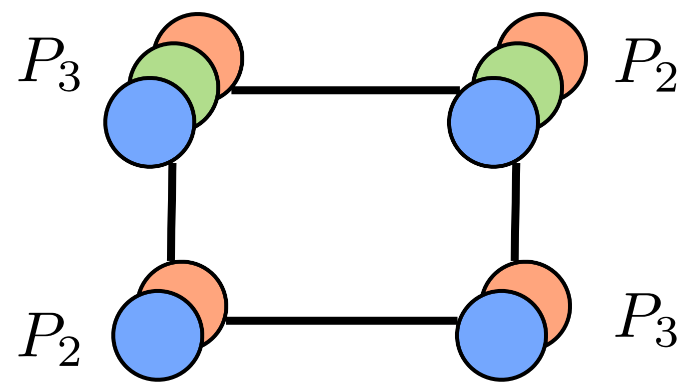
Consensus task t = 1

input



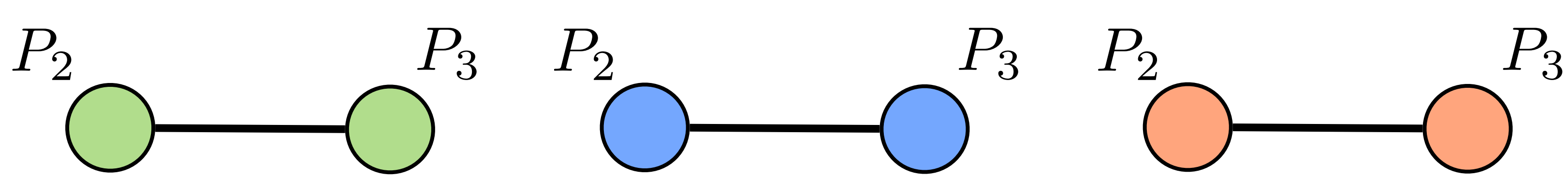
connected

round 1:
P_1 fails



connected

output



disconnected

Protocol Complexes

Consensus task <u>t = 1</u>

input

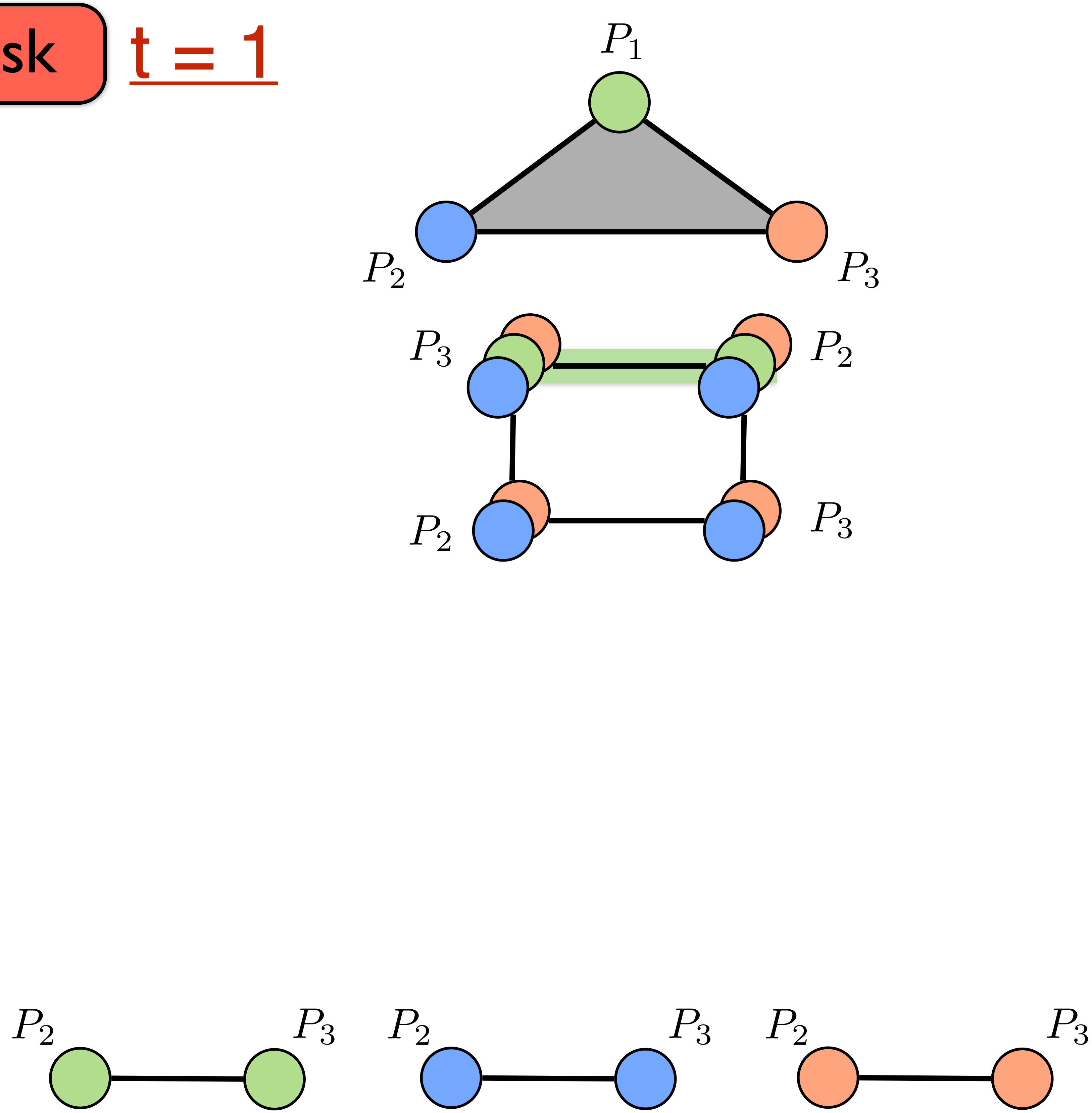
connected

round 1: P_1 fails 

connected

output

disconnected

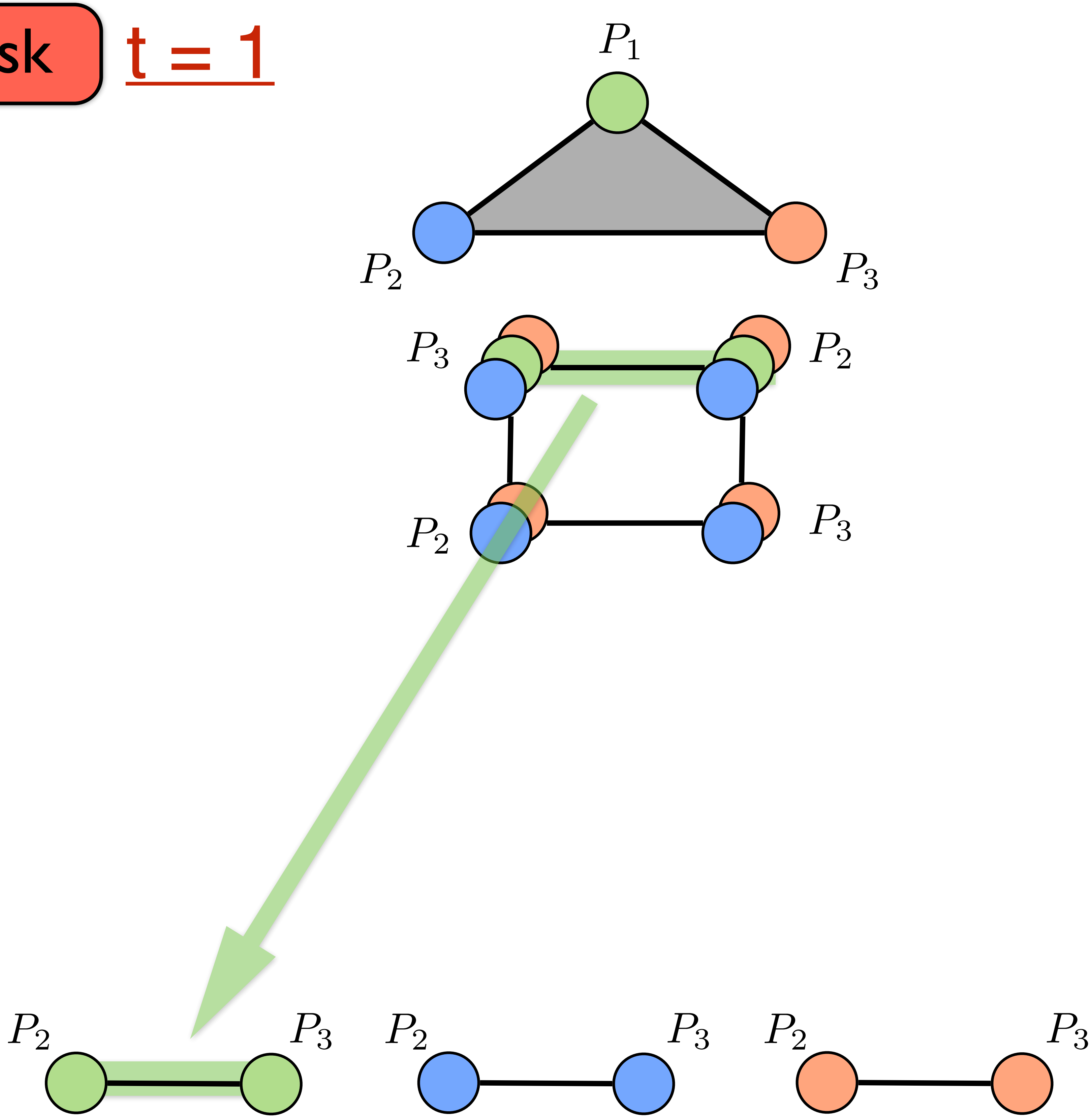


Protocol Complexes

Consensus task t = 1

input

connected



round 1:
P_1 fails

connected

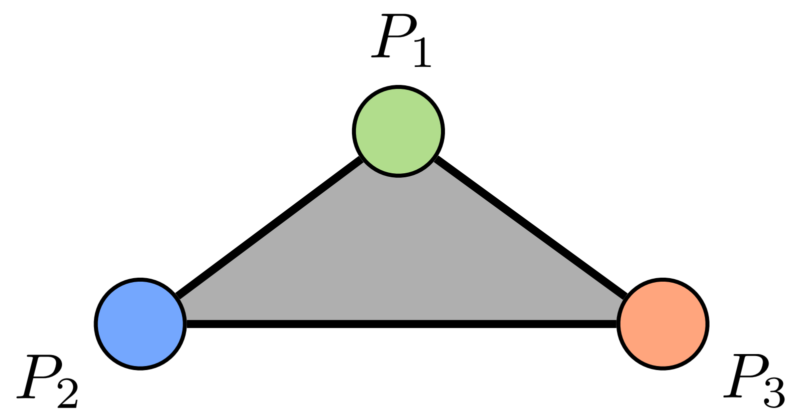
output

disconnected

Protocol Complexes

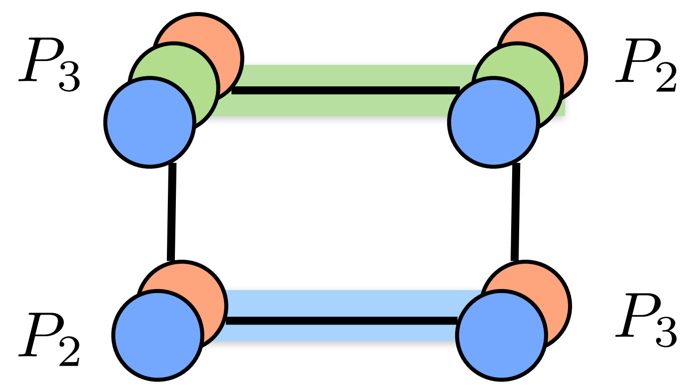
Consensus task t = 1

input



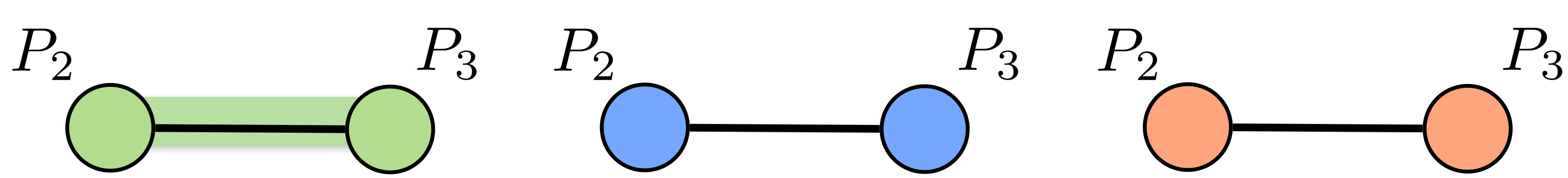
connected

round 1: P_1 fails



connected

output



disconnected

Protocol Complexes

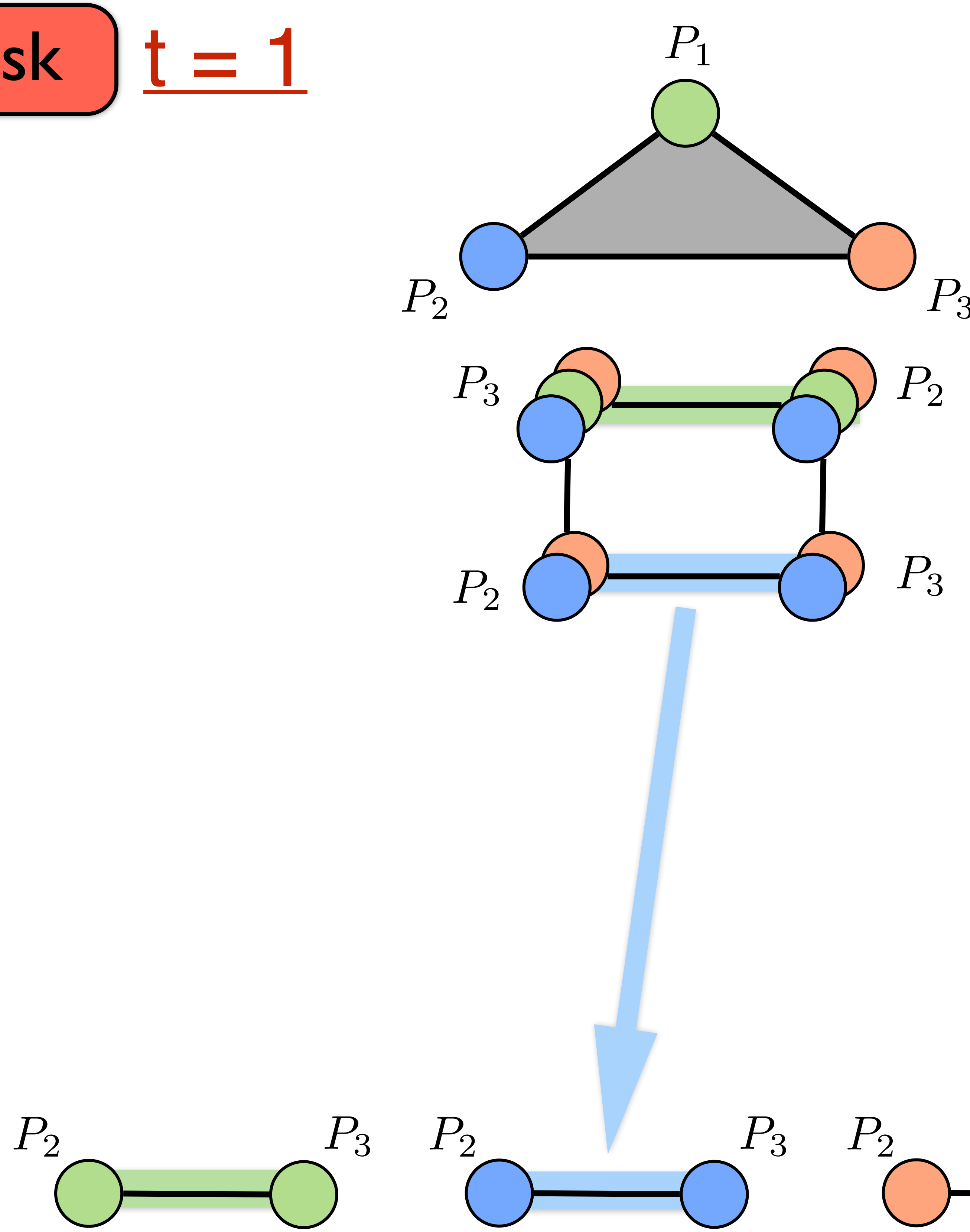
Consensus task t = 1

input

connected

round 1:
P_1 fails

connected



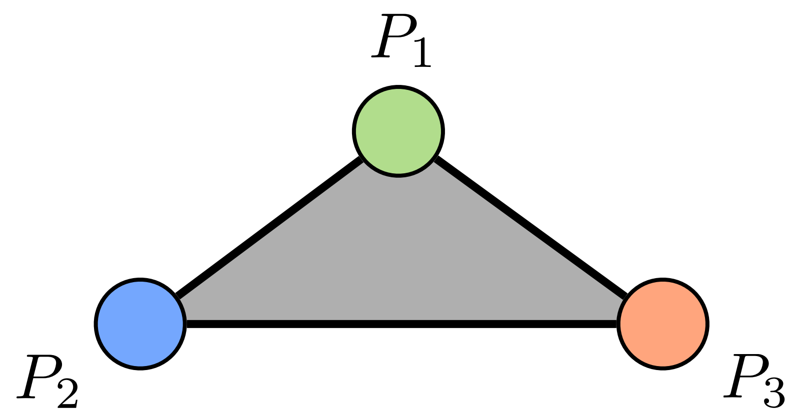
output

disconnected

Protocol Complexes

Consensus task t = 1

input



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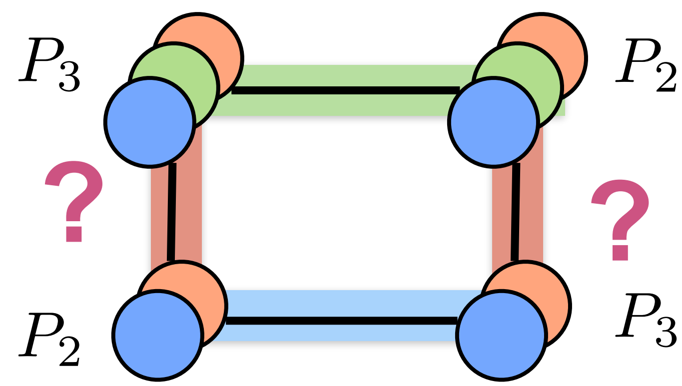
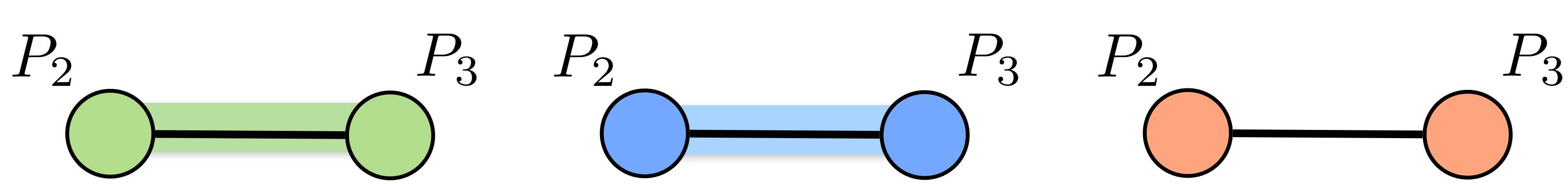


connected

output



disconnected

Protocol Complexes

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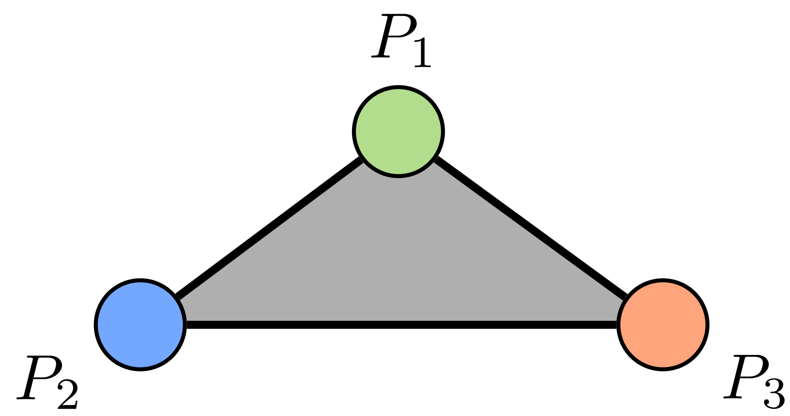
input



connected

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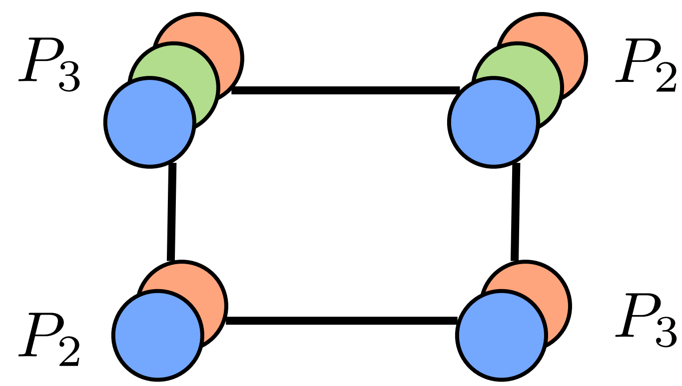
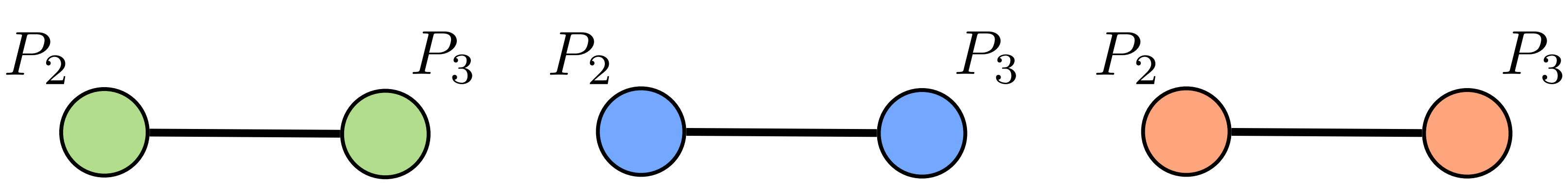
P_1 fails



connected

output

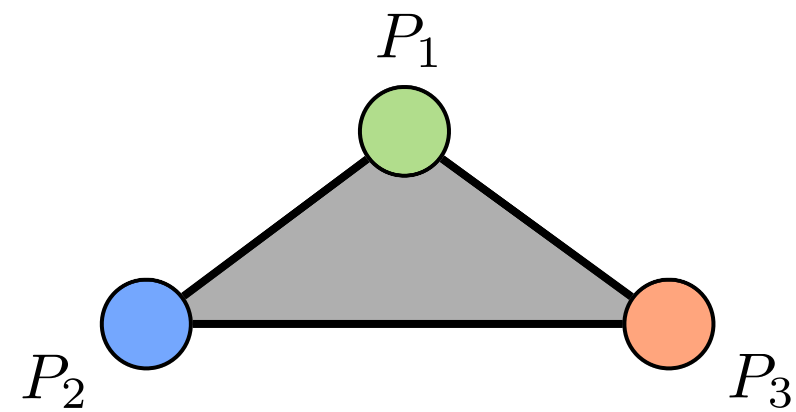


disconnected

Protocol Complexes

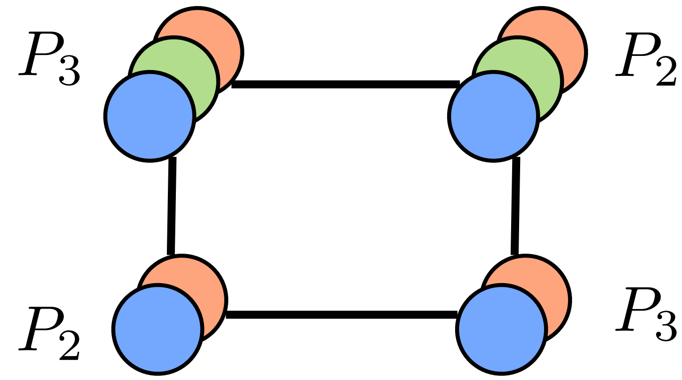
Consensus task t = 1

input



connected

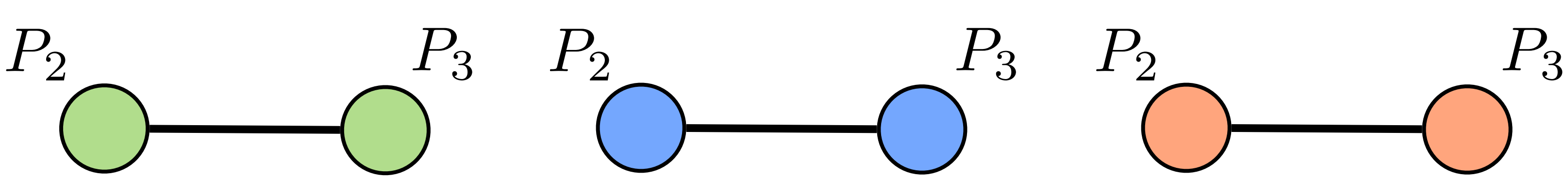
round 1:
P_1 fails



connected

round 2:
no failures

output

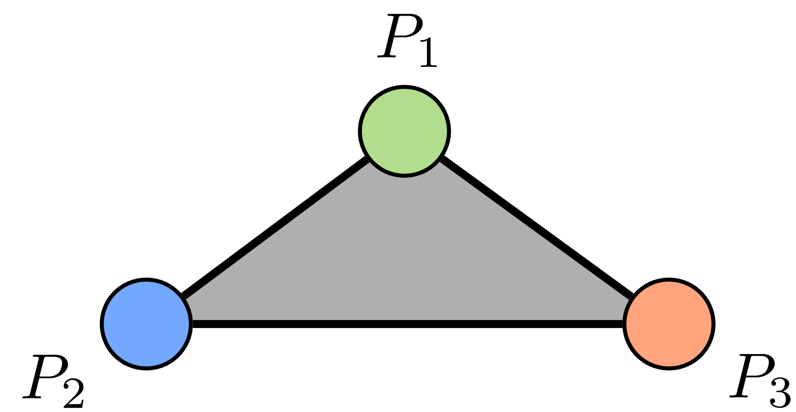


disconnected

Protocol Complexes

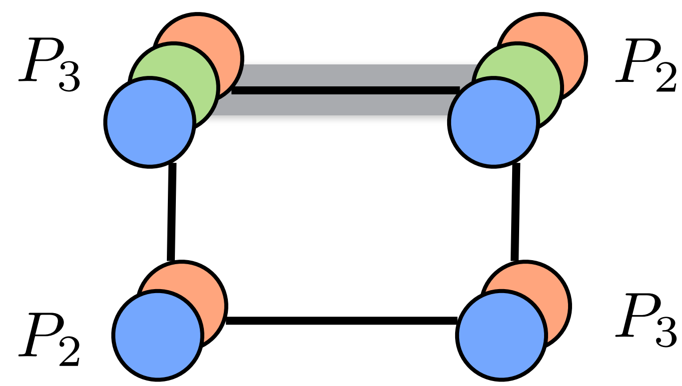
Consensus task t = 1

input



connected

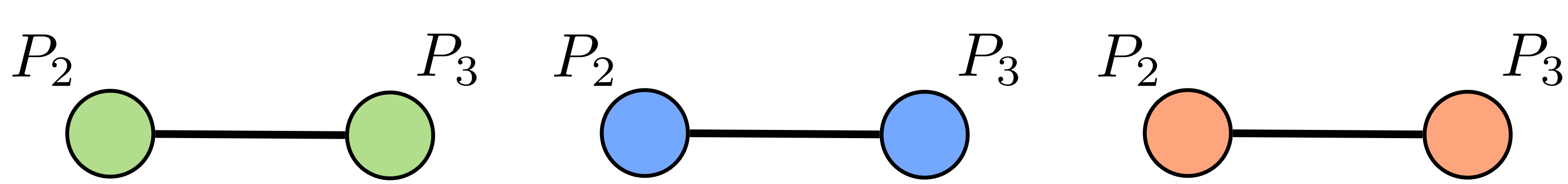
round 1:
P_1 fails



connected

round 2:
no failures

output

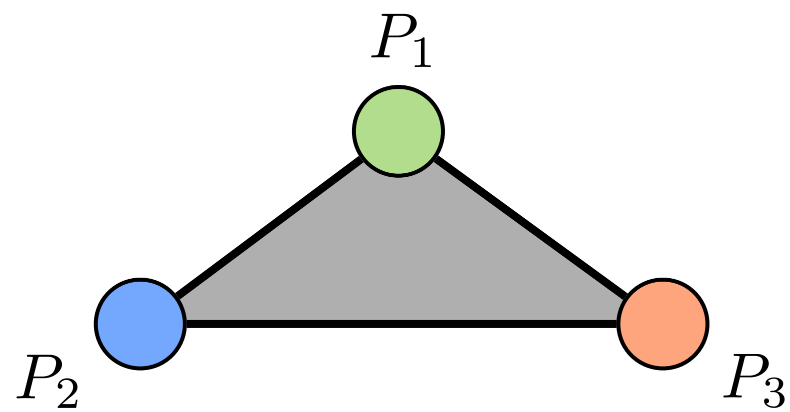


disconnected

Protocol Complexes

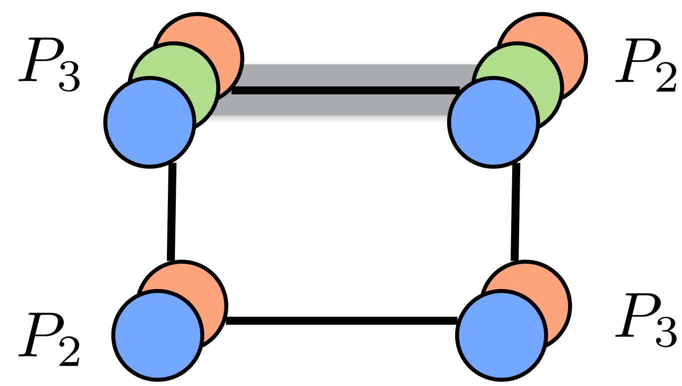
Consensus task <u>t = 1</u>

input



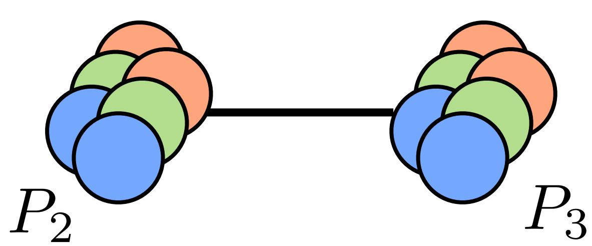
connected

round 1:
P_1 fails

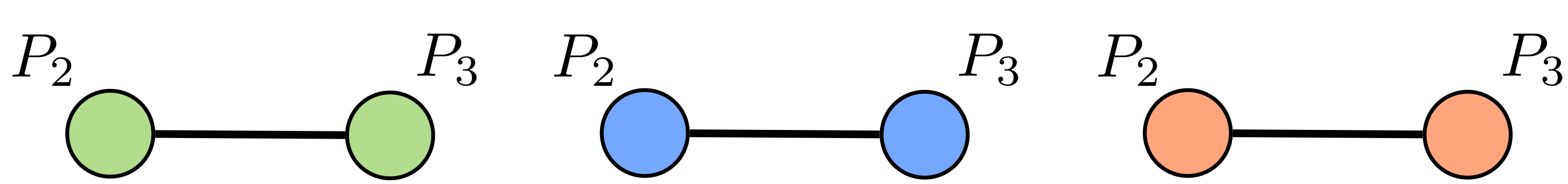


connected

round 2:
no failures



output

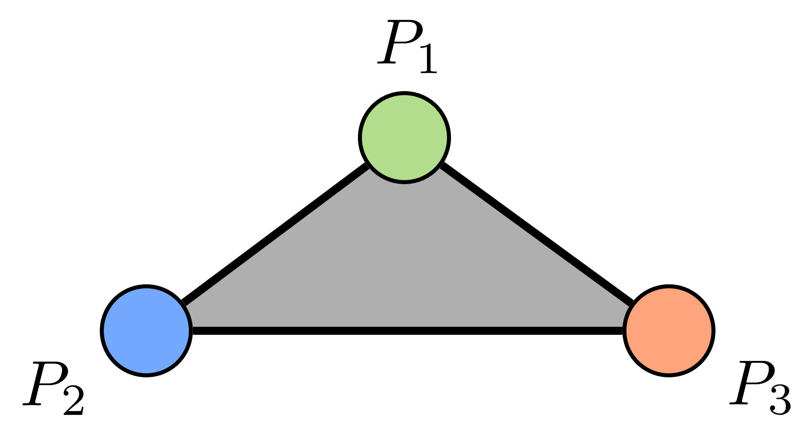


disconnected

Protocol Complexes

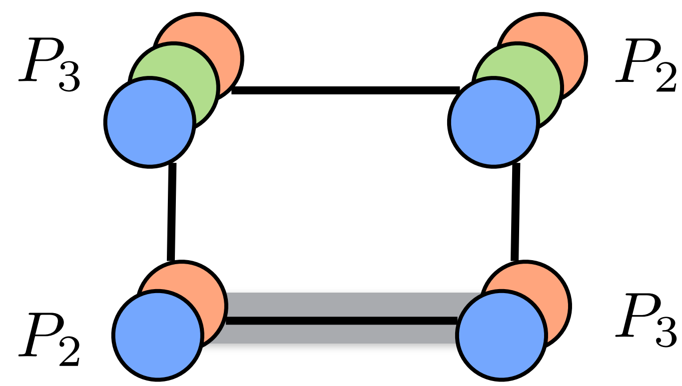
Consensus task t = 1

input



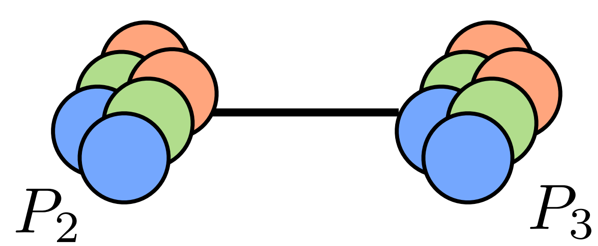
connected

round 1: P_1 fails

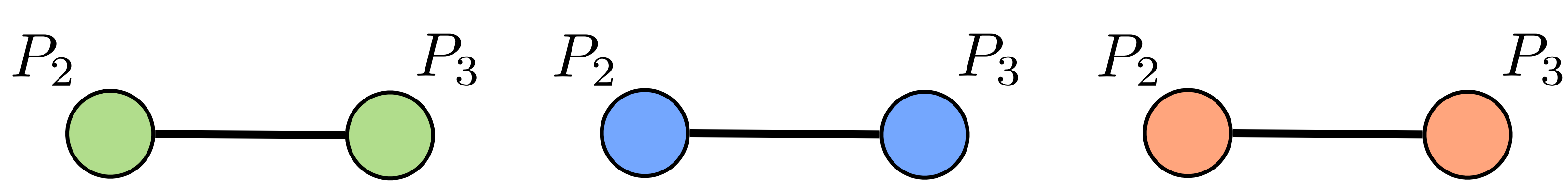


connected

round 2: no failures



output

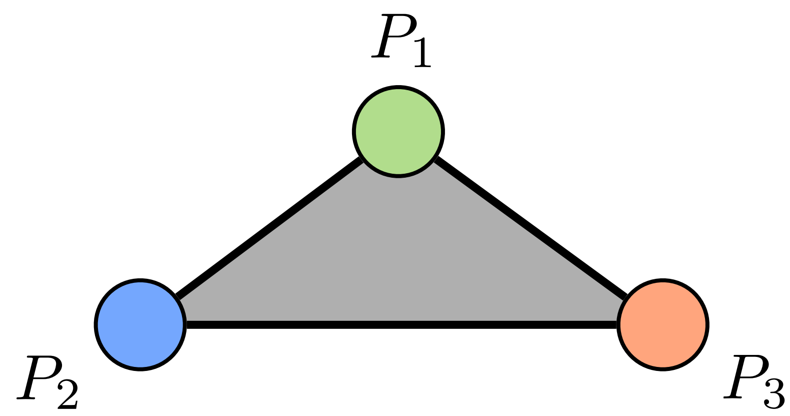


disconnected

Protocol Complexes

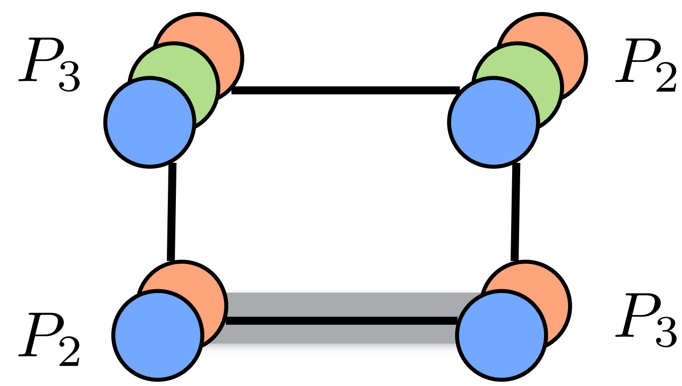
Consensus task t = 1

input



connected

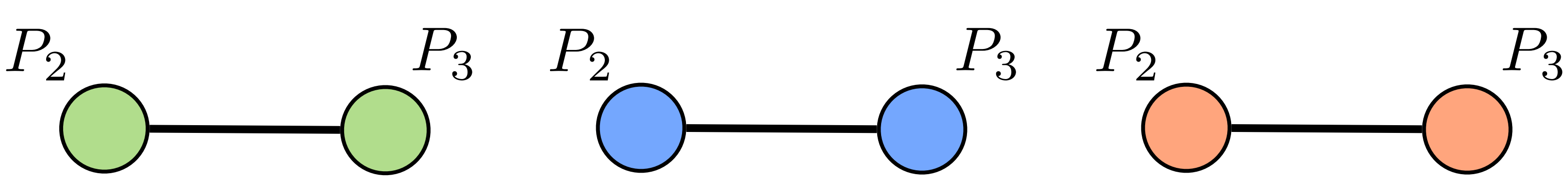
round 1:
P_1 fails



connected

round 2:
no failures

output



disconnected

Protocol Complexes

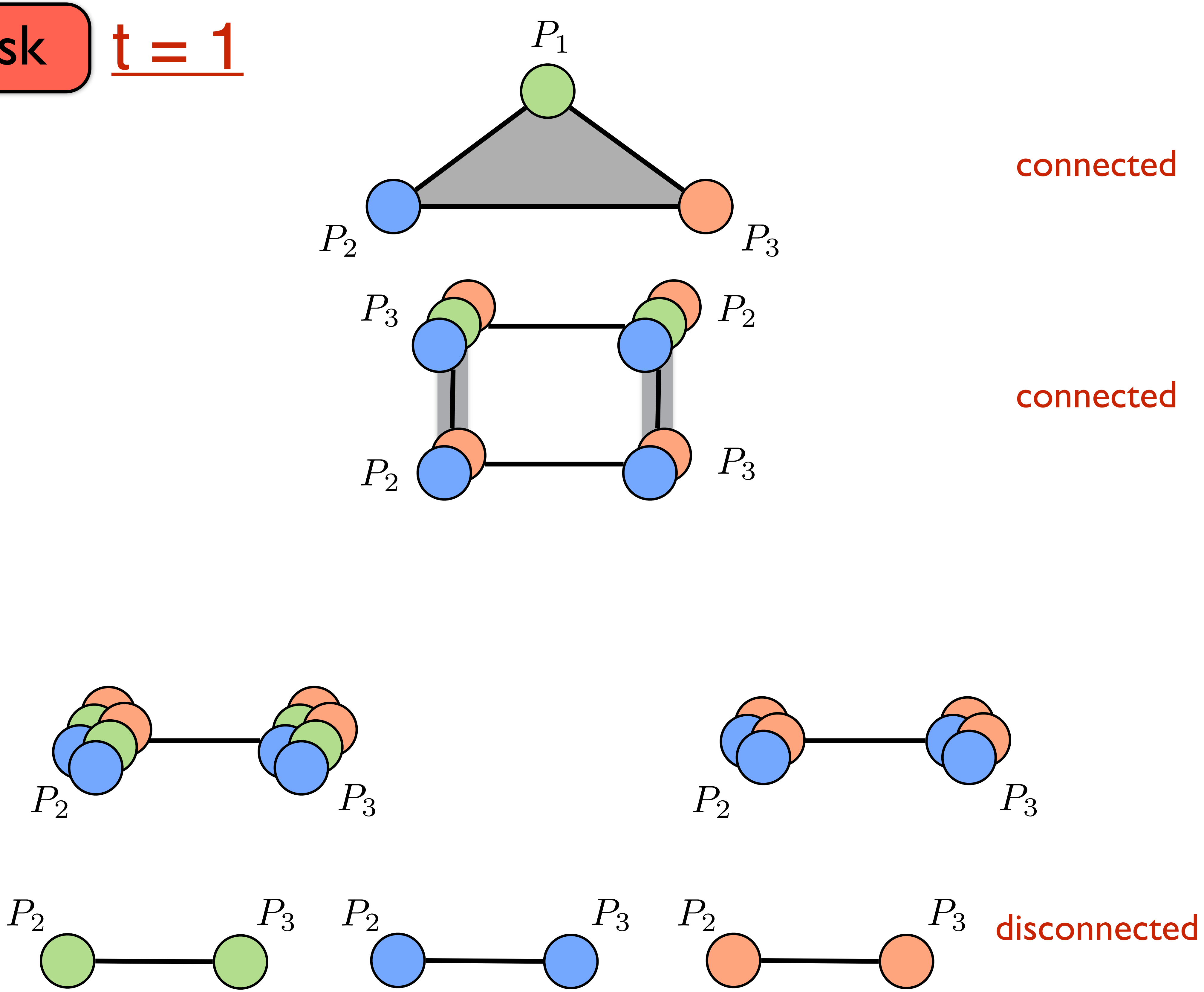
Consensus task t = 1

input

round 1:
P_1 fails

round 2:
no failures

output



Protocol Complexes

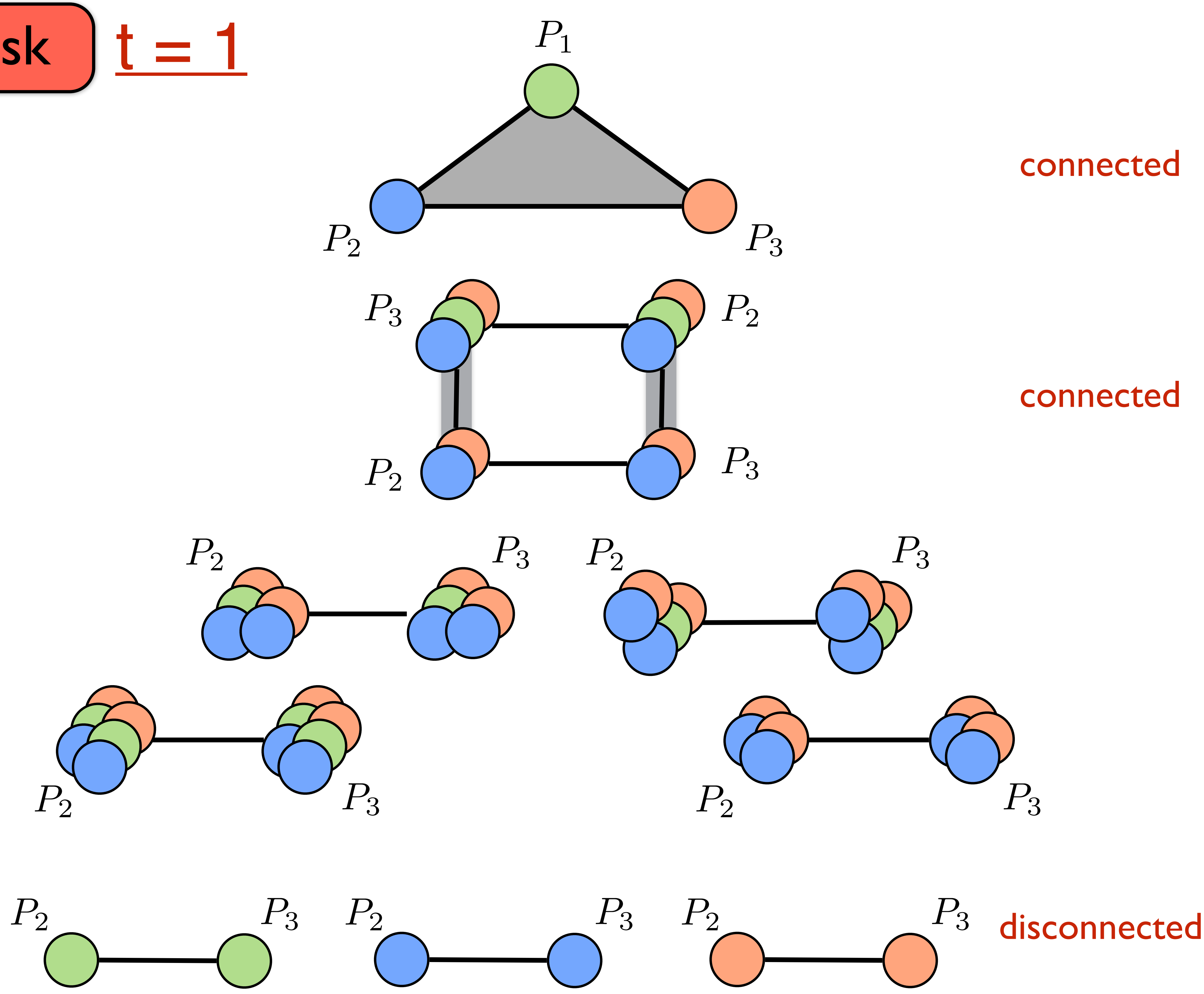
Consensus task t = 1

input

round 1: P_1 fails

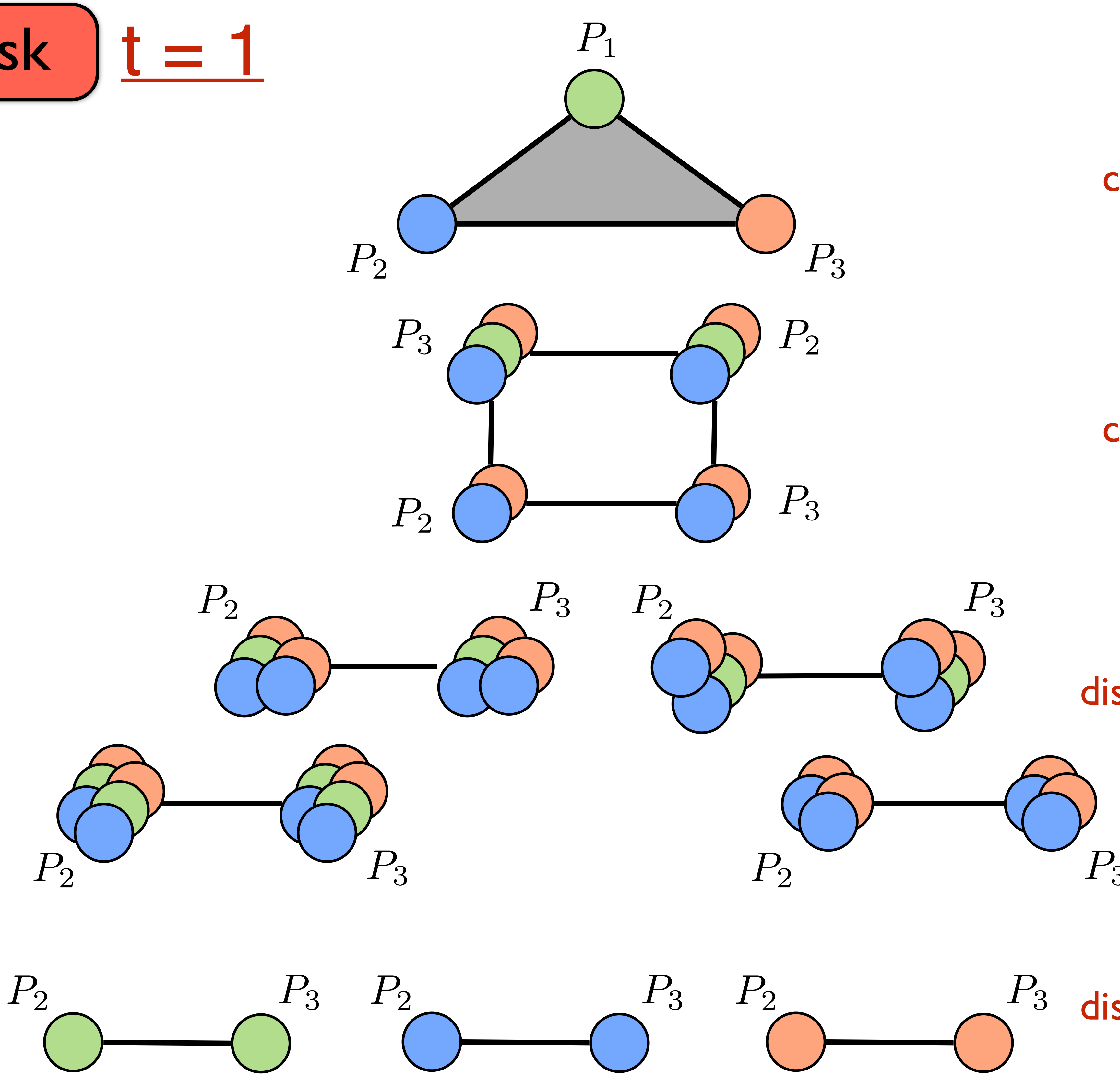
round 2: no failures

output



Protocol Complexes

Consensus task t = 1



input — connected

round 1: P_1 fails

connected

round 2: no failures — disconnected

output — disconnected

Protocol Complexes

Consensus task t = 1

input

connected

round 1: P_1 fails

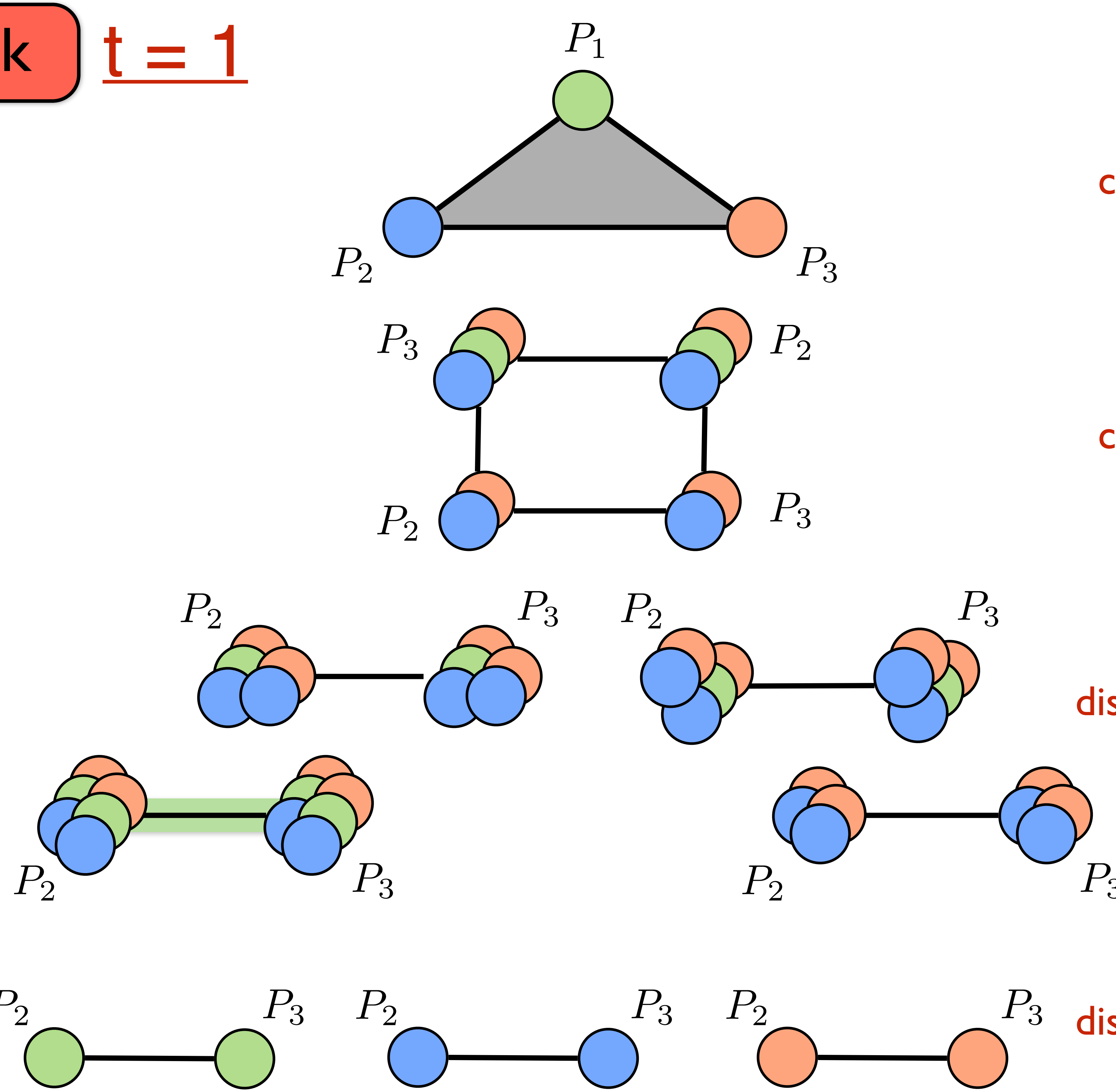
connected

round 2: no failures

disconnected

output

disconnected



Protocol Complexes

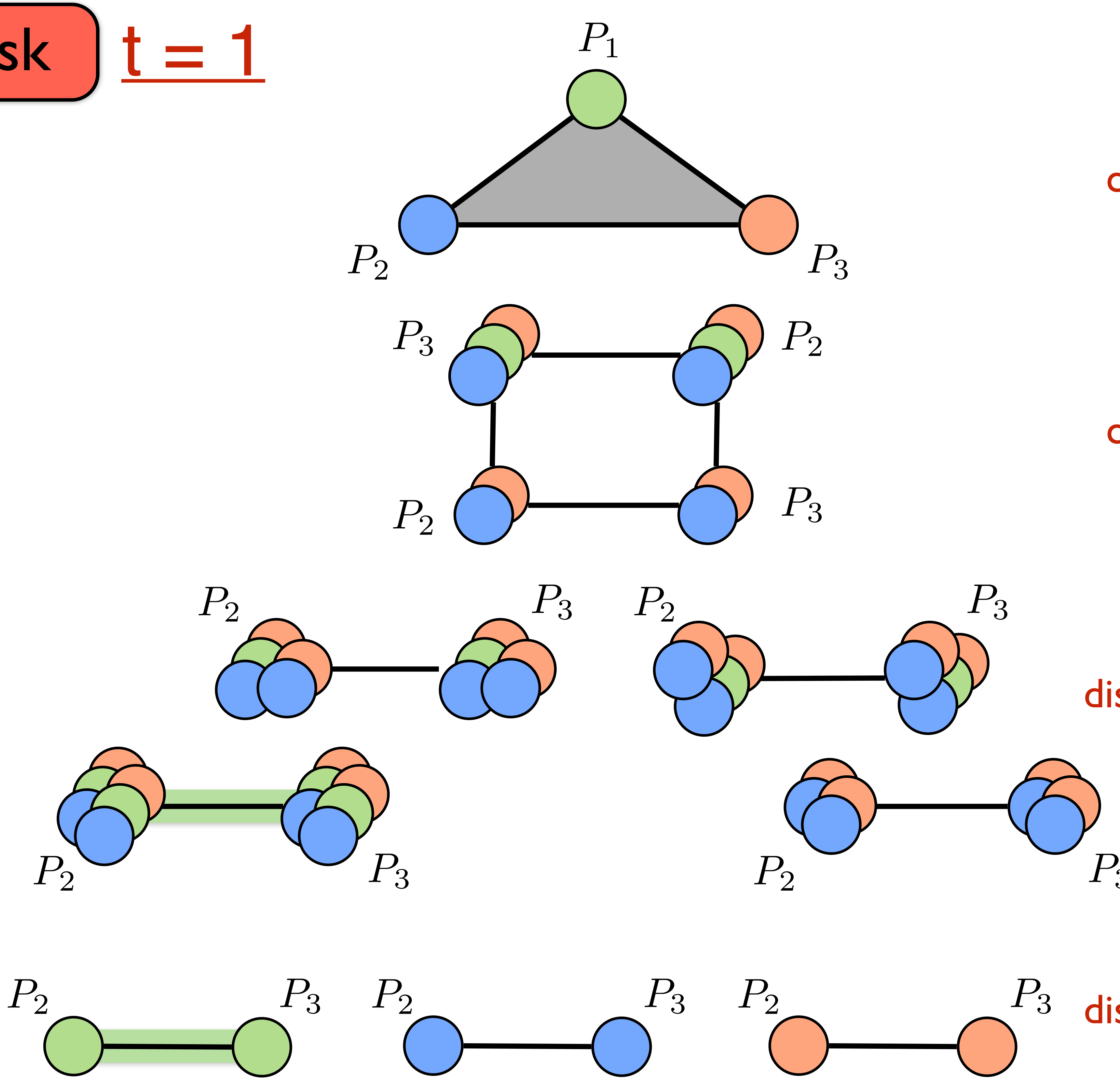
Consensus task t = 1

input

round 1: P_1 fails

round 2: no failures

output



connected

connected

disconnected

disconnected

Protocol Complexes

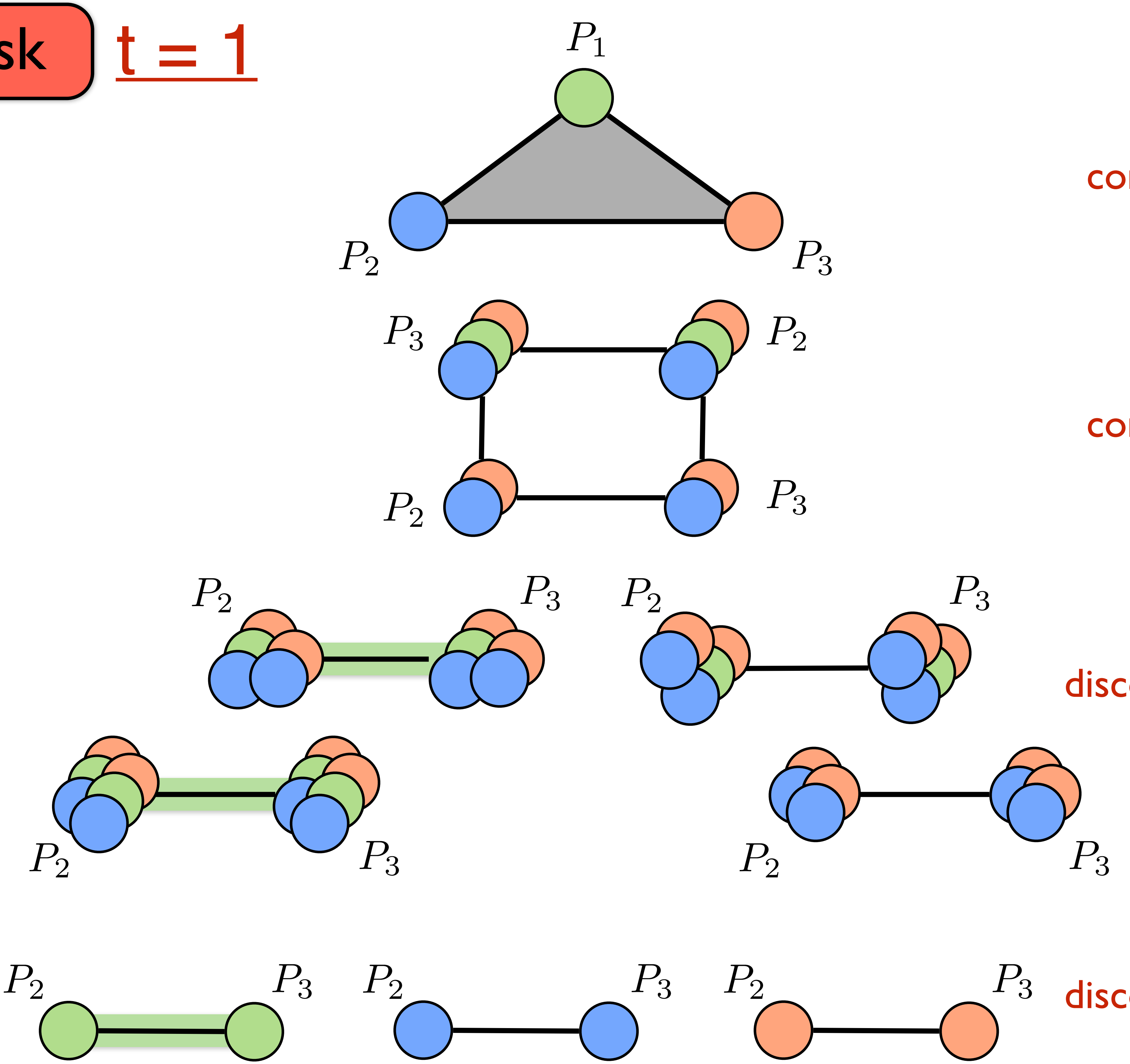
Consensus task t = 1

input — connected

round 1: P_1 fails  — connected

round 2: no failures — disconnected

output — disconnected



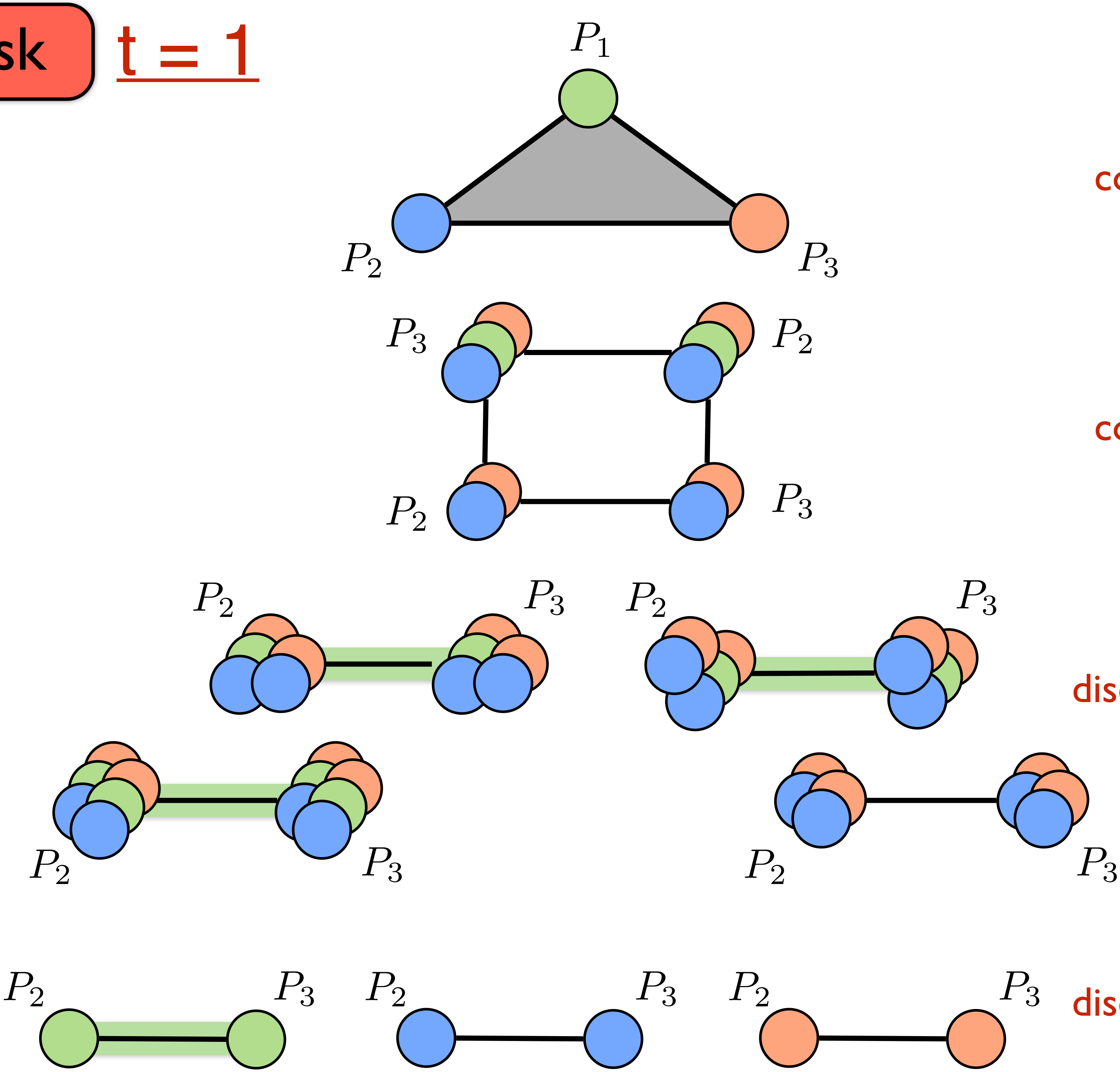
Protocol Complexes

Consensus task t = 1

input

connected

round 1:
P_1 fails



connected

round 2:
no failures

disconnected

output

disconnected

Protocol Complexes

Consensus task t = 1

input

connected

round 1:
P_1 fails

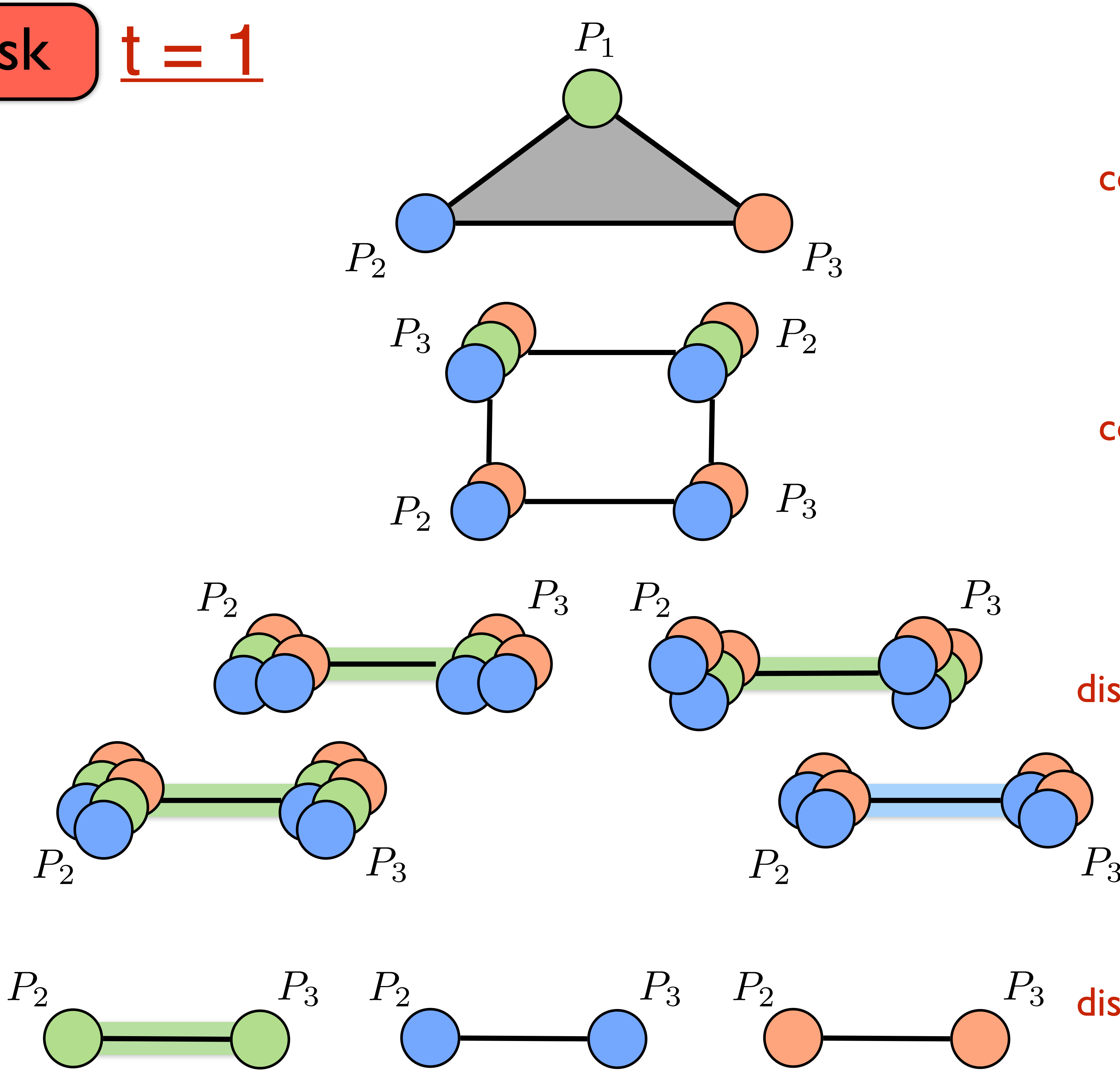
connected

round 2:
no failures

disconnected

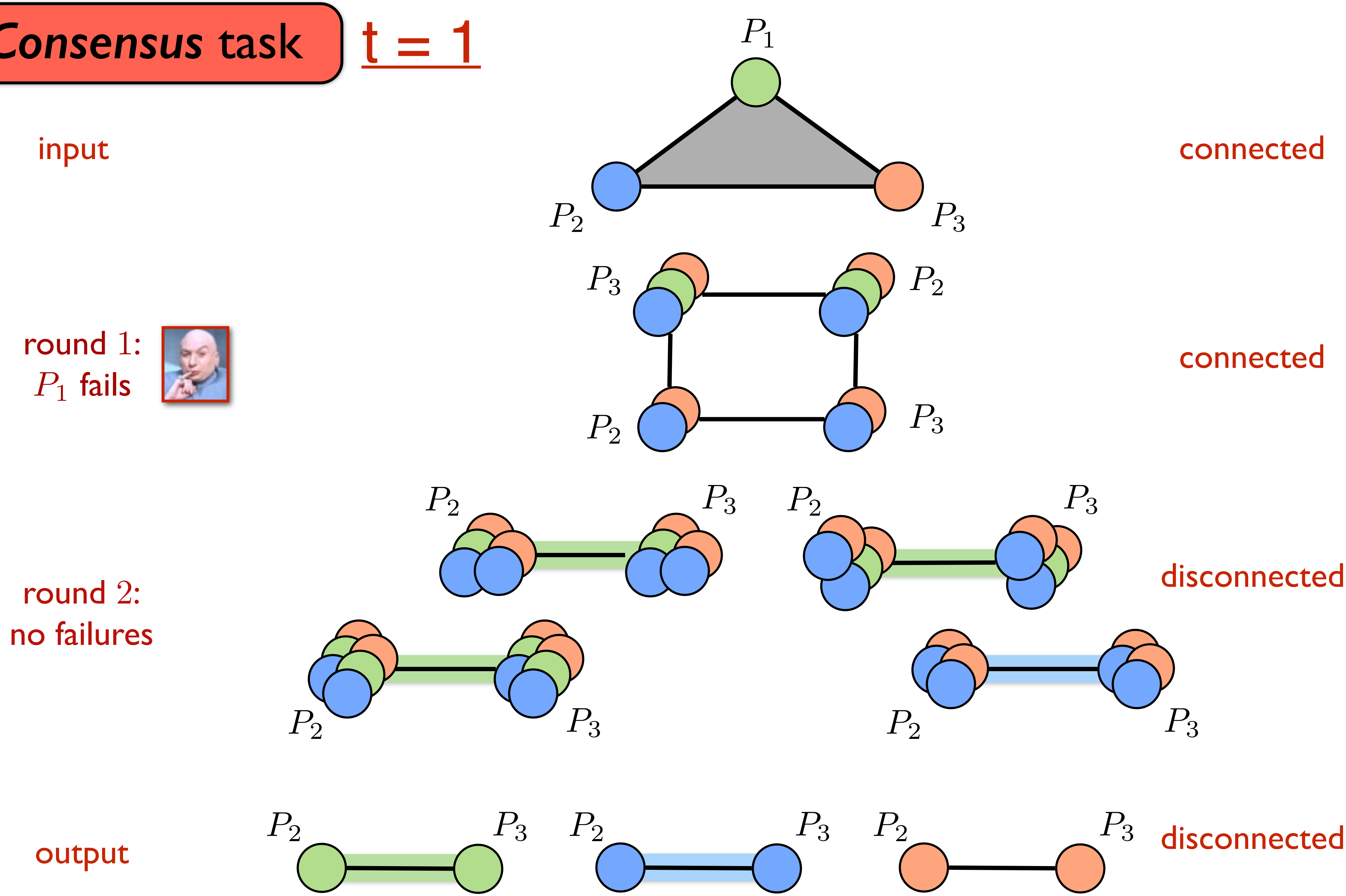
output

disconnected



Protocol Complexes

Consensus task t = 1



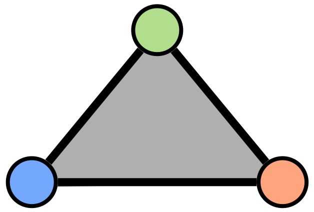
With Crash Failures…

Herlihy & Rajsbaum 2000

With Crash Failures…

Herlihy & Rajsbaum 2000

$\mathcal{K}_0 = \mathcal{I}^*$



With Crash Failures…

Herlihy & Rajsbaum 2000

adversarial execution

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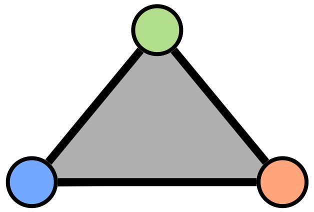

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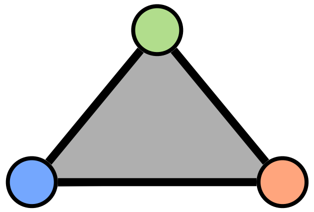

$\mathcal{K}_1 = \mathcal{R}_c(\mathcal{K}_0, k)$

$(k-1)$-connected

With Crash Failures…

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adversarial execution

k failures per round

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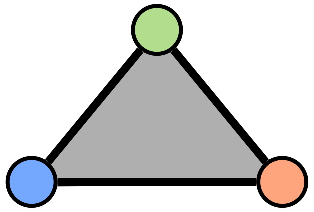

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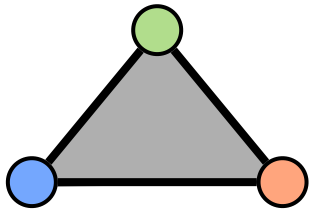

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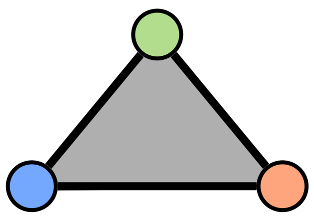

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$(k-1)$-connected

$\mathcal{K}_2 = \mathcal{R}_c(\mathcal{K}_1, k)$

$(k-1)$-connected

$\mathcal{K}_3 = \mathcal{R}_c(\mathcal{K}_2, k)$

$(k-1)$-connected

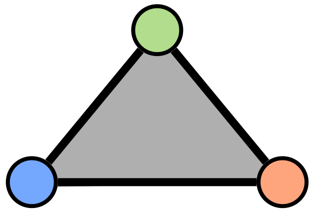
With Crash Failures…

Herlihy & Rajsbaum 2000

adversarial execution

k failures per round

$\mathcal{K}_0 = \mathcal{I}^*$ $\quad \mathcal{K}_1 = \mathcal{R}_c(\mathcal{K}_0, k)$ $\quad \mathcal{K}_2 = \mathcal{R}_c(\mathcal{K}_1, k)$ $\quad \mathcal{K}_3 = \mathcal{R}_c(\mathcal{K}_2, k)$



…

$(k-1)$-connected $\quad (k-1)$-connected $\quad (k-1)$-connected

With Crash Failures…

Herlihy & Rajsbaum 2000

adversarial execution

k failures per round

$\mathcal{K}_0 = \mathcal{I}^*$ $\mathcal{K}_1 = \mathcal{R}_c(\mathcal{K}_0, k)$ $\mathcal{K}_2 = \mathcal{R}_c(\mathcal{K}_1, k)$ $\mathcal{K}_3 = \mathcal{R}_c(\mathcal{K}_2, k)$

…

$(k-1)$-connected $(k-1)$-connected $(k-1)$-connected

Theorem:

While the protocol complex is $(k-1)$-connected, we cannot solve the k-set agreement task

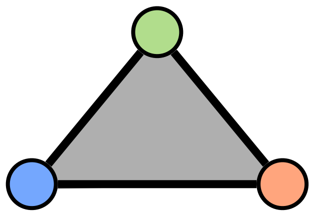
With Crash Failures…

Herlihy & Rajsbaum 2000

adversarial execution

k failures per round

$\mathcal{K}_0 = \mathcal{I}^*$ $\mathcal{K}_1 = \mathcal{R}_c(\mathcal{K}_0, k)$ $\mathcal{K}_2 = \mathcal{R}_c(\mathcal{K}_1, k)$ $\mathcal{K}_3 = \mathcal{R}_c(\mathcal{K}_2, k)$



…

$(k-1)$-connected $(k-1)$-connected $(k-1)$-connected

With Crash Failures…

Herlihy & Rajsbaum 2000

adversarial execution

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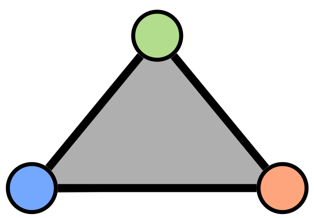

…

$(k-1)$-connected $(k-1)$-connected $(k-1)$-connected

Theorem:

We cannot solve k-set agreement with t failures in $\lfloor t/k \rfloor$ or less rounds.

With Crash Failures…

Herlihy & Rajsbaum 2000

adversarial execution

k failures per round

$\mathcal{K}_0 = \mathcal{I}^*$ $\mathcal{K}_1 = \mathcal{R}_c(\mathcal{K}_0, k)$ $\mathcal{K}_2 = \mathcal{R}_c(\mathcal{K}_1, k)$ $\mathcal{K}_3 = \mathcal{R}_c(\mathcal{K}_2, k)$

$(k-1)$-connected $(k-1)$-connected $(k-1)$-connected

Theorem:

We cannot solve k-set agreement with t failures in $\lfloor t/k \rfloor$ or less rounds.

We have a $\lfloor t/k \rfloor + 1$ protocol

Byzantine Equivocation

Is equivocation a problem in synchronous systems?

Byzantine Equivocation

Is equivocation a problem in synchronous systems? *sort of*

Byzantine Equivocation

Is equivocation a problem in synchronous systems? sort of

Ex.: 4 *processes,* 1 *failure*

Byzantine Equivocation

Is equivocation a problem in synchronous systems? sort of

you know this

Ex.: 4 *processes,* 1 *failure*

Byzantine Equivocation

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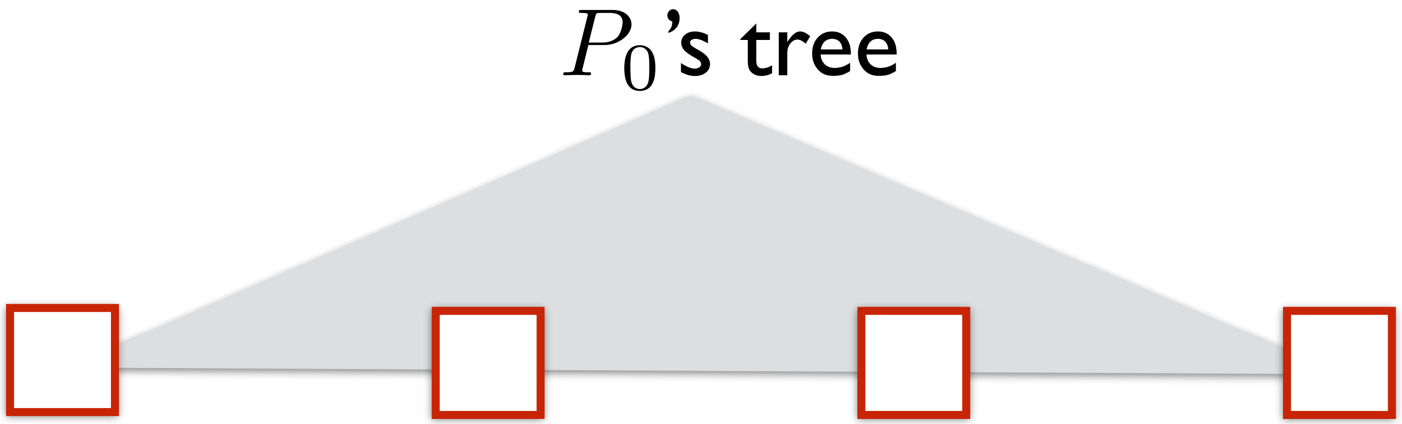
Ex.: 4 *processes,* 1 *failure*



Byzantine Equivocation

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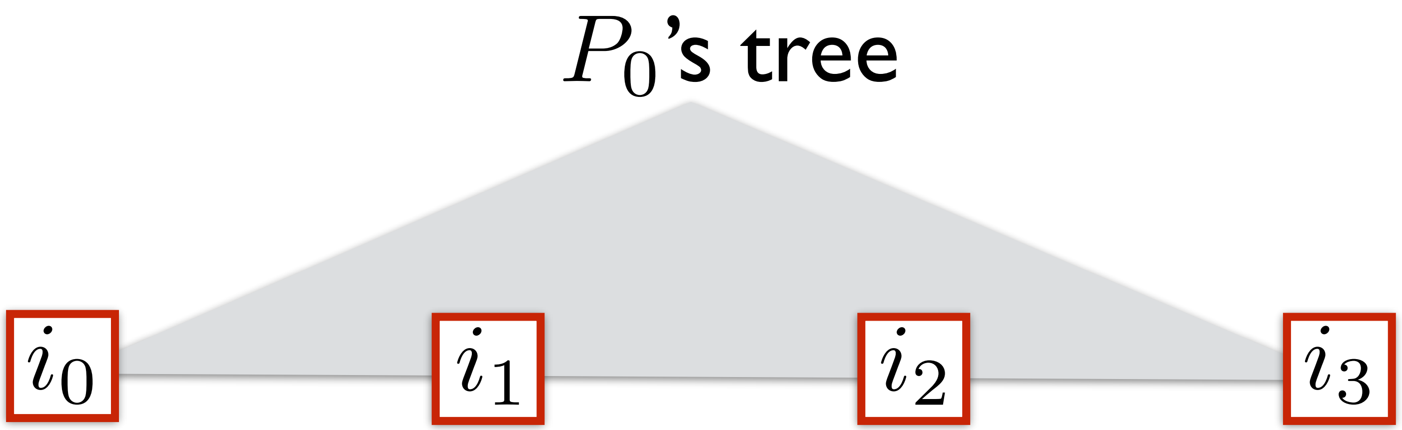

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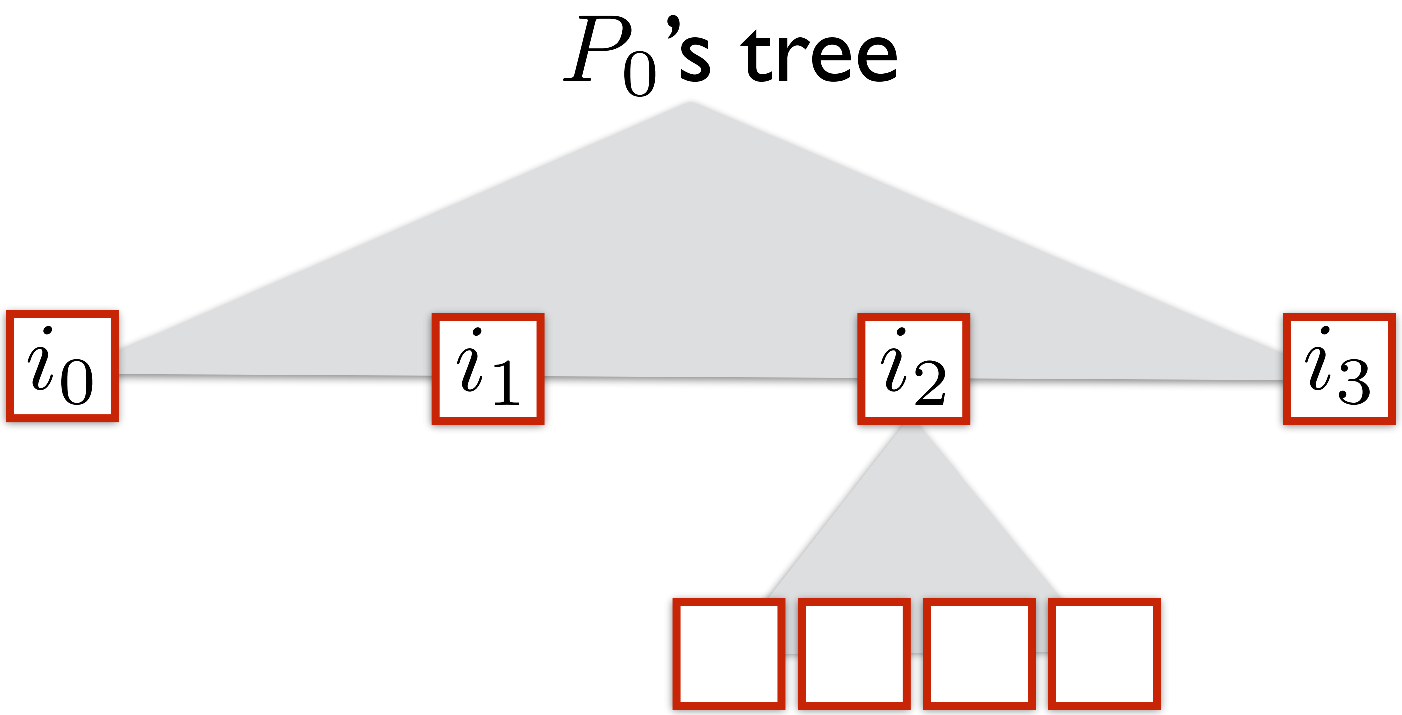

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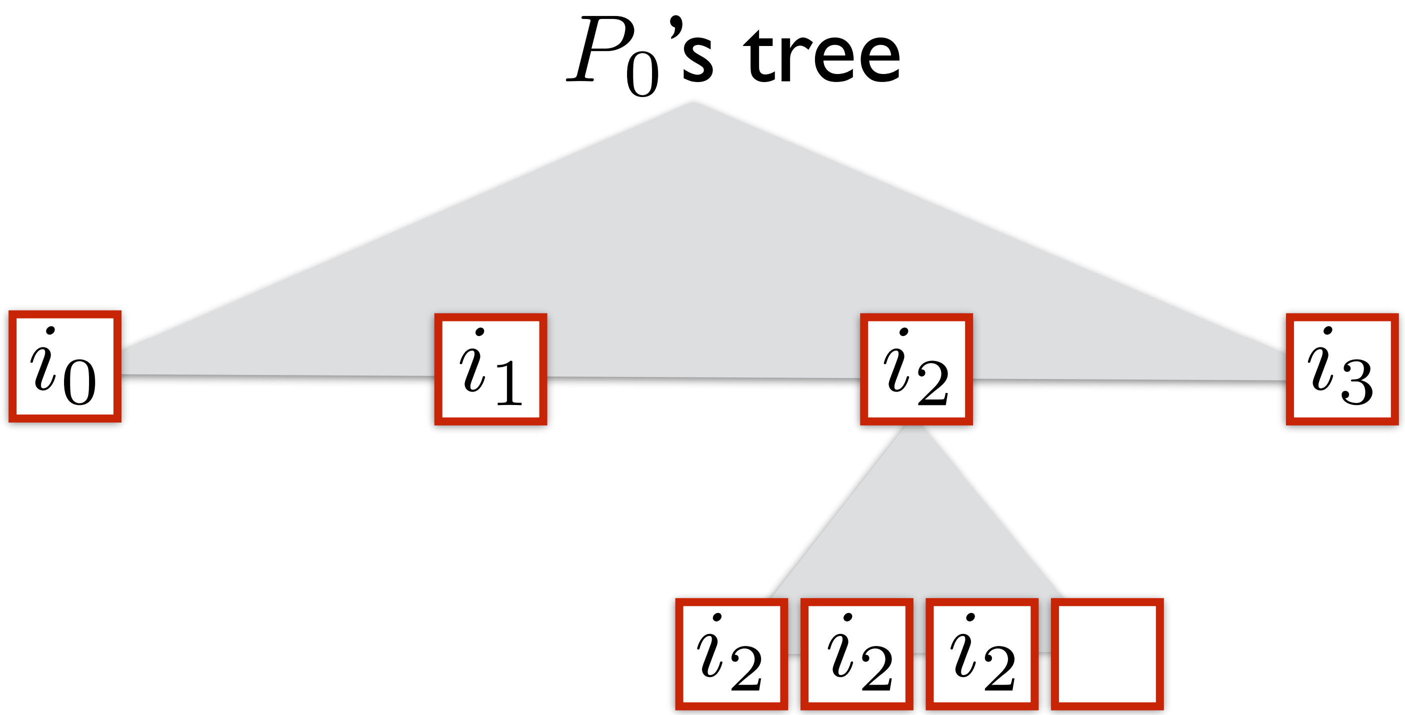

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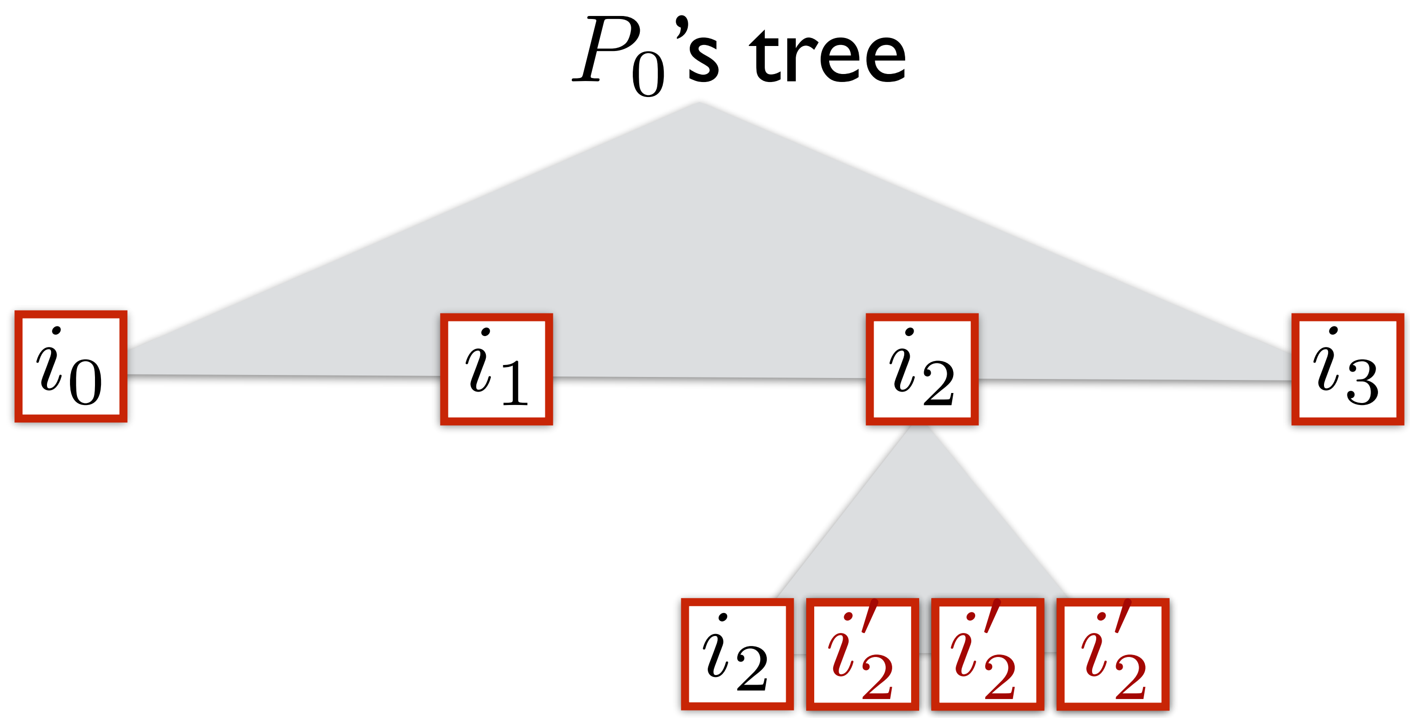

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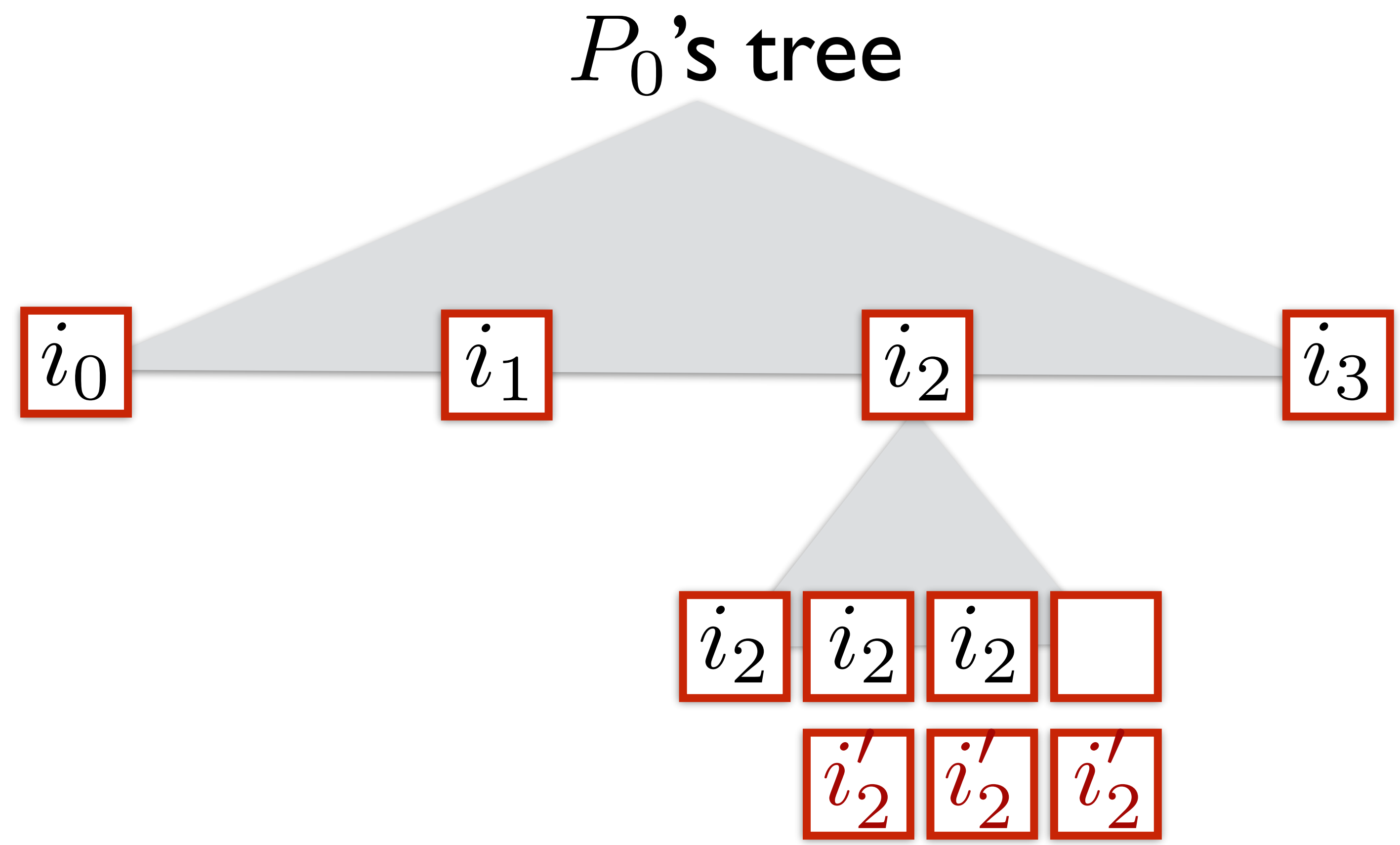

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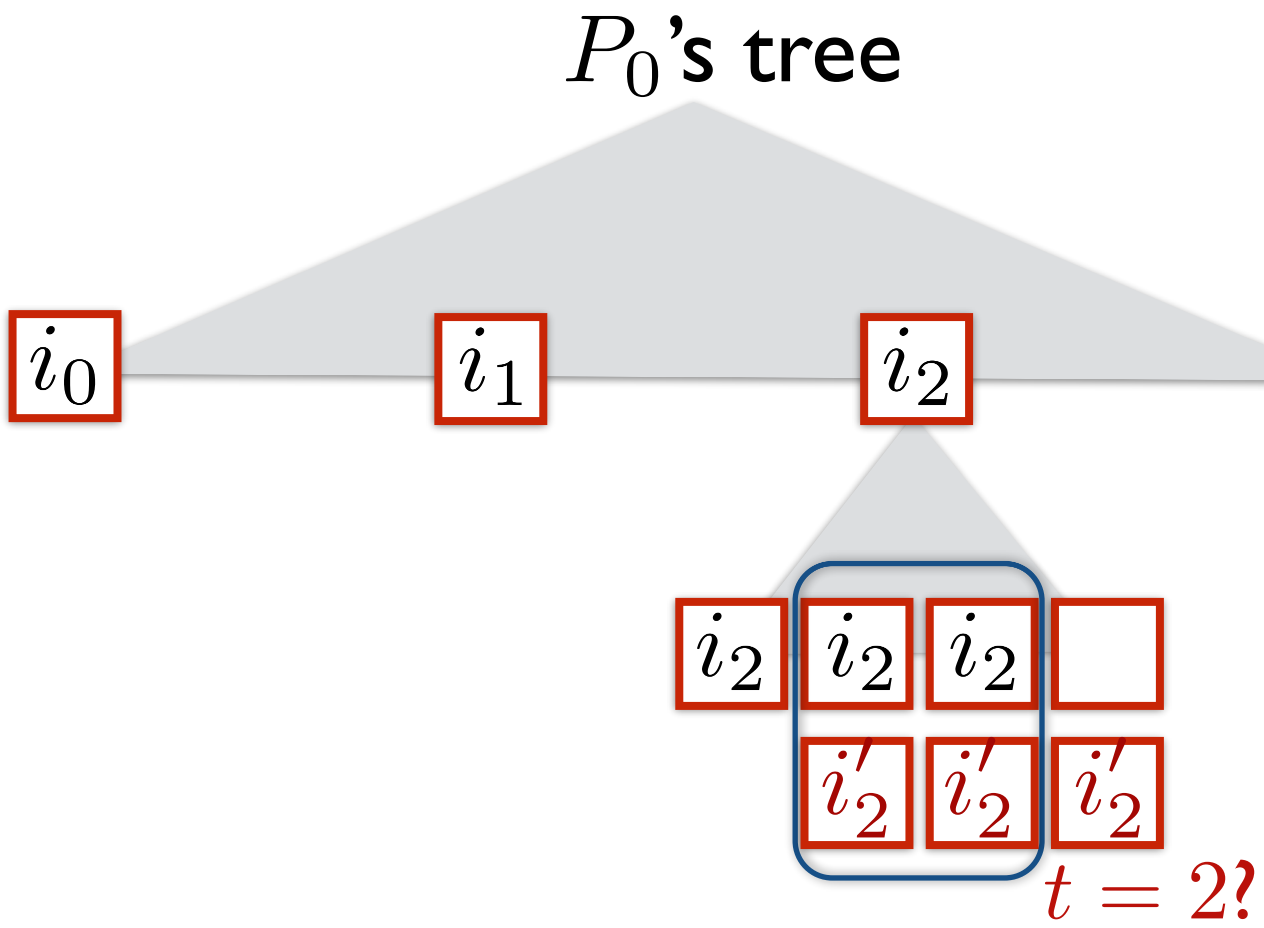

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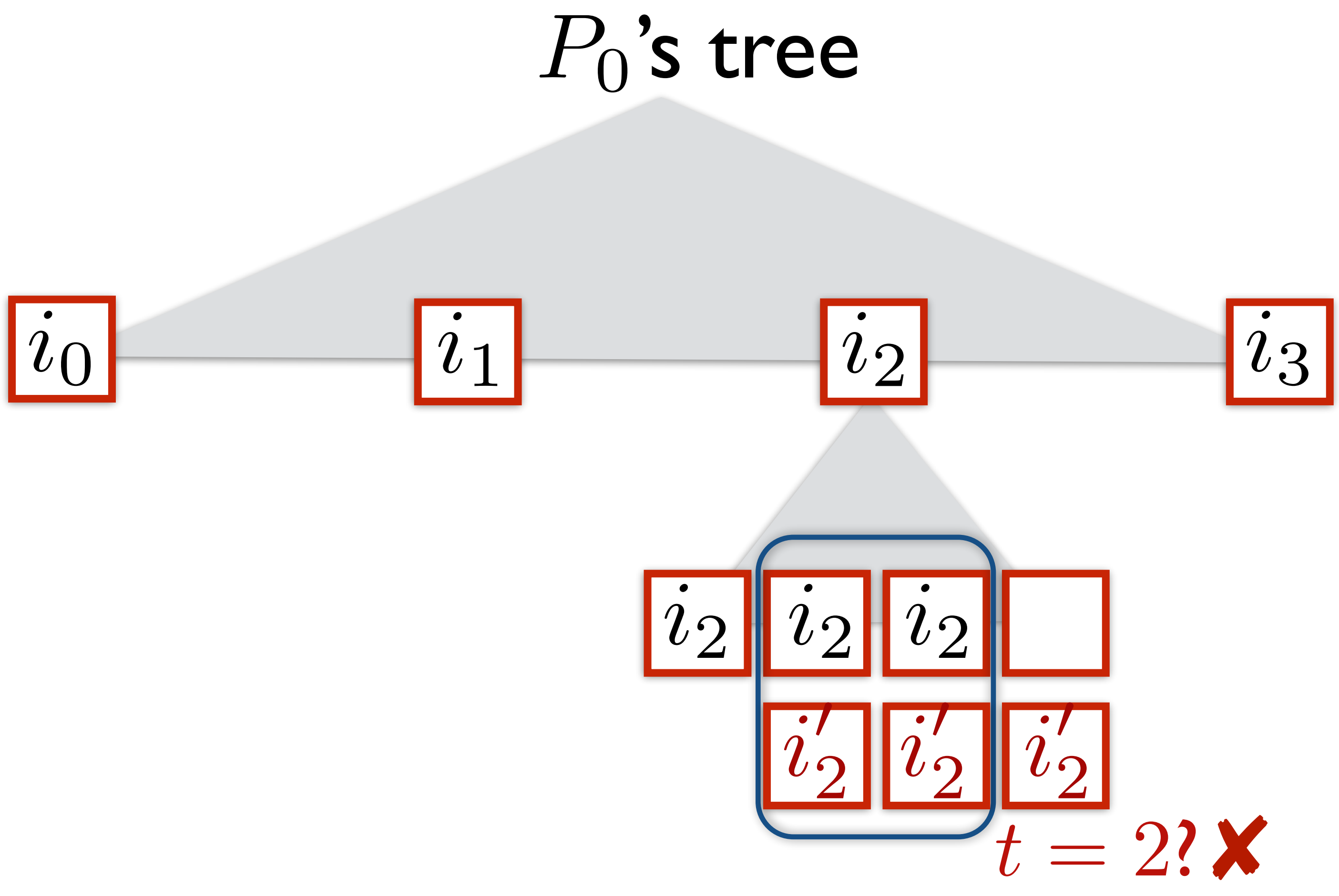

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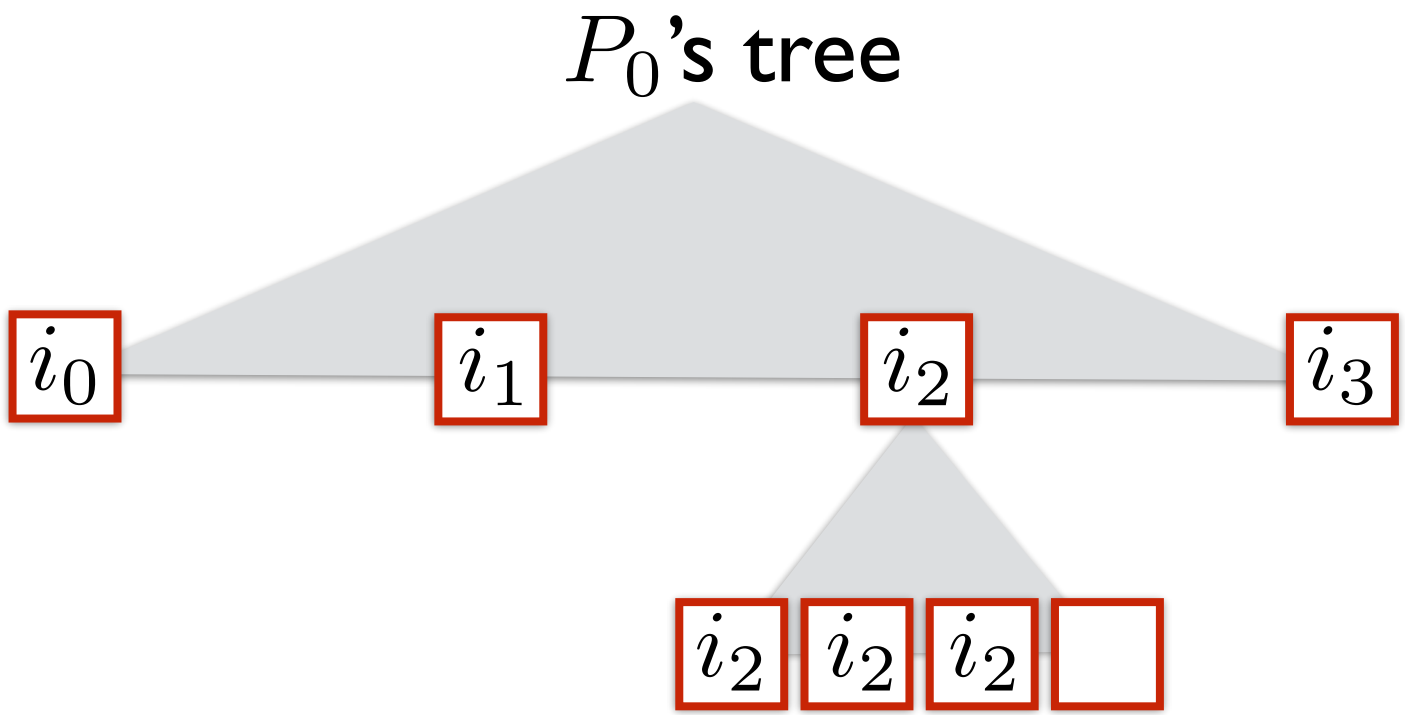

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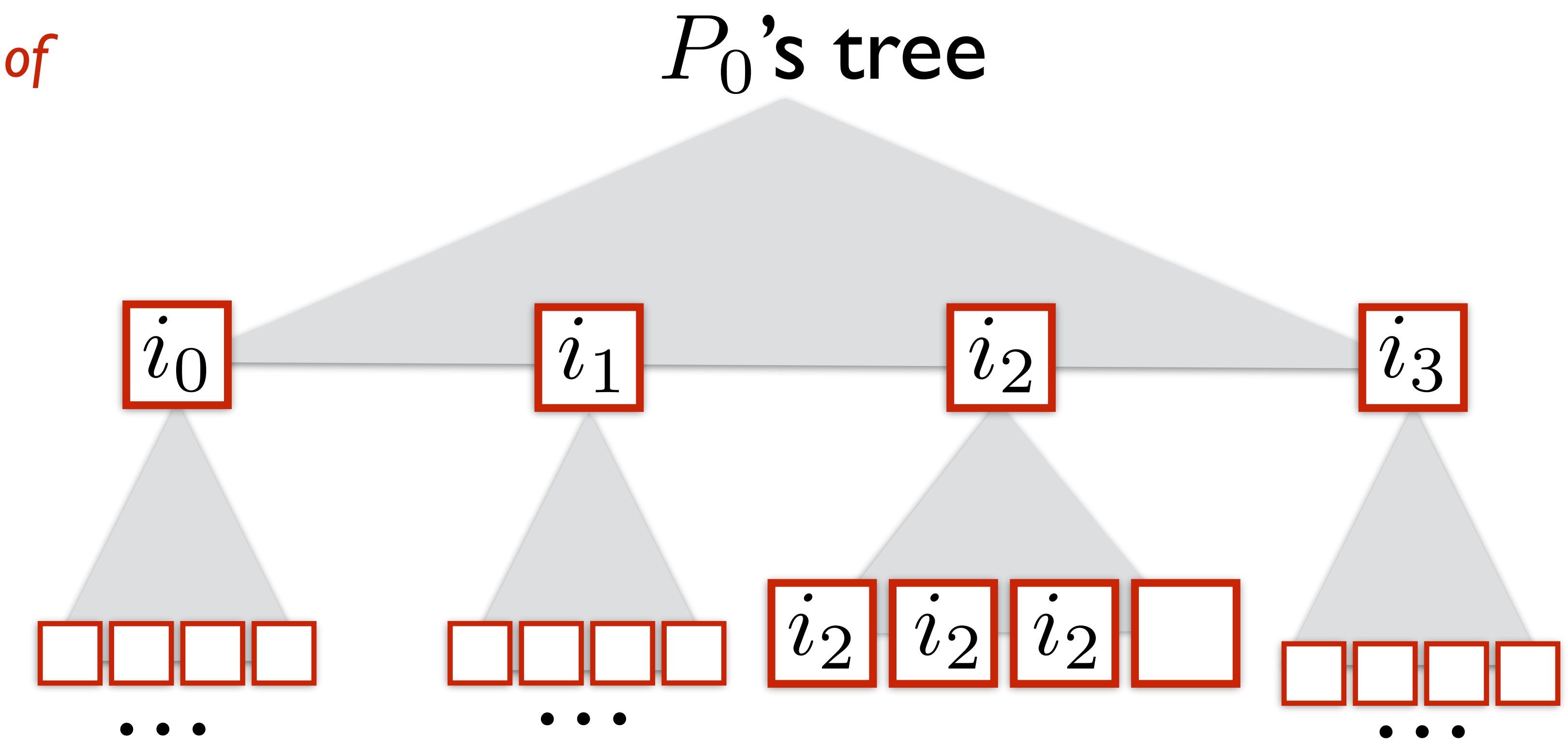
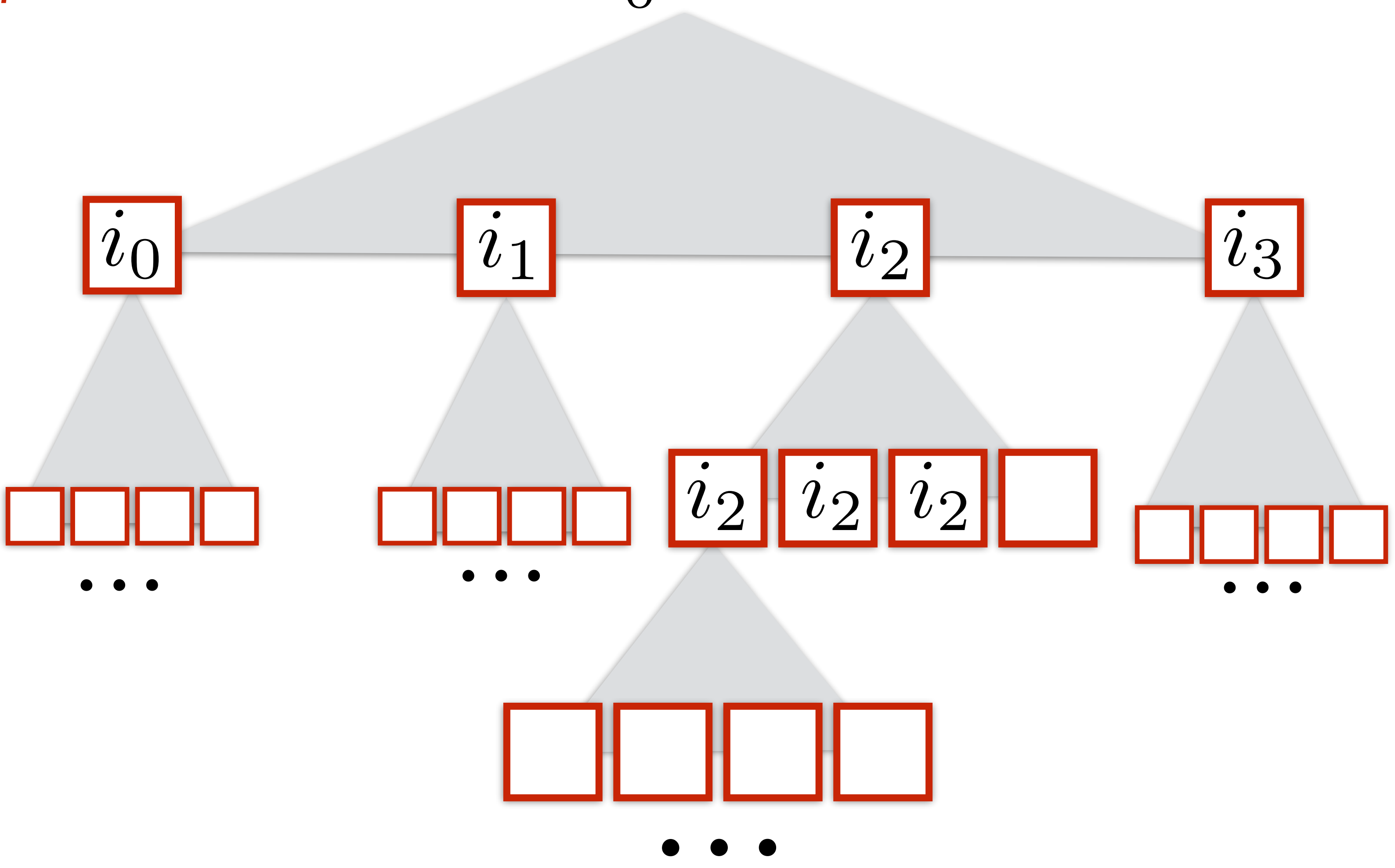


Byzantine Equivocation

Is equivocation a problem in synchronous systems? *sort of*

Ex.: 4 *processes,* 1 *failure*

P_0's tree

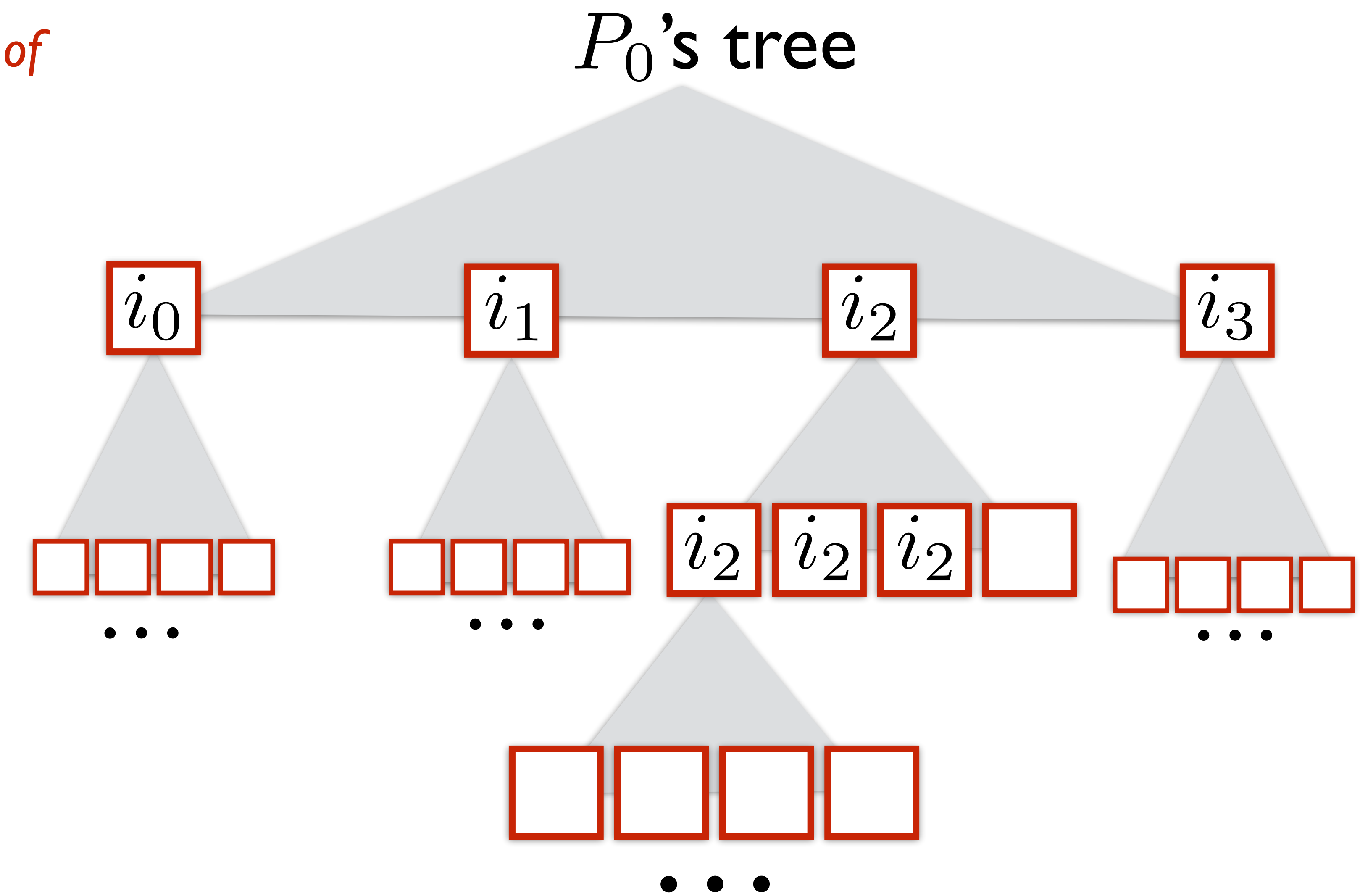


Byzantine Equivocation

Is equivocation a problem in synchronous systems? *sort of*

Ex.: 4 processes, 1 failure

Inner nodes:



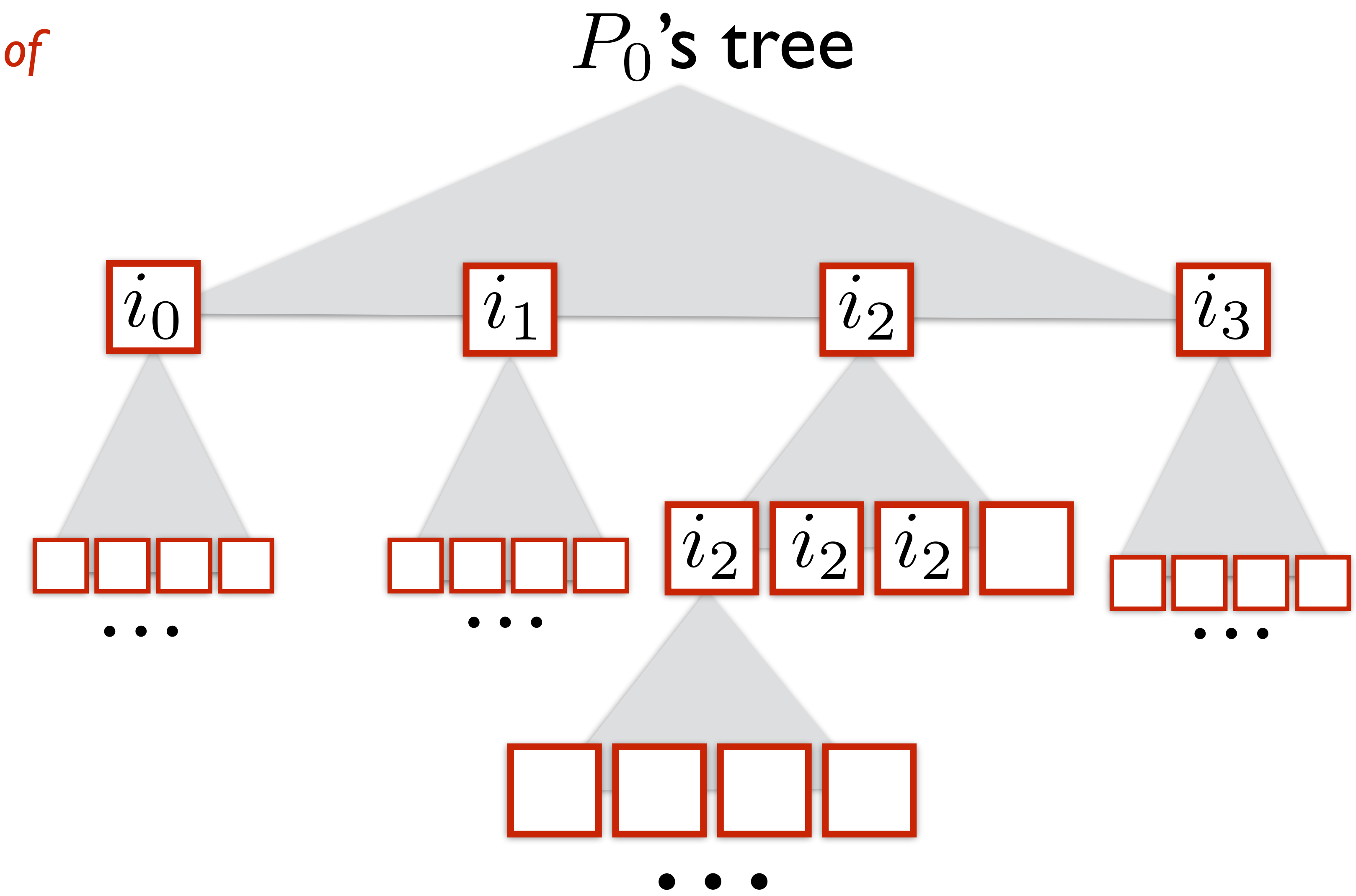
Byzantine Equivocation

Is equivocation a problem in synchronous systems? *sort of*

Ex.: 4 processes, 1 failure

Inner nodes:

validated



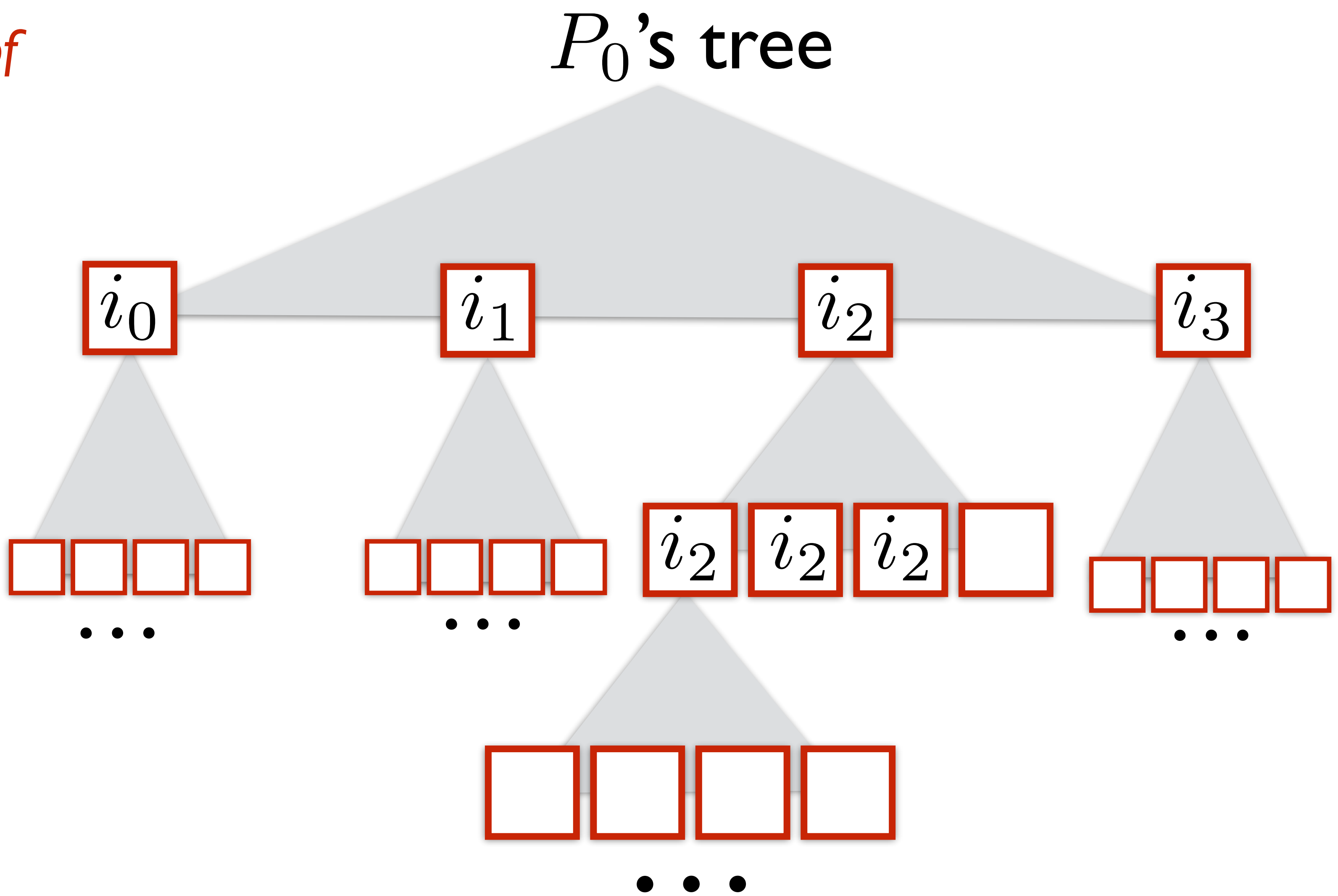
Byzantine Equivocation

Is equivocation a problem in synchronous systems? *sort of*

Ex.: 4 processes, 1 failure

Inner nodes: validated

Leaf nodes:



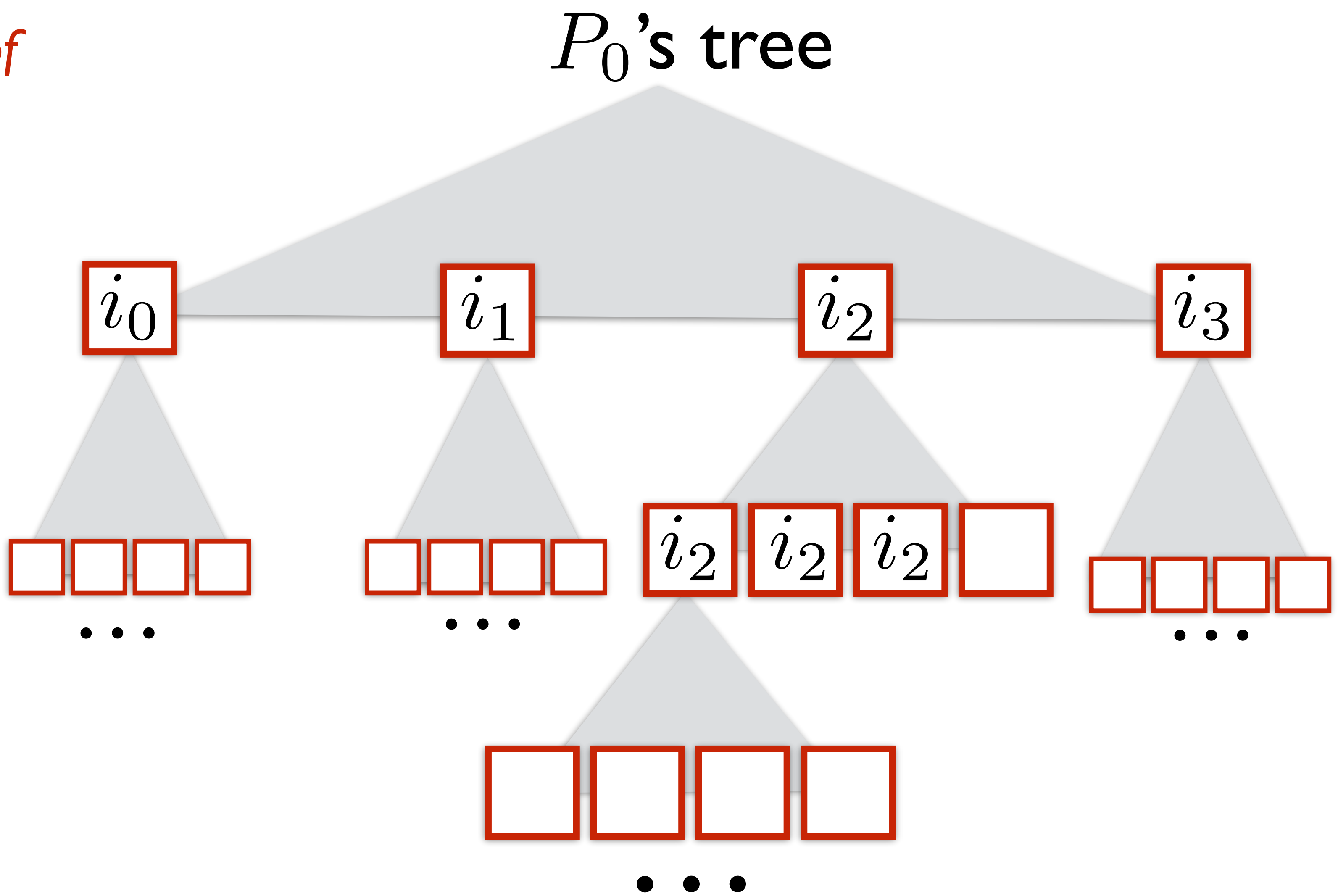
Byzantine Equivocation

Is equivocation a problem in synchronous systems? *sort of*

Ex.: 4 processes, 1 failure

Inner nodes: validated

Leaf nodes: not validated



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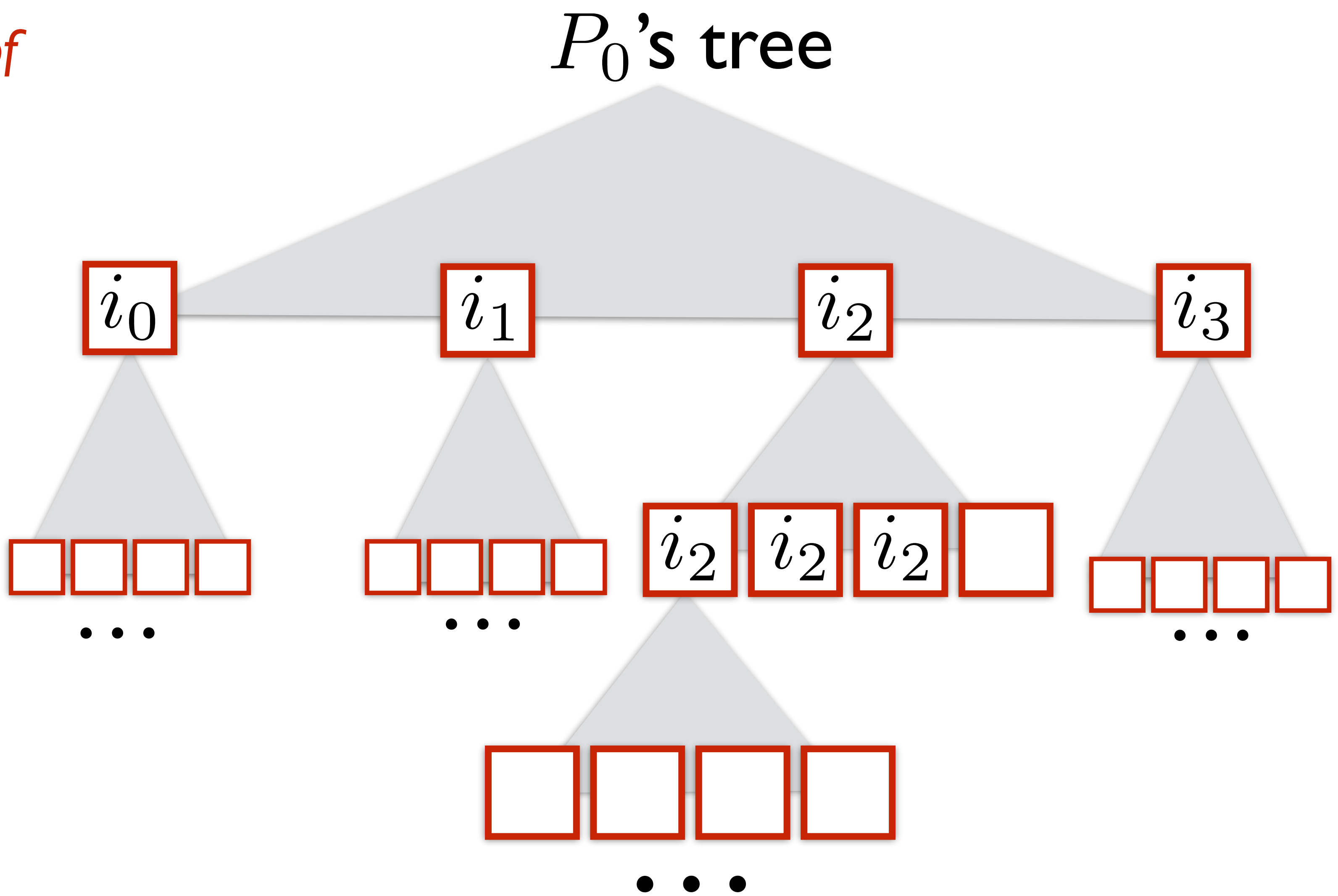

Inner nodes: validated

Leaf nodes: not validated

Last-round equivocation is problematic…

Byzantine Equivocation

Is equivocation a problem in synchronous systems? *sort of*

Ex.: 4 processes, 1 failure

Inner nodes: validated

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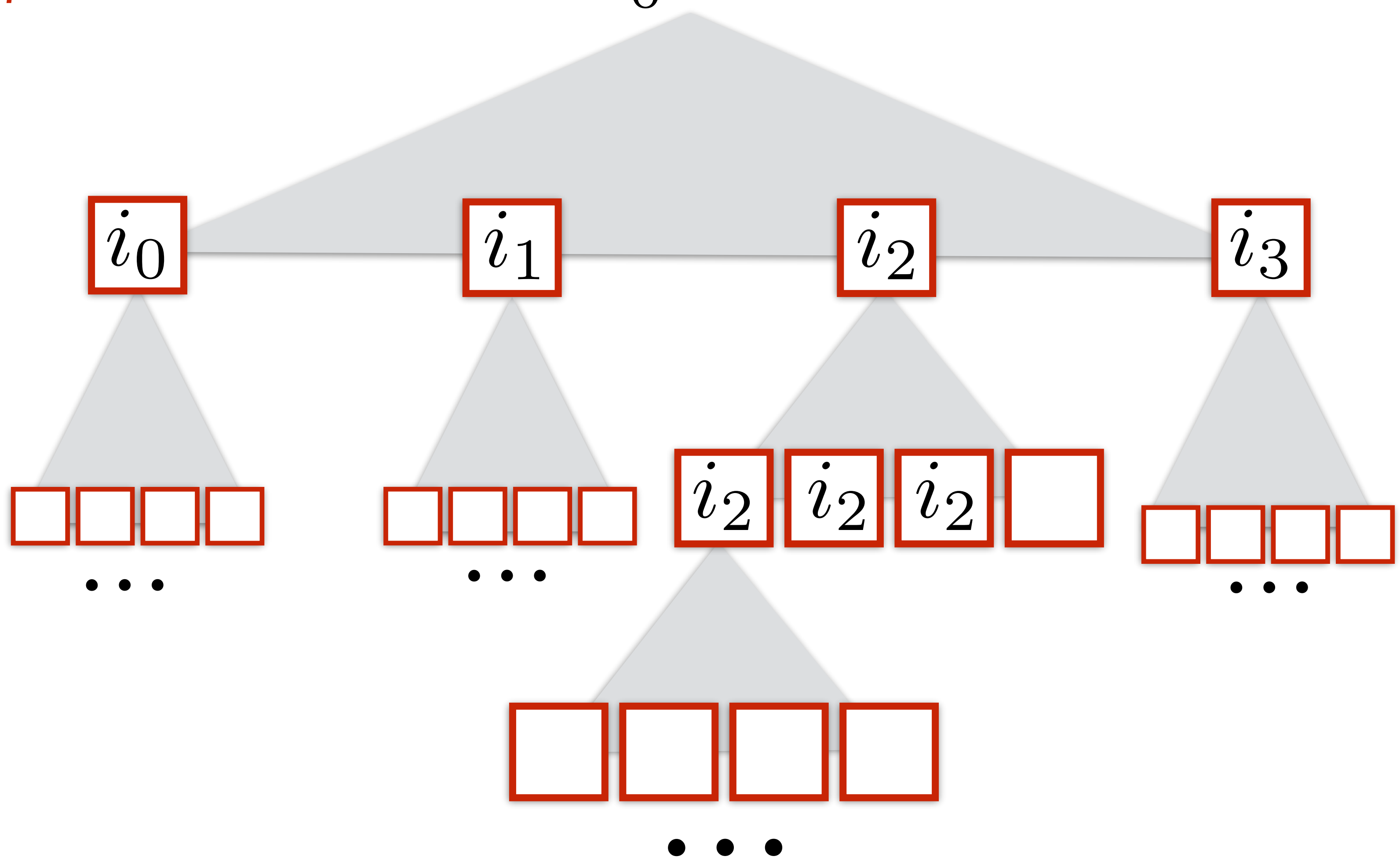
P_0's tree



Last-round equivocation is problematic…

… if Byzantine processes do not “reveal" themselves

Strategy

Strategy

We define a ***new*** round operator

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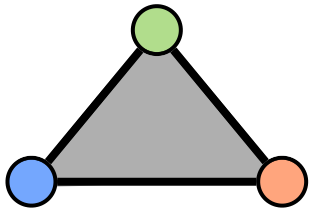
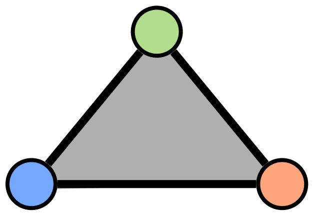
$\mathcal{K}_0 = \mathcal{I}^*$



Strategy

We define a ***new*** round operator

$\mathcal{K}_0 = \mathcal{I}^*$ $\quad$ $\mathcal{K}_1 = \mathcal{R}_c(\mathcal{K}_0, k)$ $\quad$ $\mathcal{K}_2 = \mathcal{R}_c(\mathcal{K}_1, k)$



...

$(k-1)$**-connected** $\quad$ $(k-1)$**-connected**

We define a *new* round operator

$\mathcal{K}_0 = \mathcal{I}^*$ $\mathcal{K}_1 = \mathcal{R}_c(\mathcal{K}_0, k)$ $\mathcal{K}_2 = \mathcal{R}_c(\mathcal{K}_1, k)$

$(k-1)$-connected $(k-1)$-connected

$\lfloor t/k \rfloor$ rounds

Strategy

We define a ***new*** round operator

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...

$(k-1)$-connected $\quad$ $(k-1)$-connected

$\lfloor t/k \rfloor$ rounds $\quad$ if $t \bmod k \neq 0$

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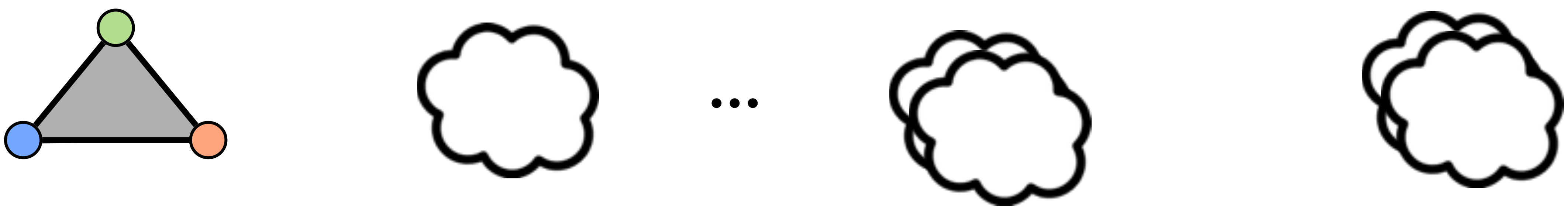

$(k-1)$-connected $(k-1)$-connected $(k-1)$-connected

$\lfloor t/k \rfloor$ rounds

if t mod $k \neq 0$

Strategy

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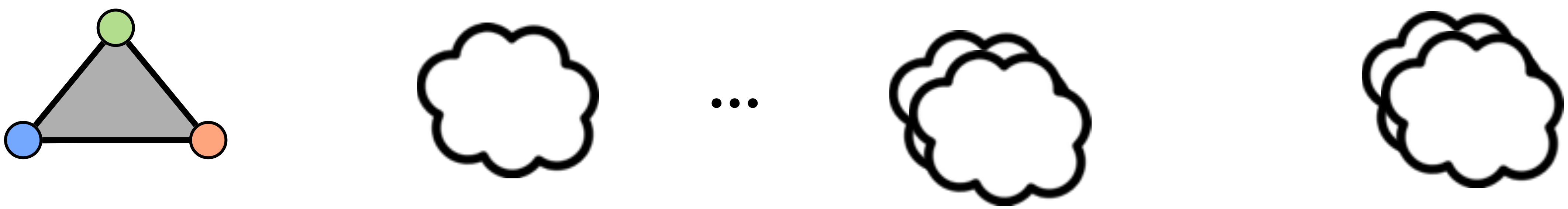


$(k-1)$-connected $(k-1)$-connected $(k-1)$-connected

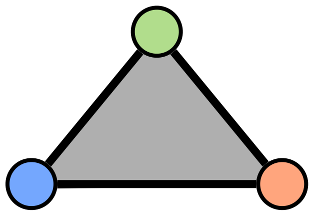
$\lfloor t/k \rfloor$ rounds if t mod $k \neq 0$

$\lceil t/k \rceil$ rounds

Strategy

We define a *new* round operator

$\mathcal{K}_0 = \mathcal{I}^*$ | $\mathcal{K}_1 = \mathcal{R}_c(\mathcal{K}_0, k)$ | ... | $\mathcal{K}_2 = \mathcal{R}_c(\mathcal{K}_1, k)$ | $\mathcal{K}_3 = \mathcal{R}_e(\mathcal{K}_2)$



$(k-1)$-connected | $(k-1)$-connected | $(k-1)$-connected

$\lfloor t/k \rfloor$ rounds

if $t \bmod k \neq 0$

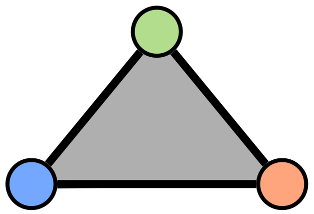
$\lceil t/k \rceil$ rounds

k-set agreement protocol

Strategy

We define a ***new*** round operator

$\mathcal{K}_0 = \mathcal{I}^*$



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$(k-1)$-connected

...

$\mathcal{K}_2 = \mathcal{R}_c(\mathcal{K}_1, k)$

$(k-1)$-connected

$\mathcal{K}_3 = \mathcal{R}_e(\mathcal{K}_2)$

$(k-1)$-connected

$\lfloor t/k \rfloor$ rounds

if $t \bmod k \neq 0$

$\lceil t/k \rceil$ rounds

k-set agreement protocol

Generalize the consensus protocol

The Equivocation Operator

The Equivocation Operator

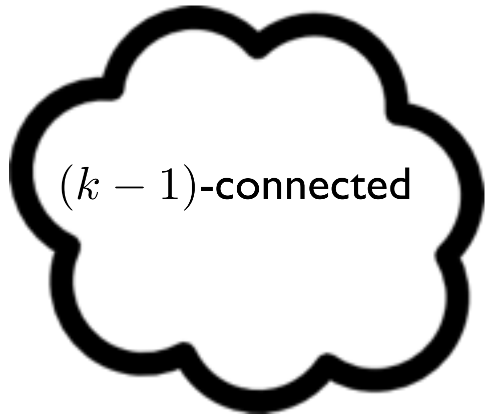
$\lfloor t/k \rfloor$ rounds,
k failures/round

$\cdots$

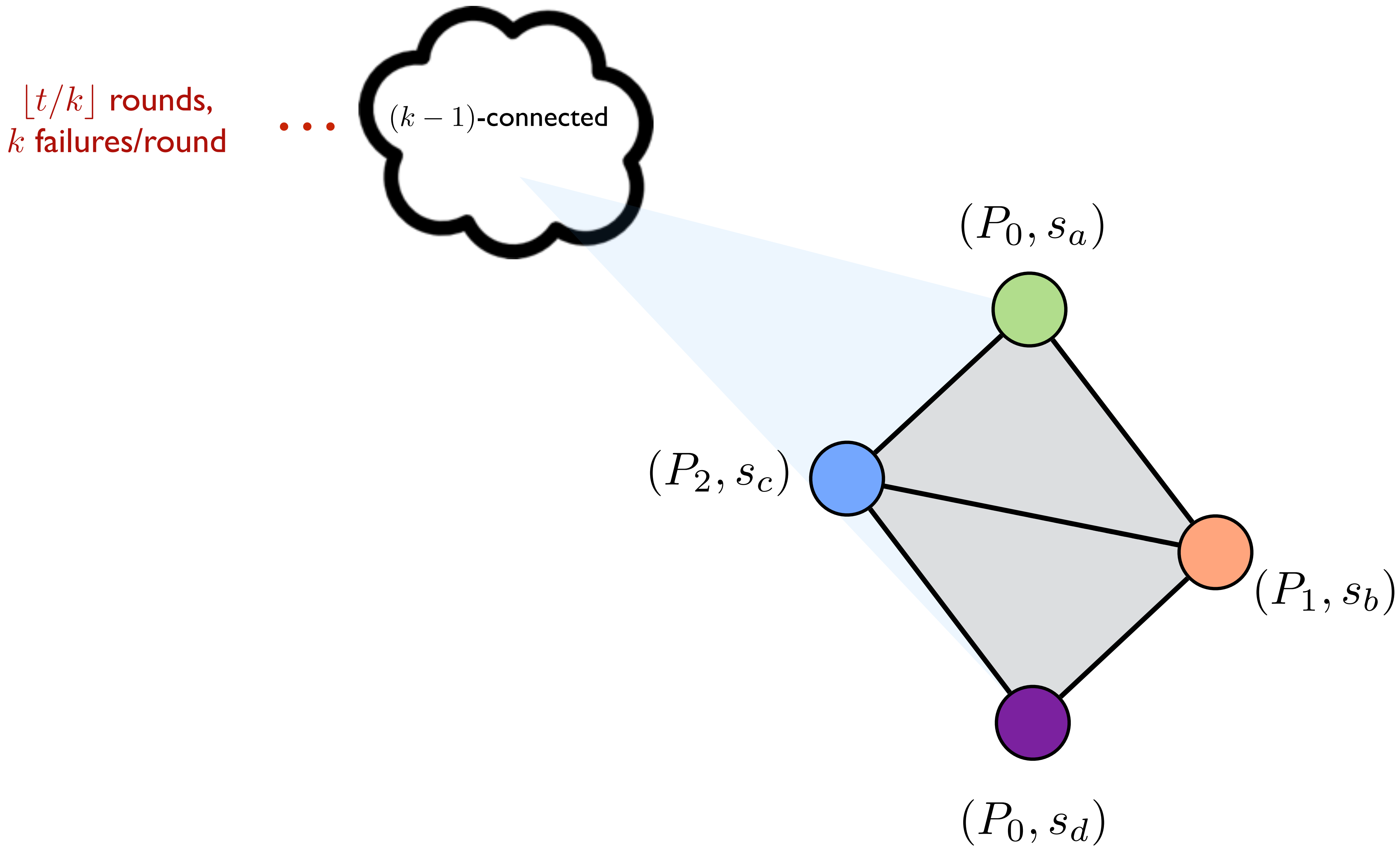
The Equivocation Operator

$\lfloor t/k \rfloor$ rounds,
k failures/round

. . .



The Equivocation Operator



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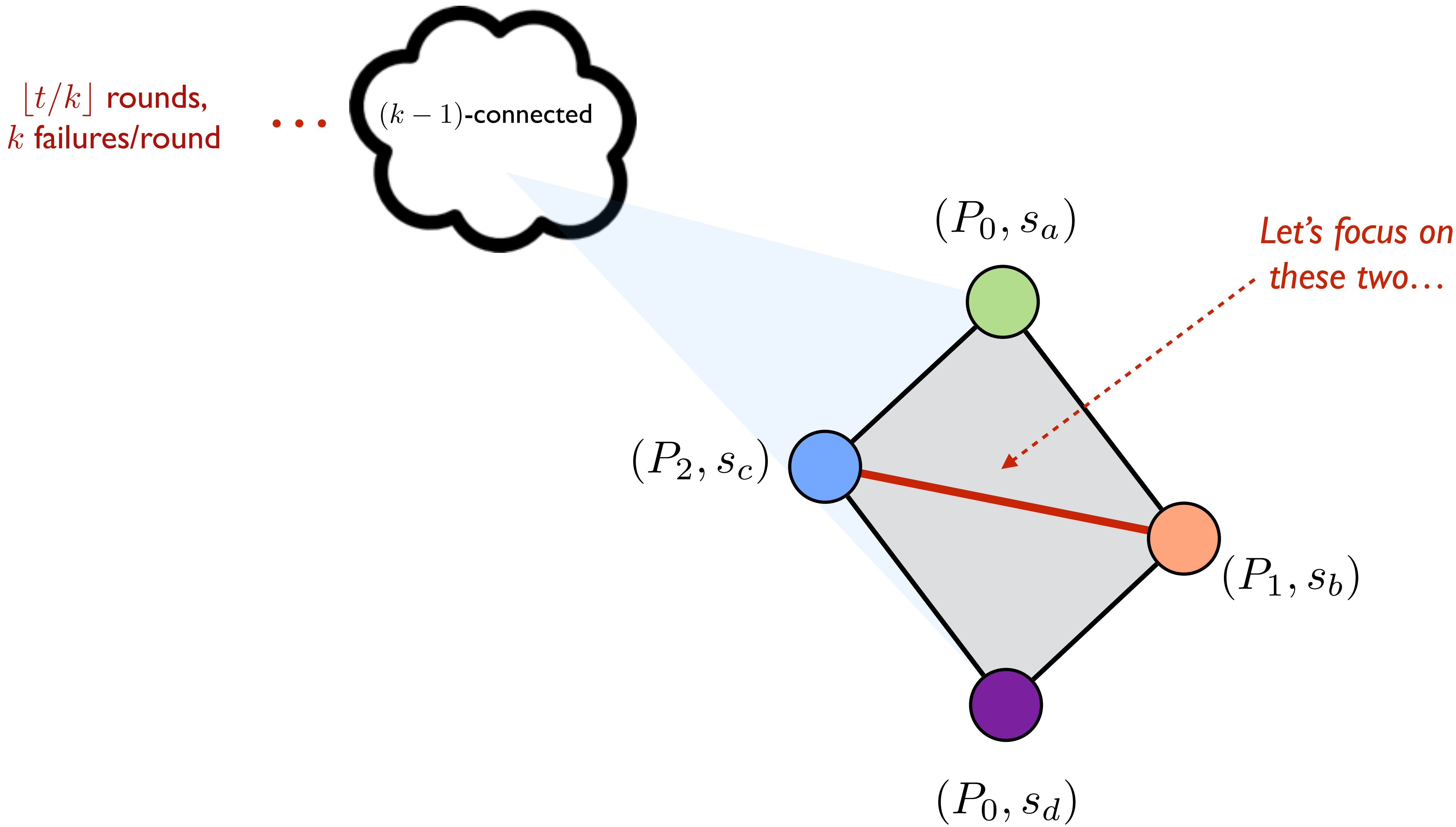
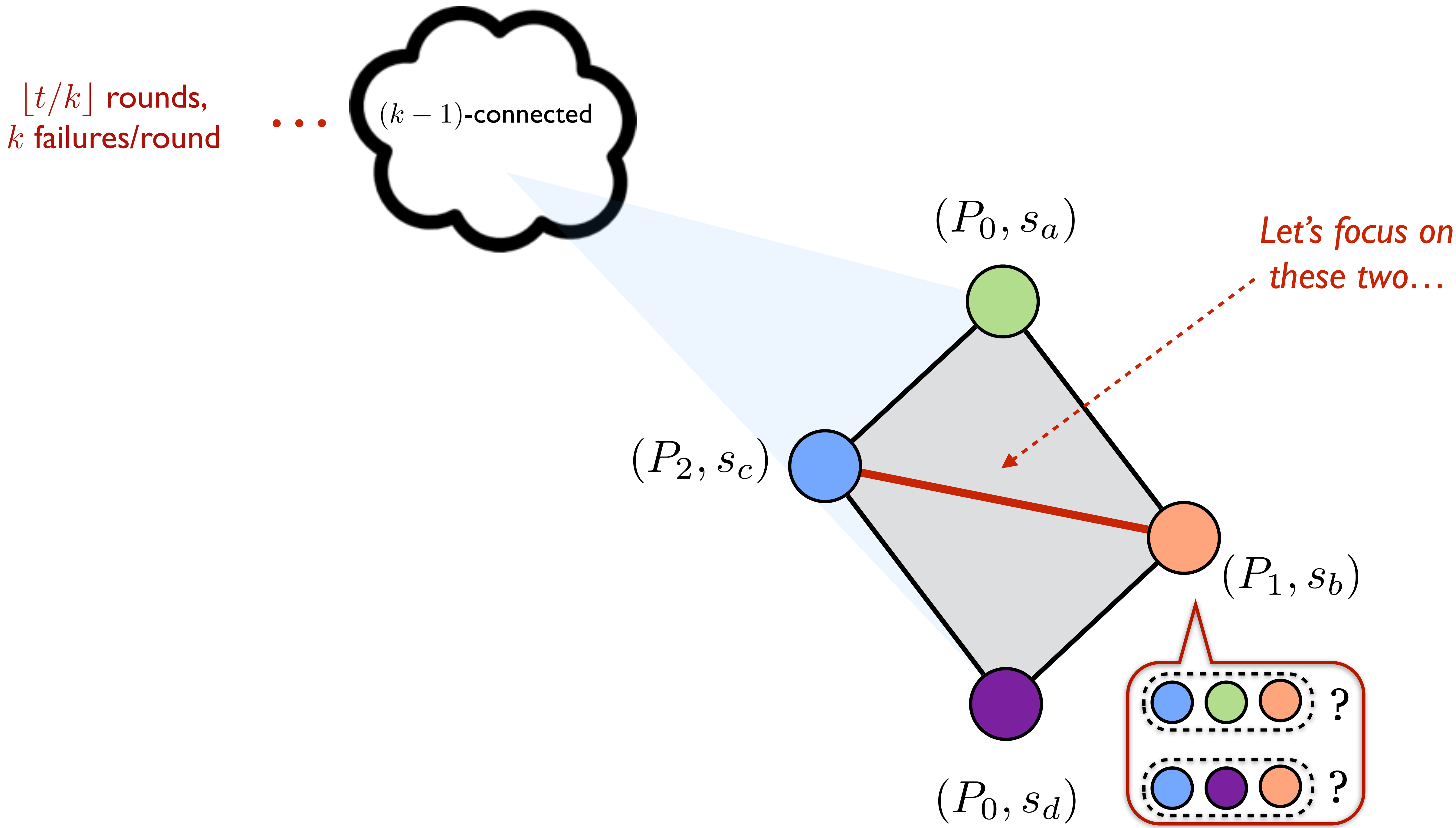


The Equivocation Operator



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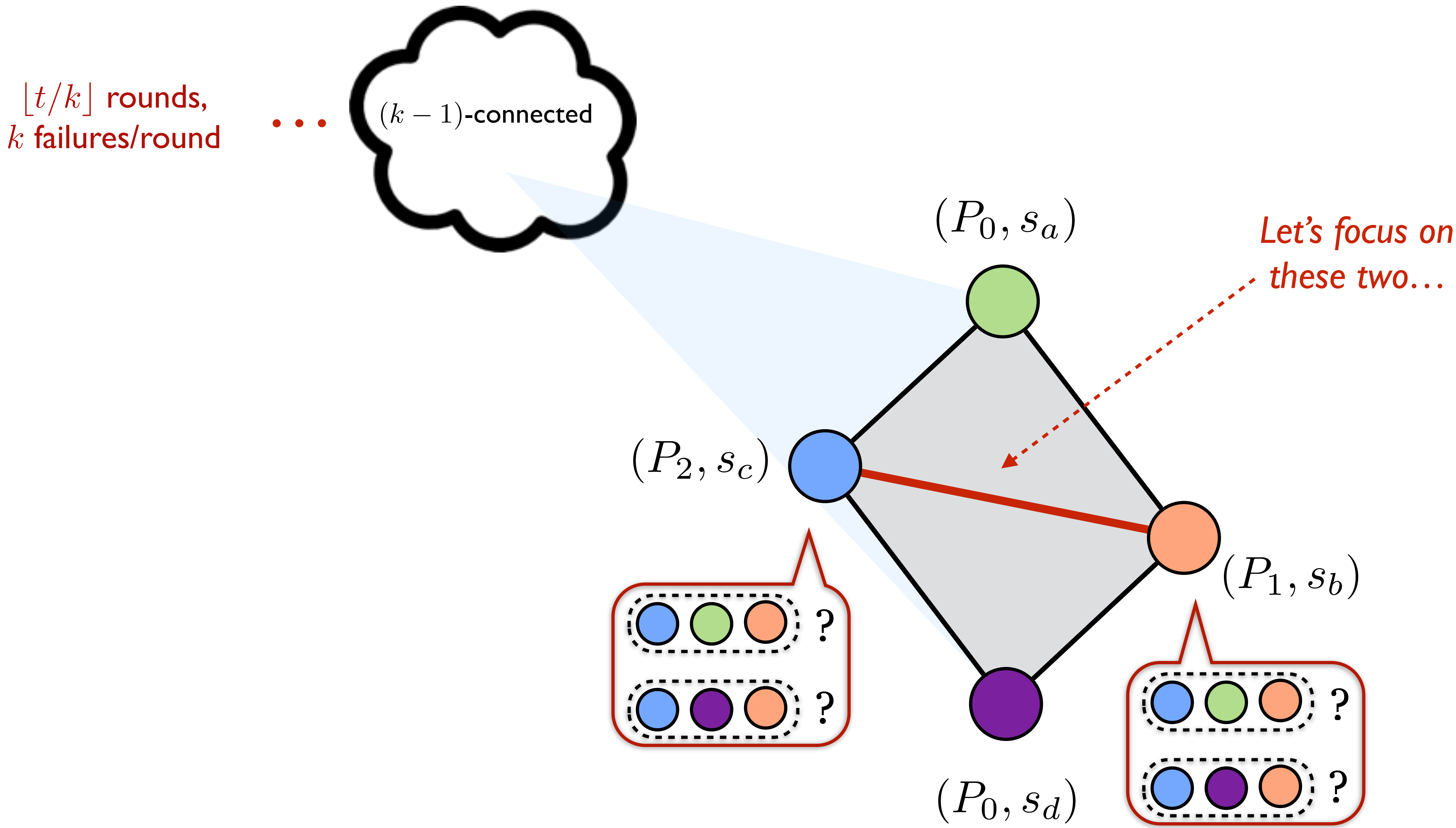
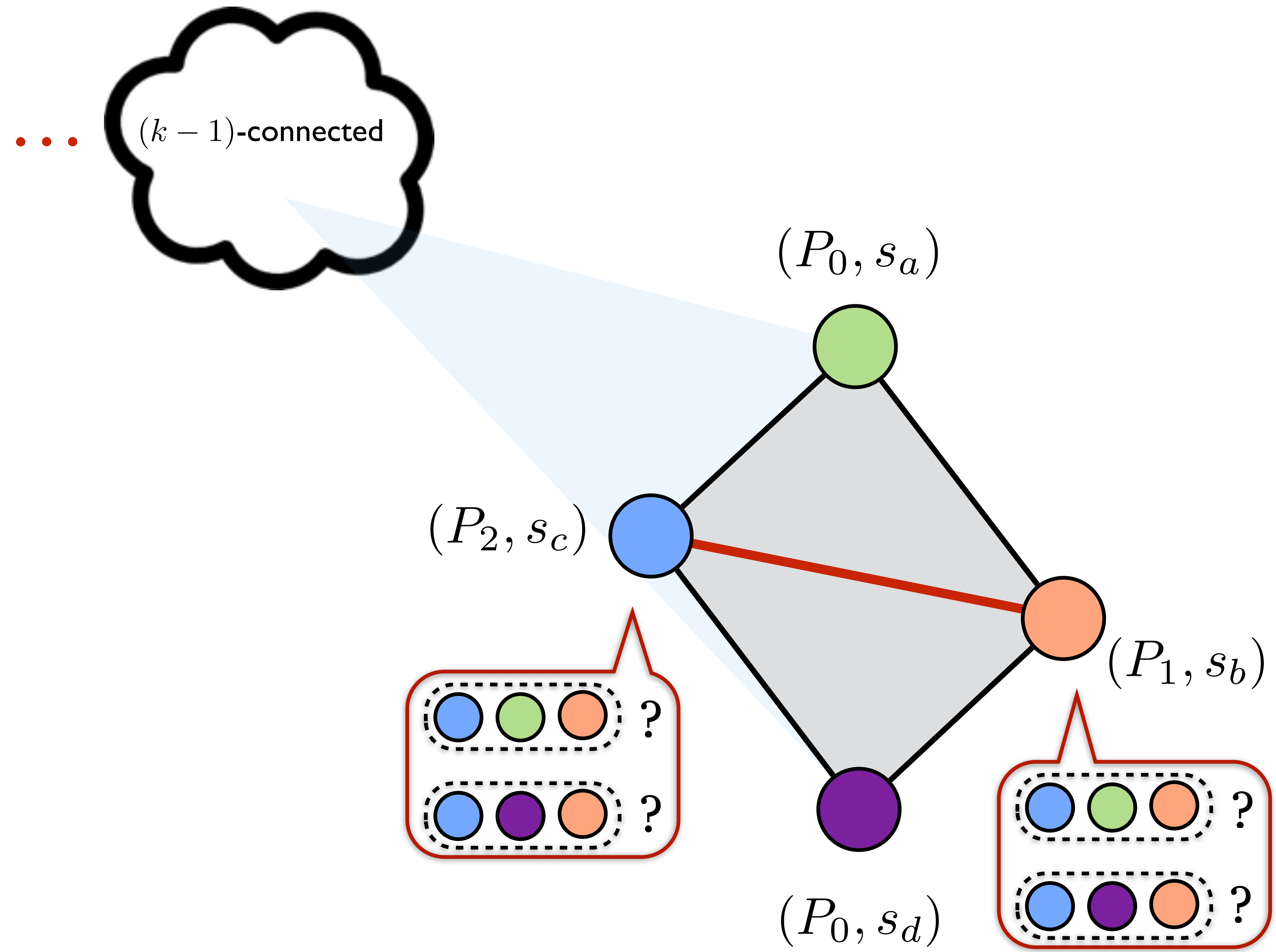


The Equivocation Operator

$\lfloor t/k \rfloor$ rounds,
k failures/round



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$\lfloor t/k \rfloor$ rounds,
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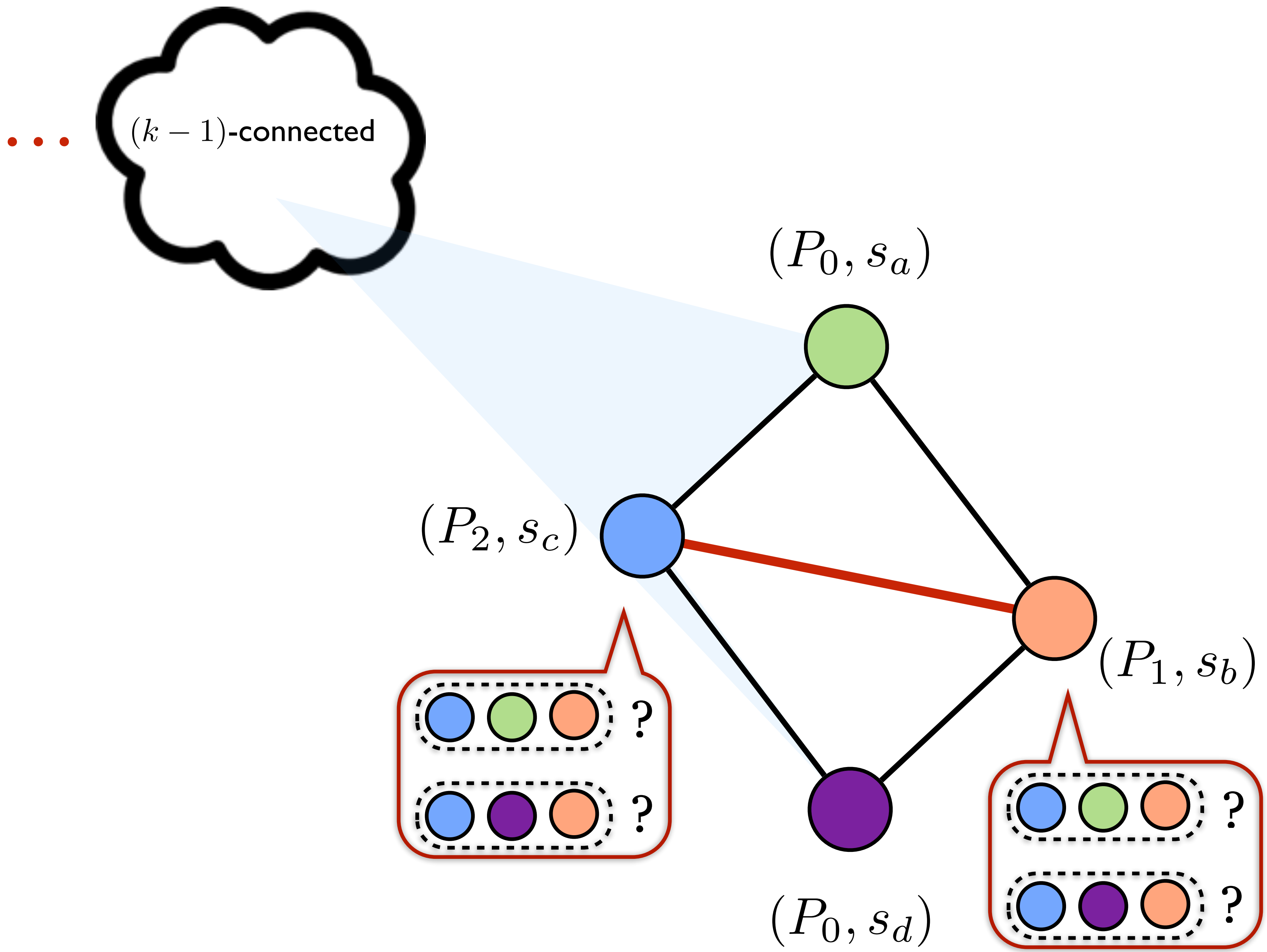
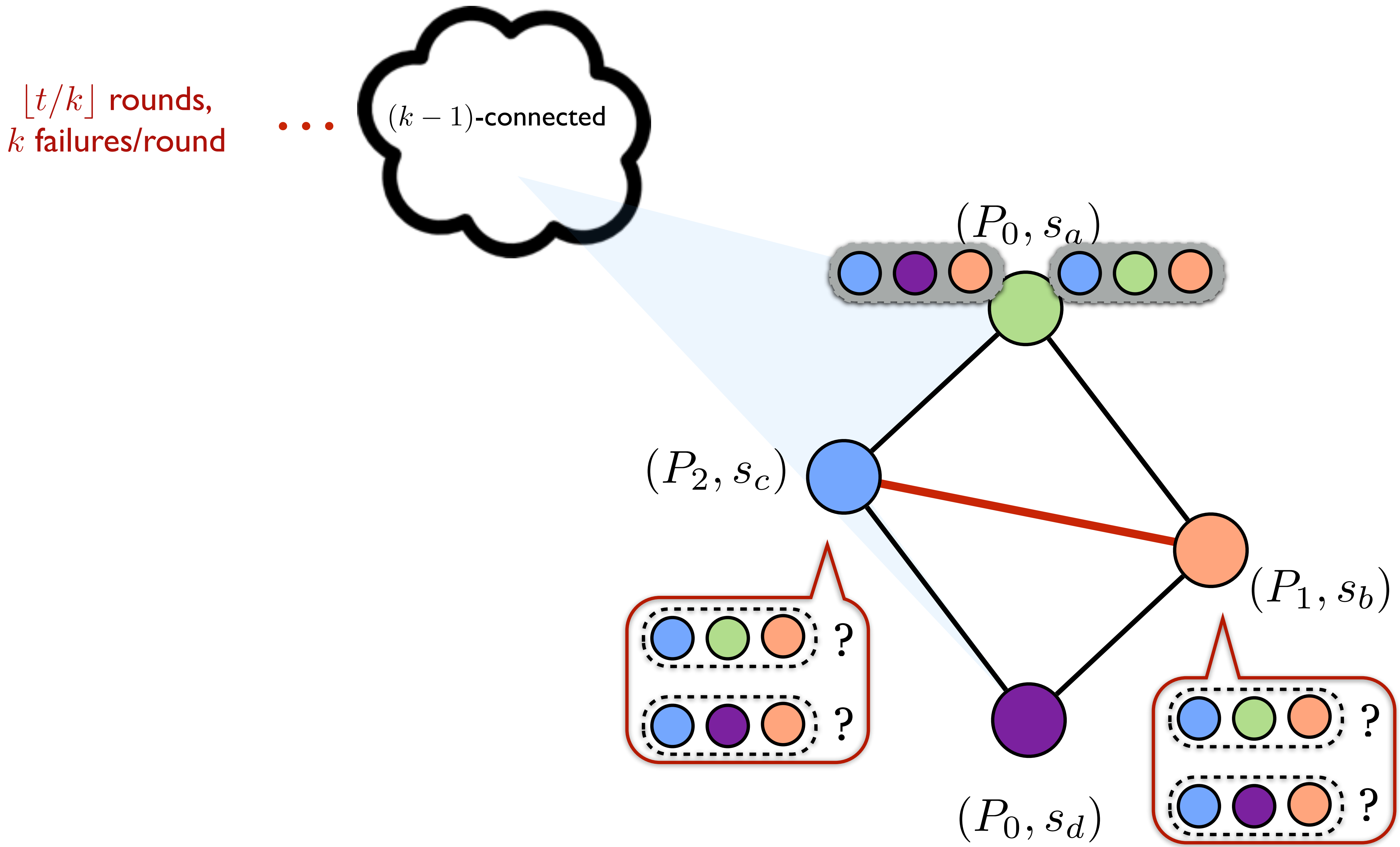


The Equivocation Operator



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$\lfloor t/k \rfloor$ rounds,
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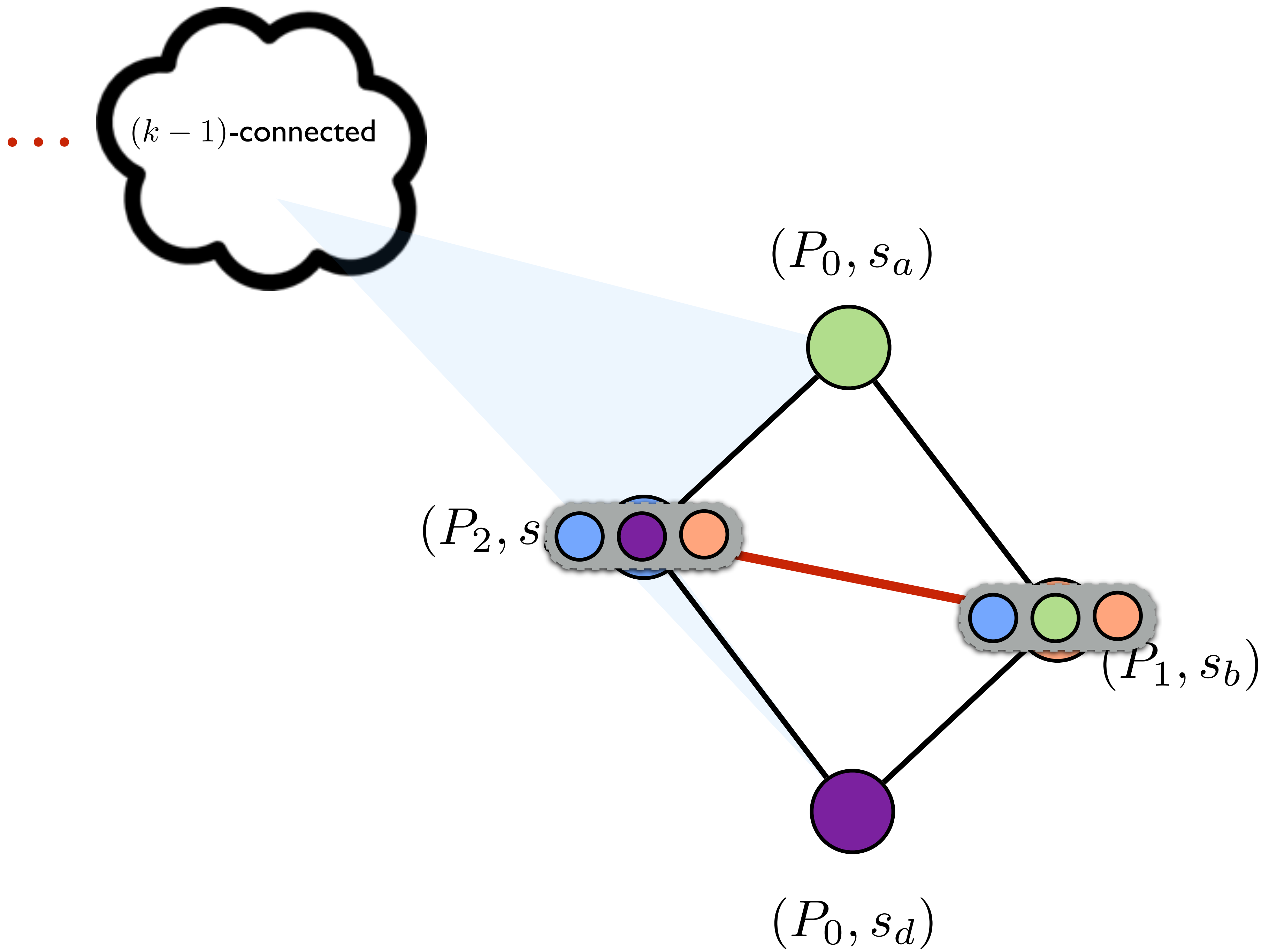

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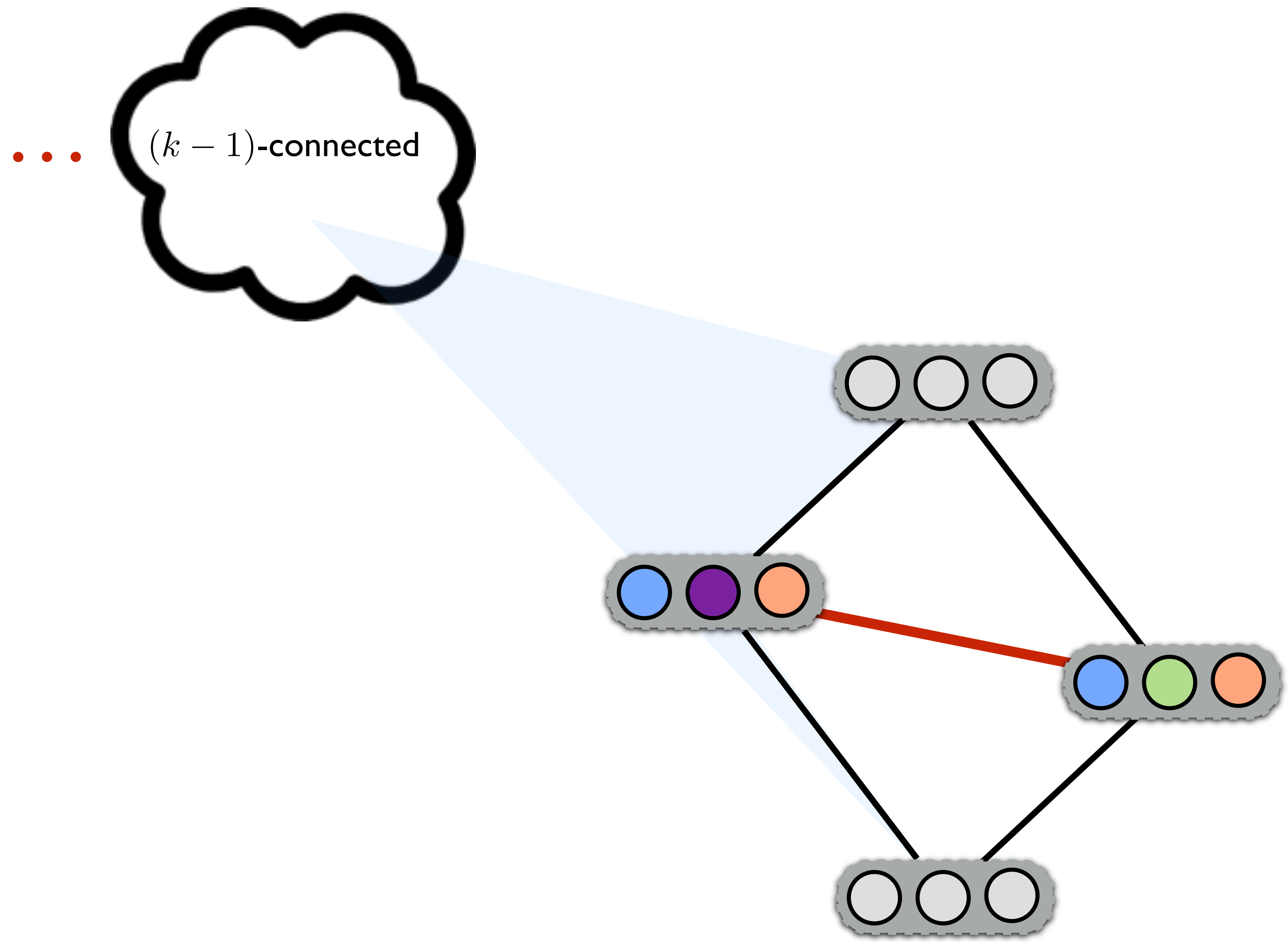

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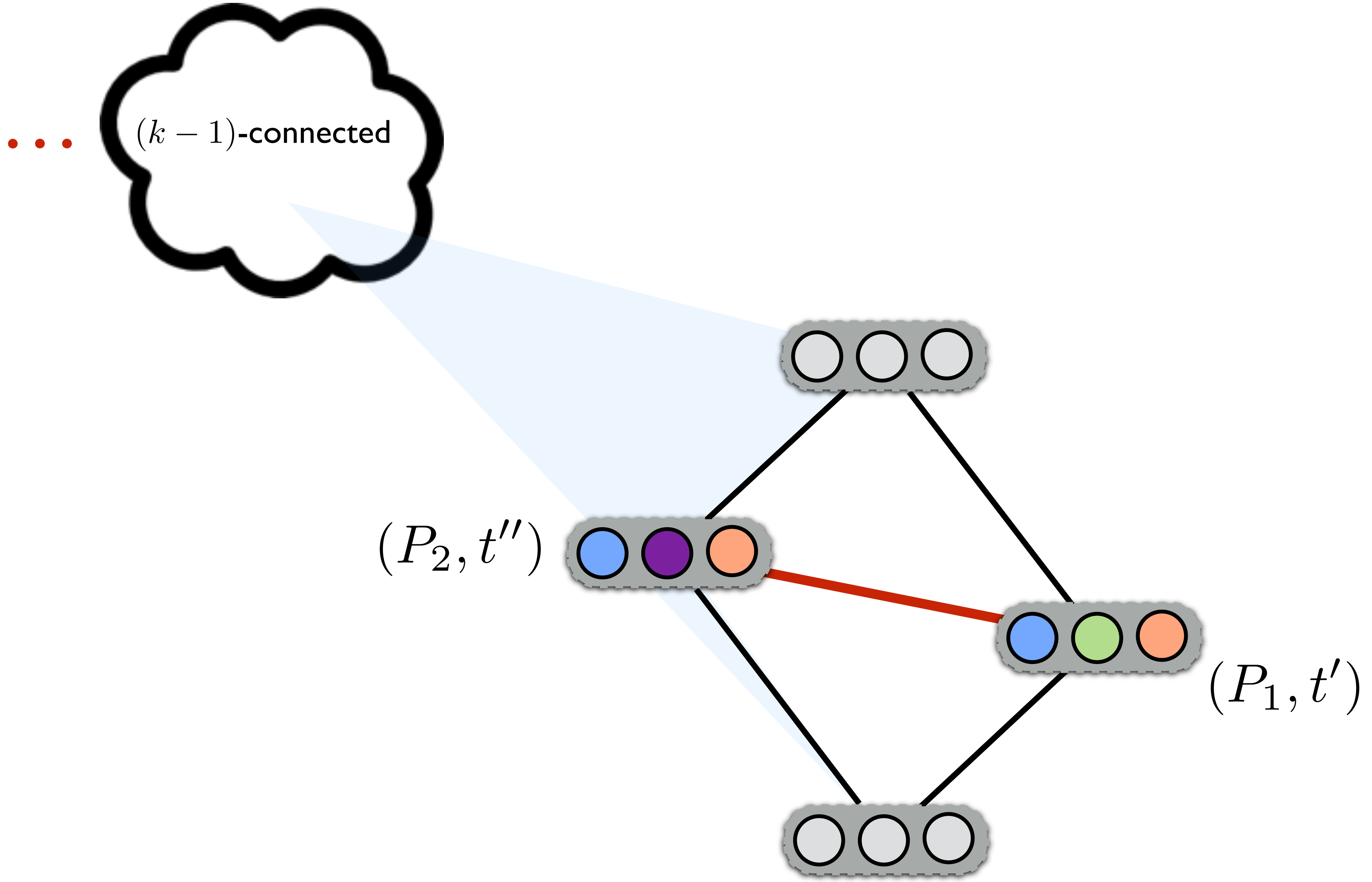

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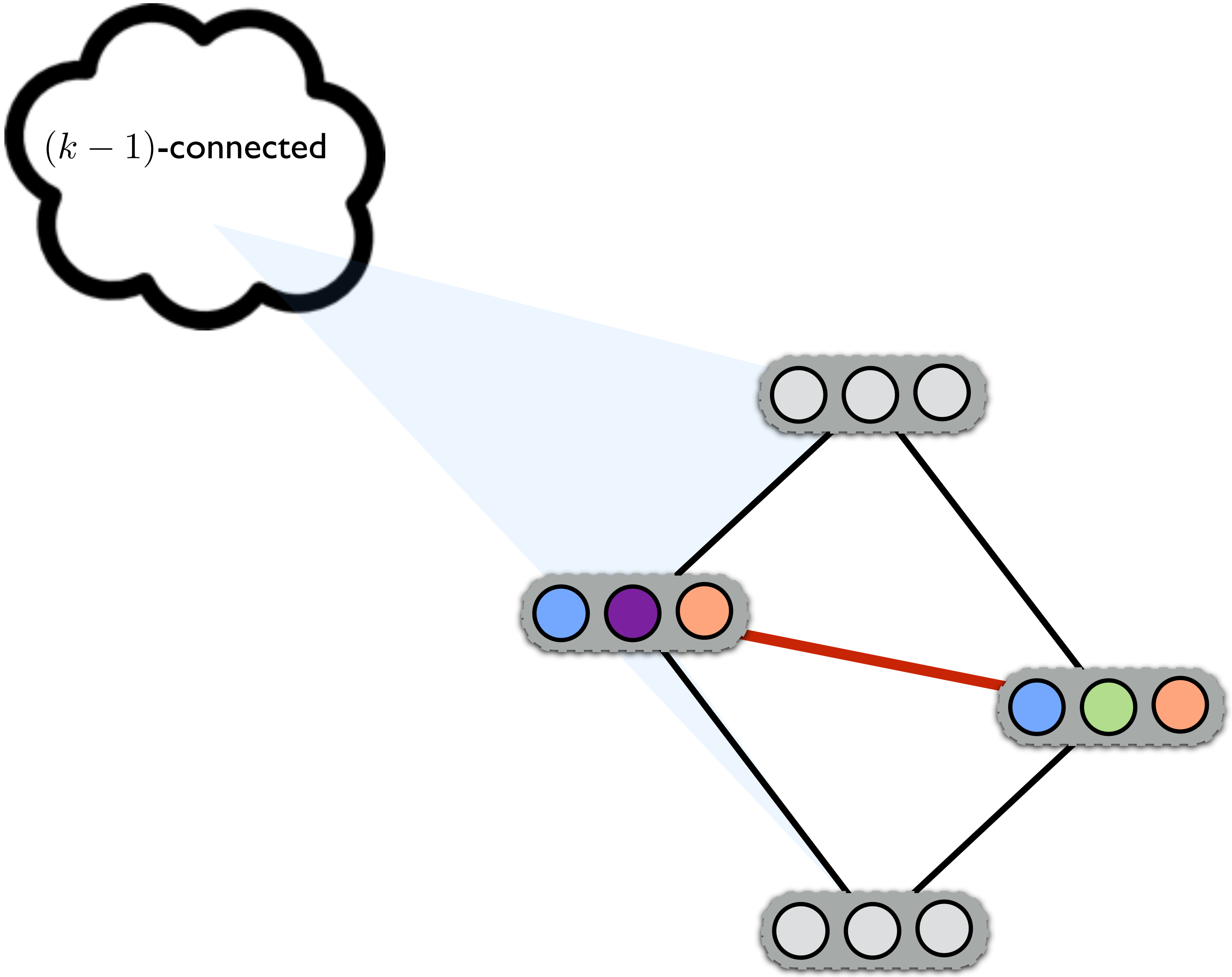

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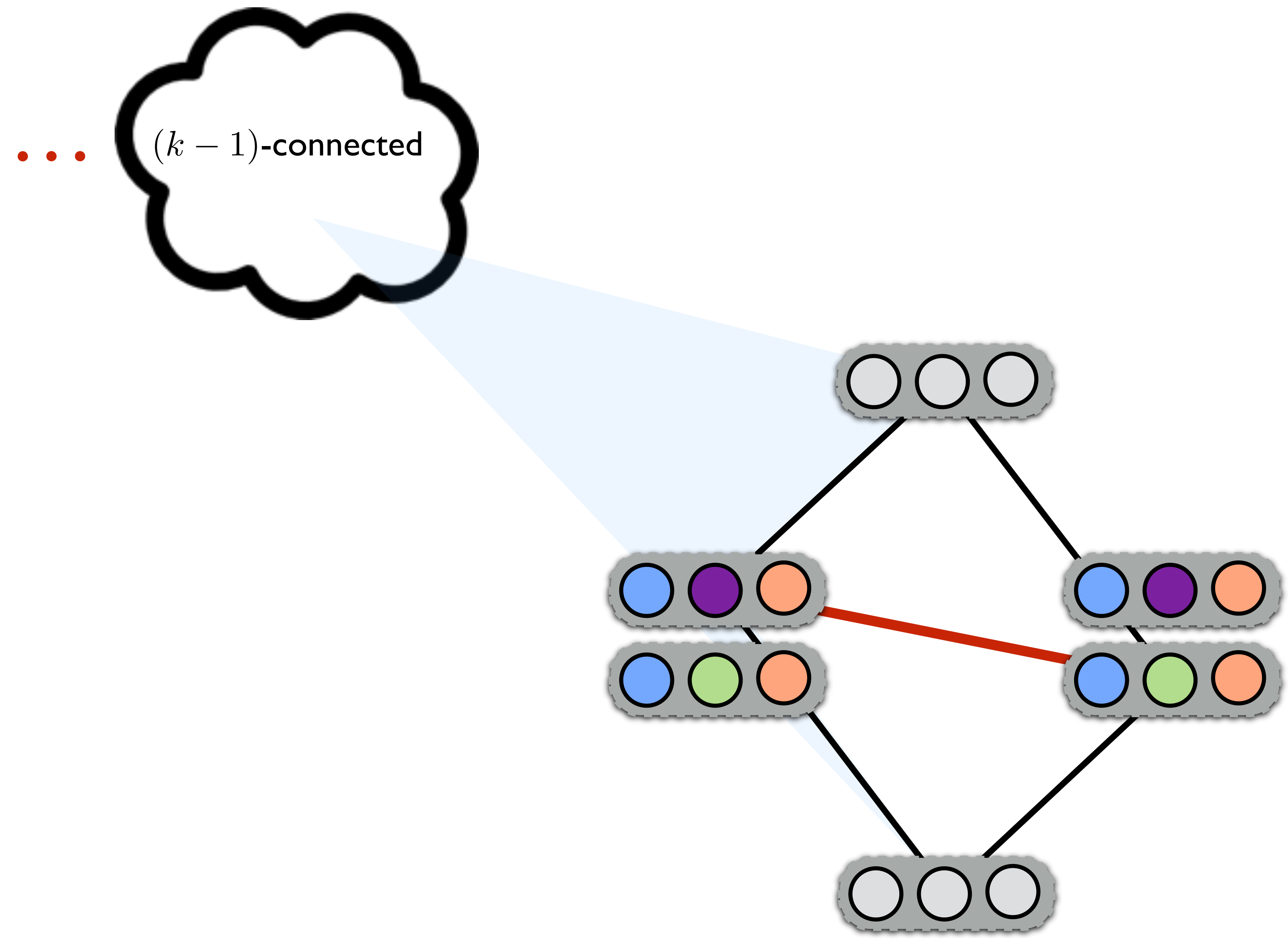

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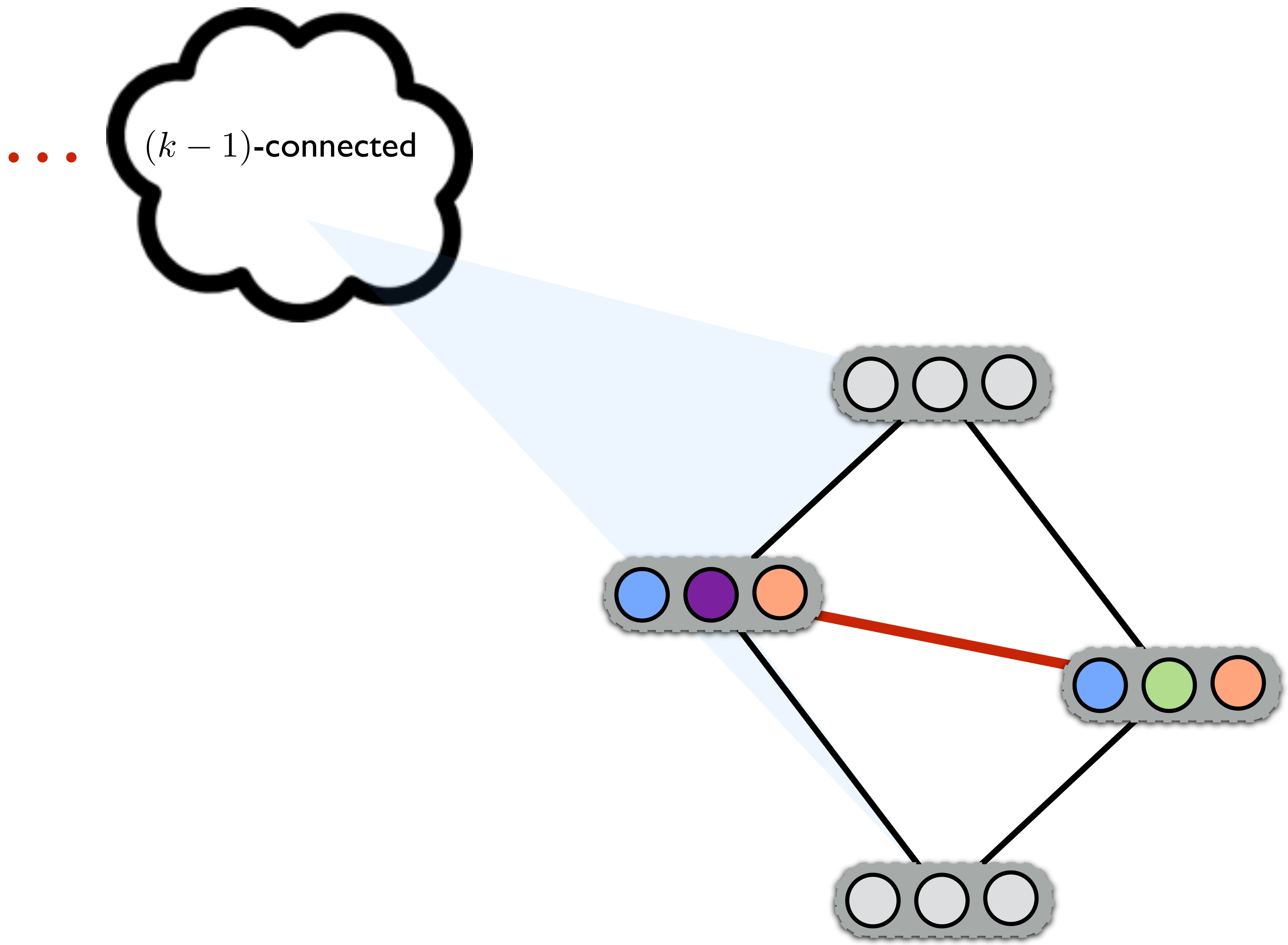

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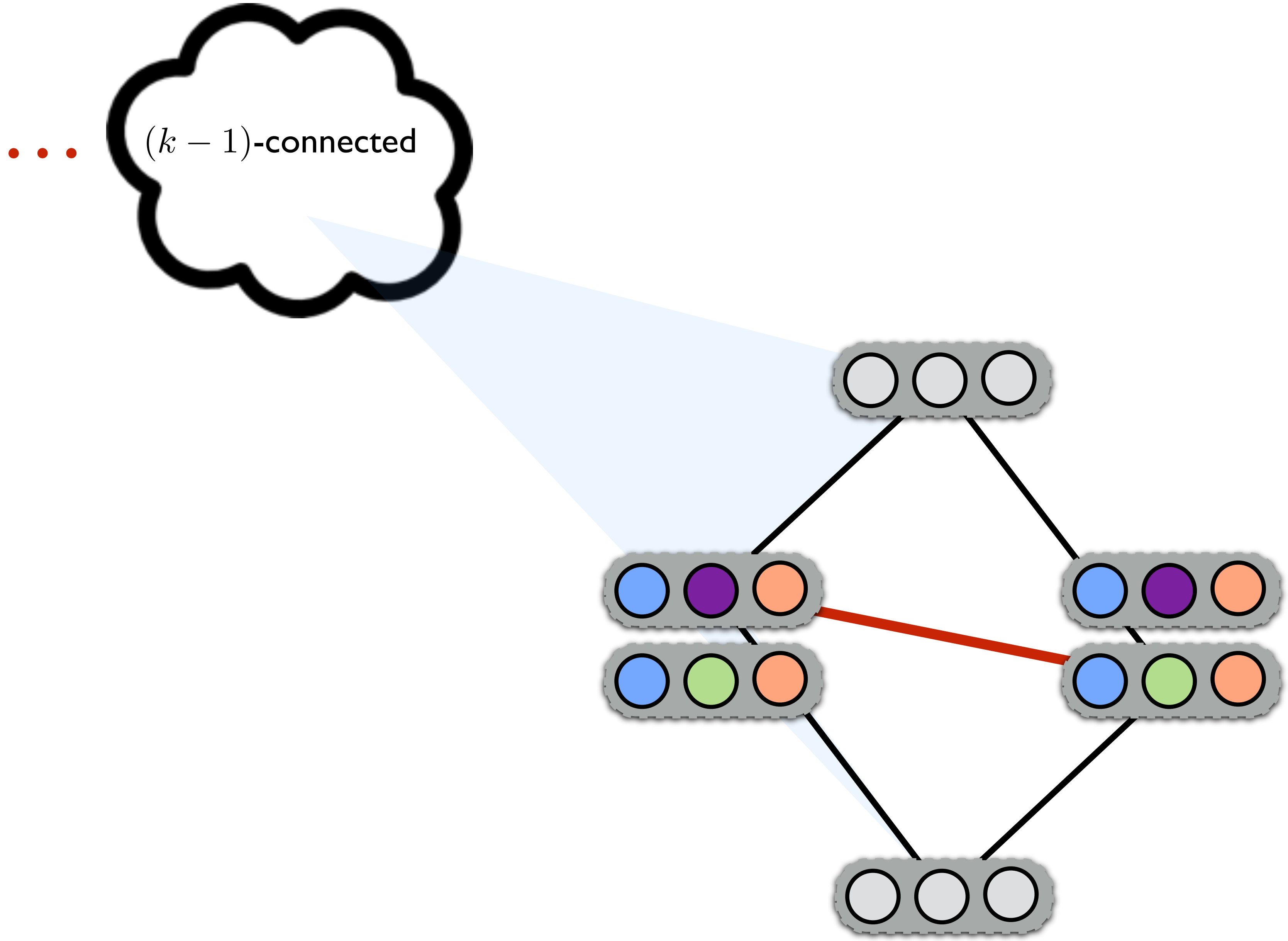

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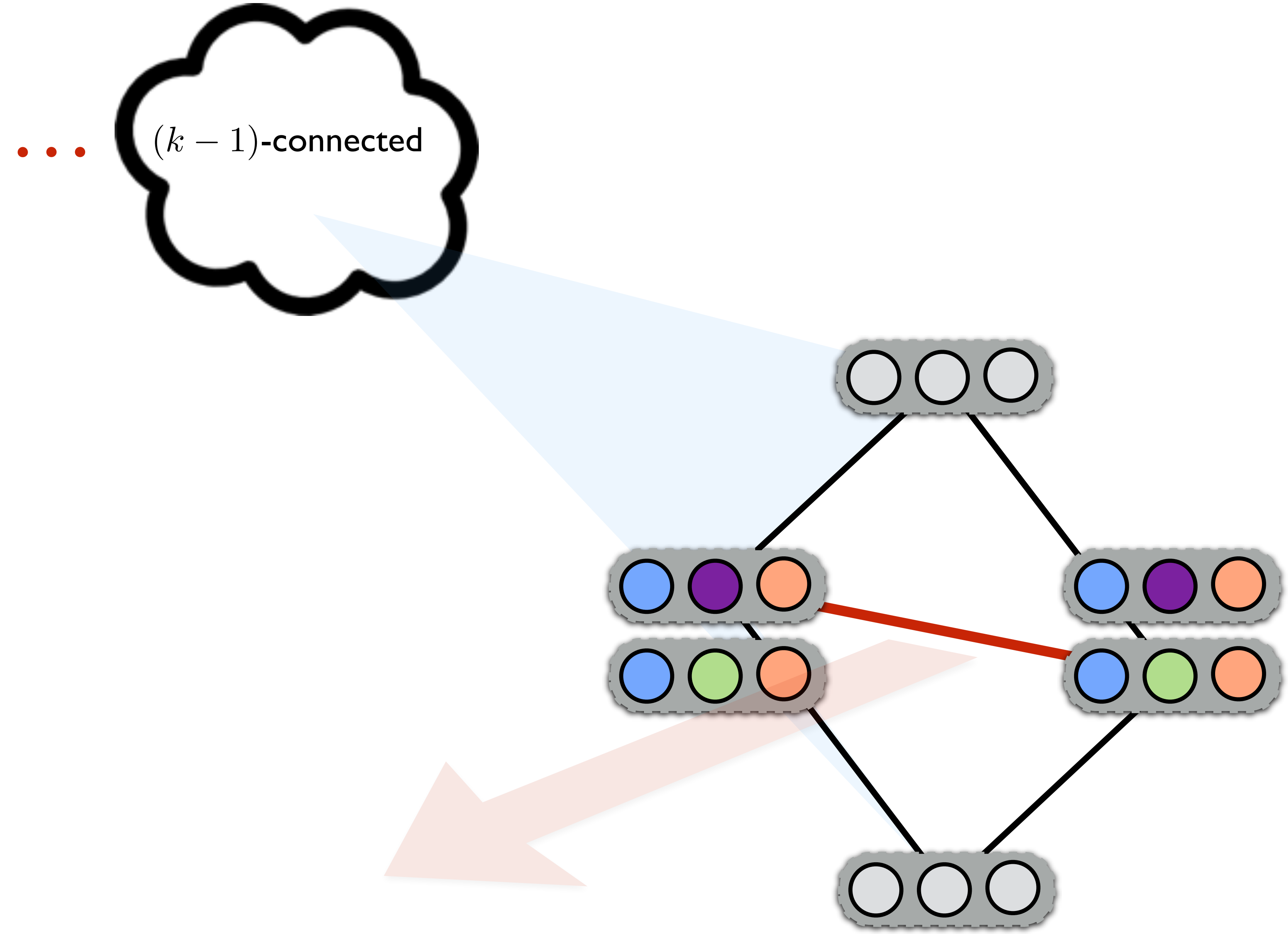

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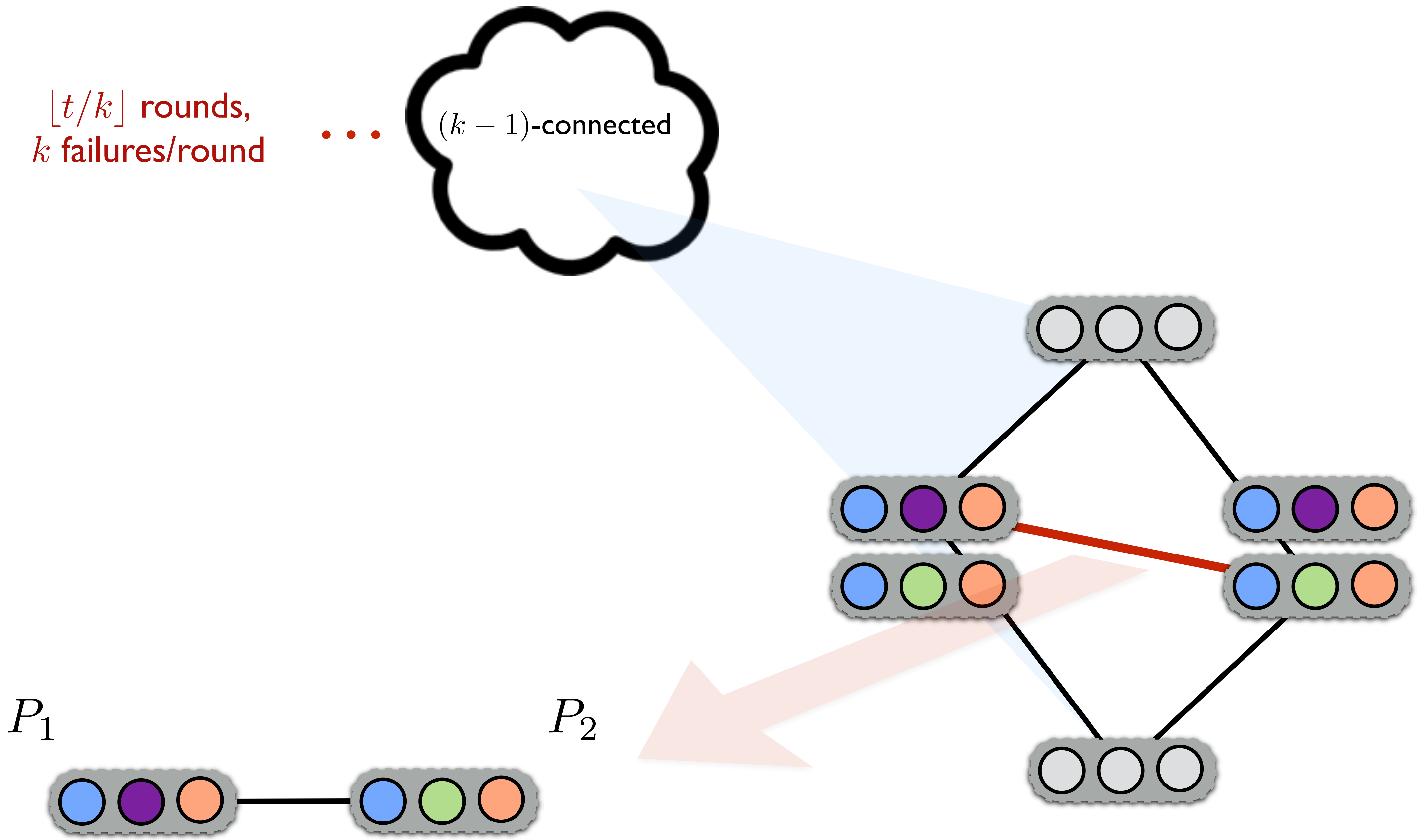

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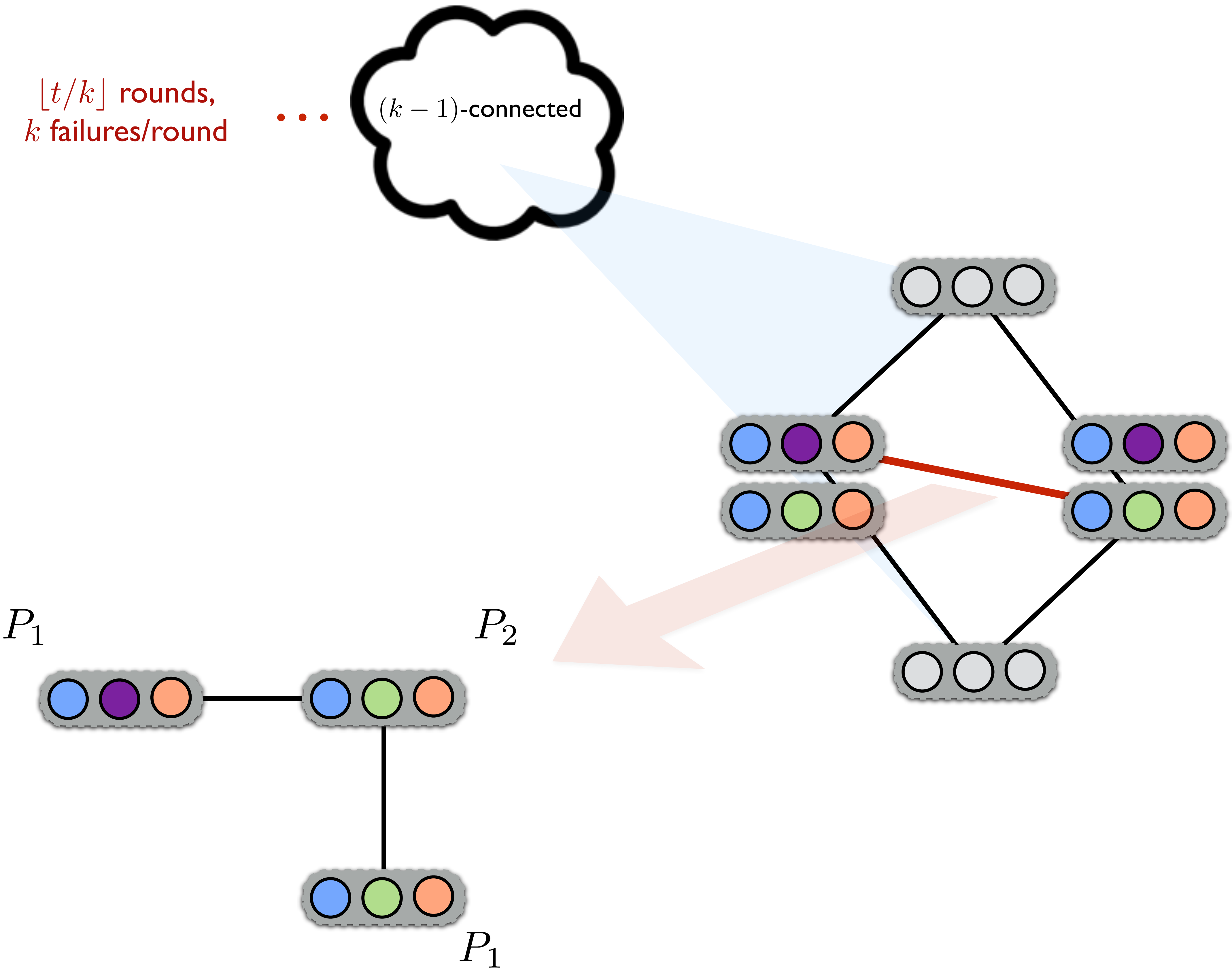

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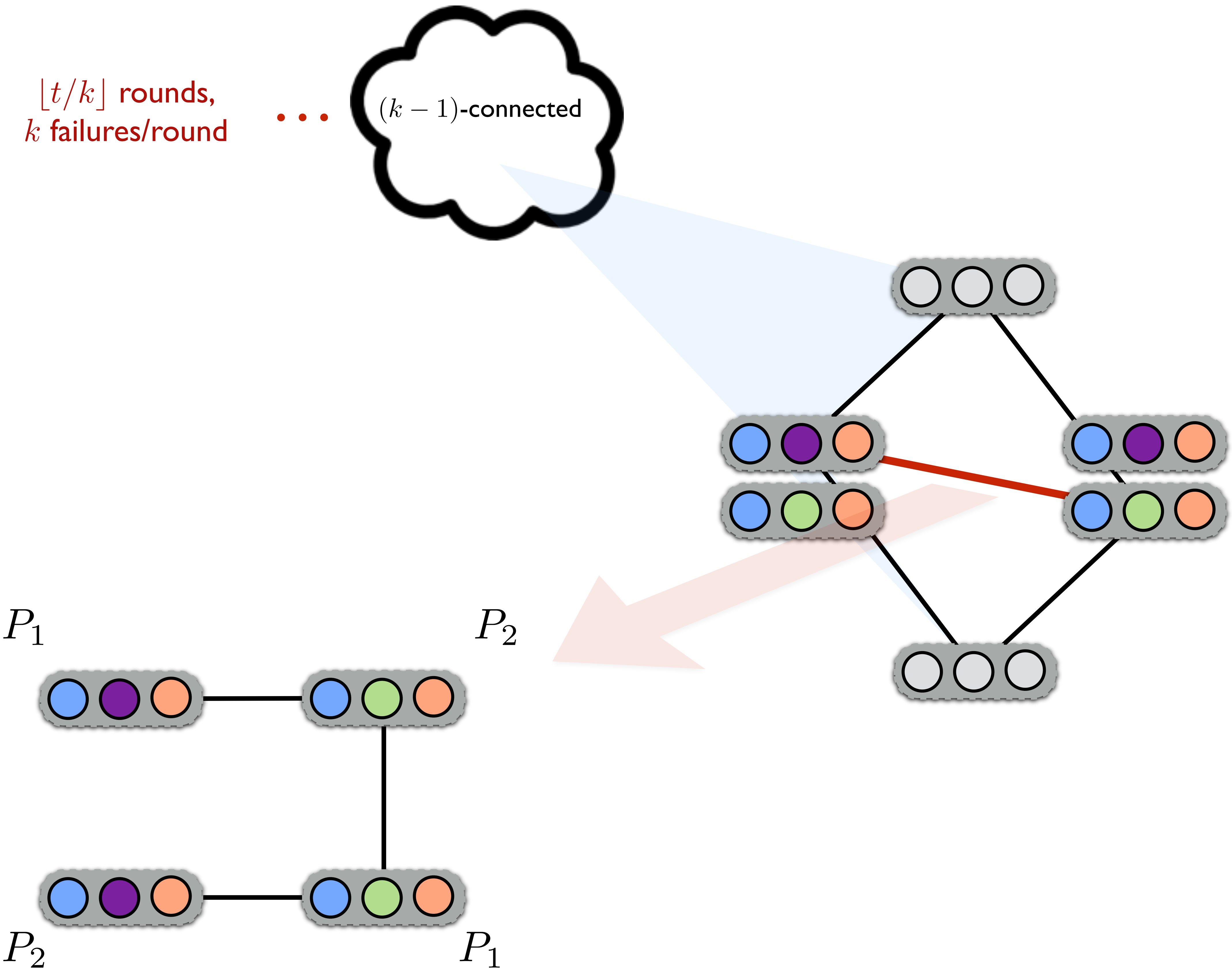

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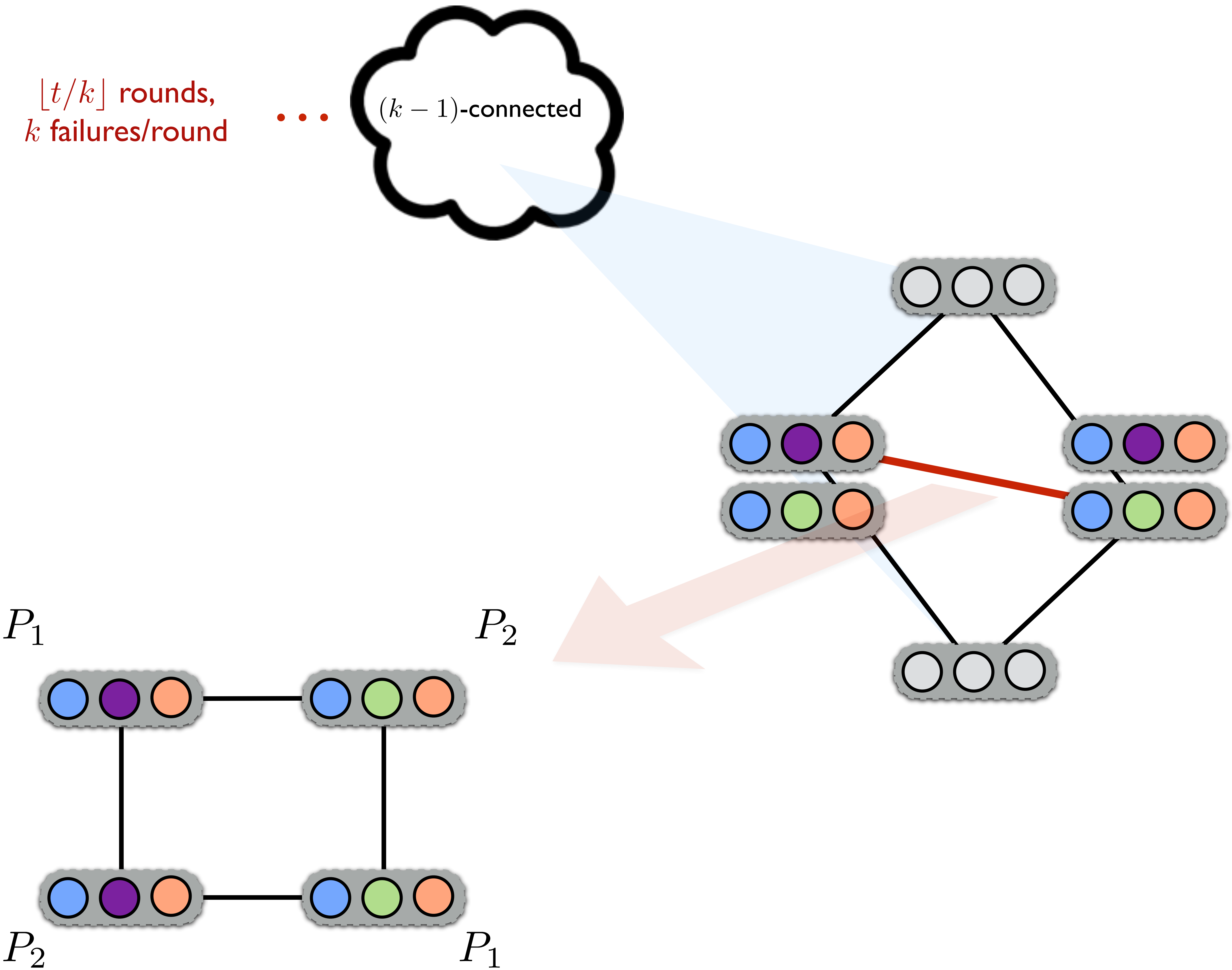

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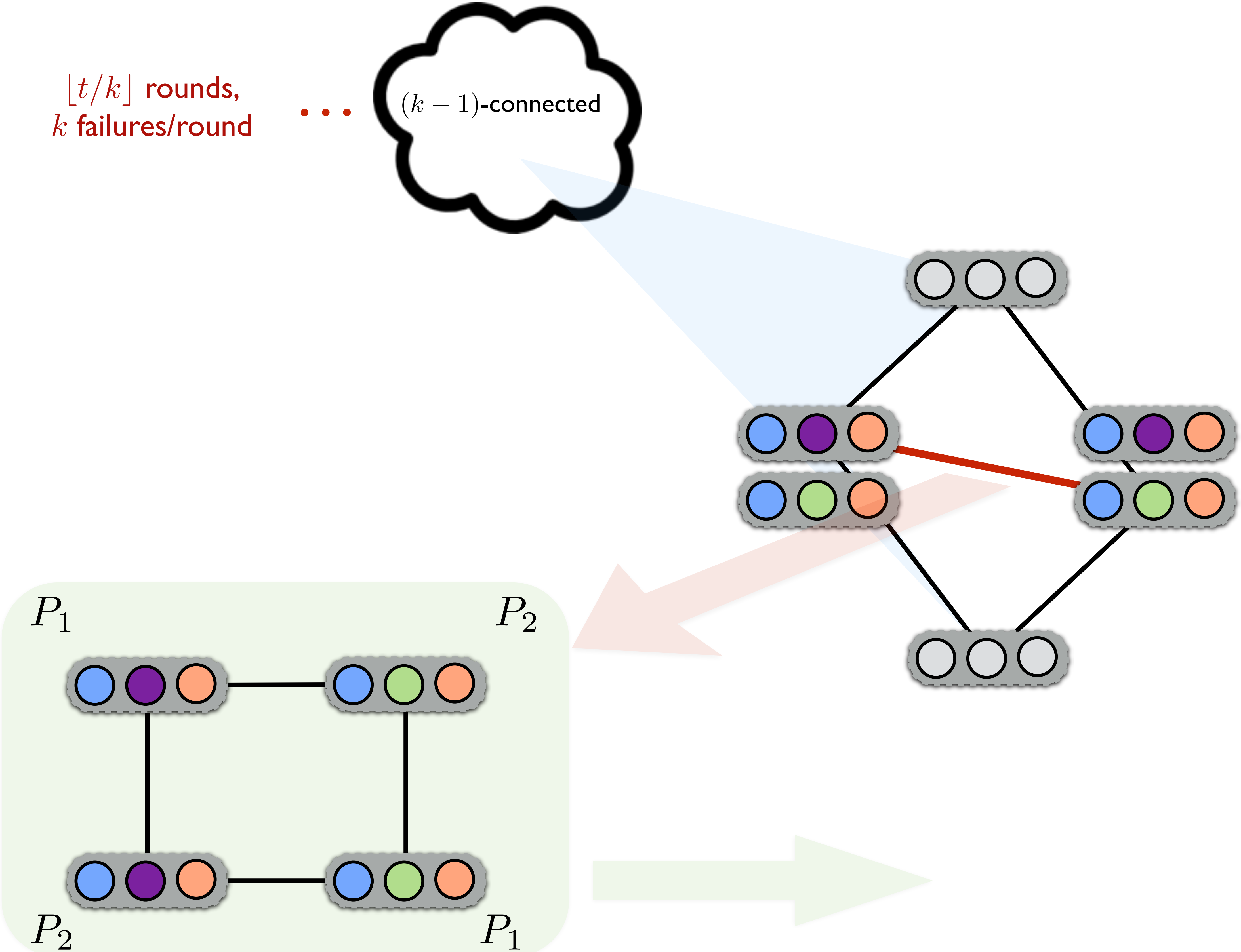

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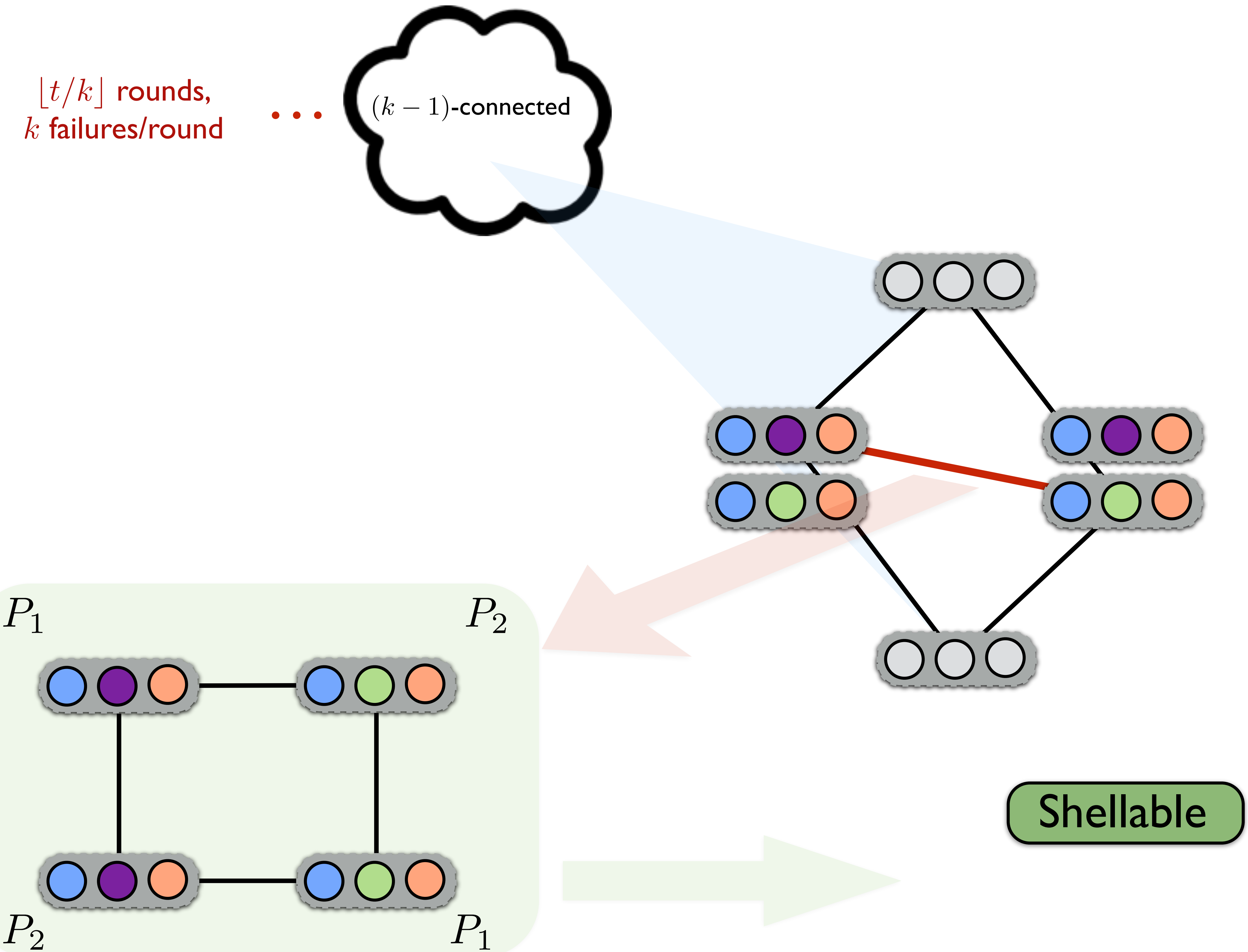

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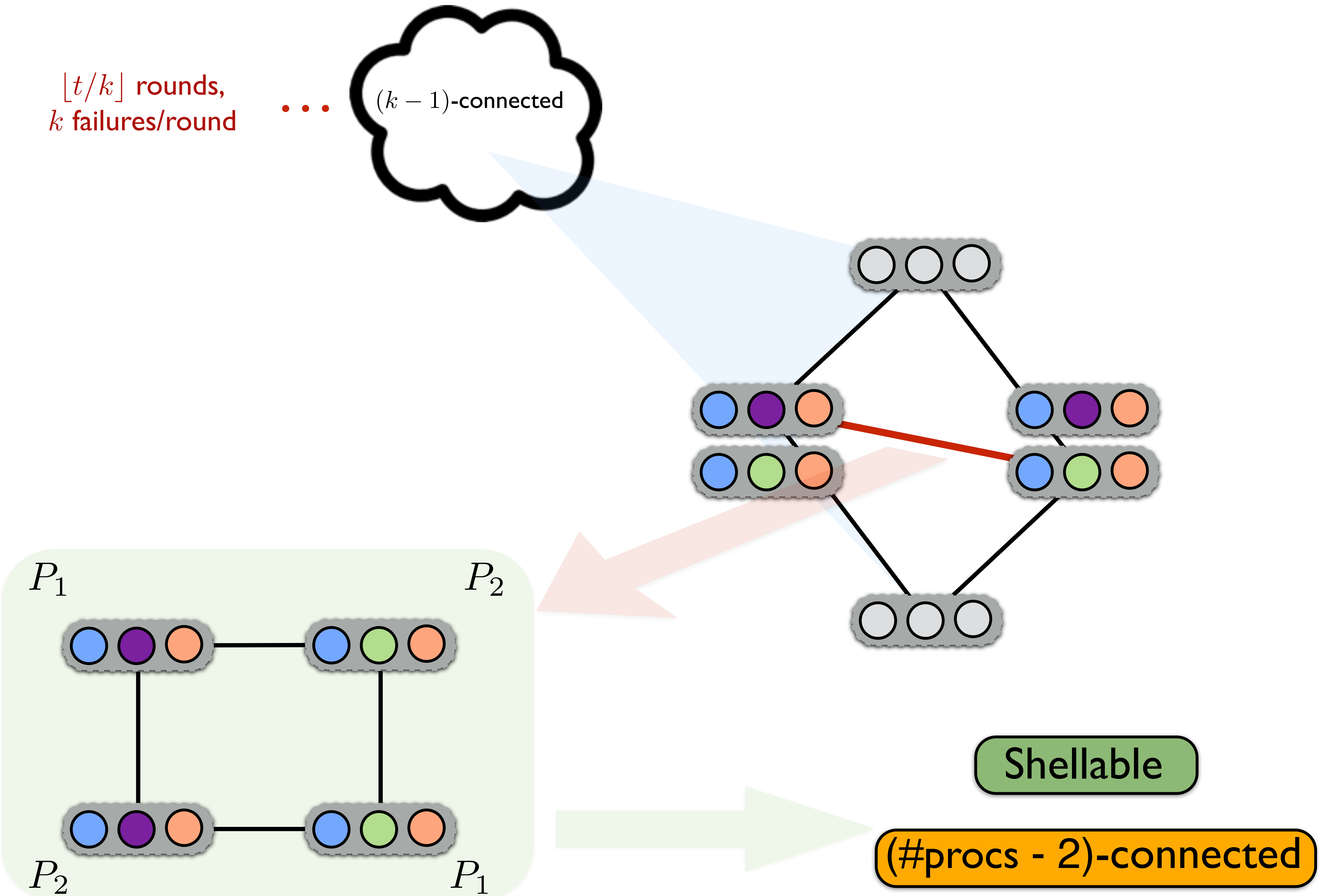

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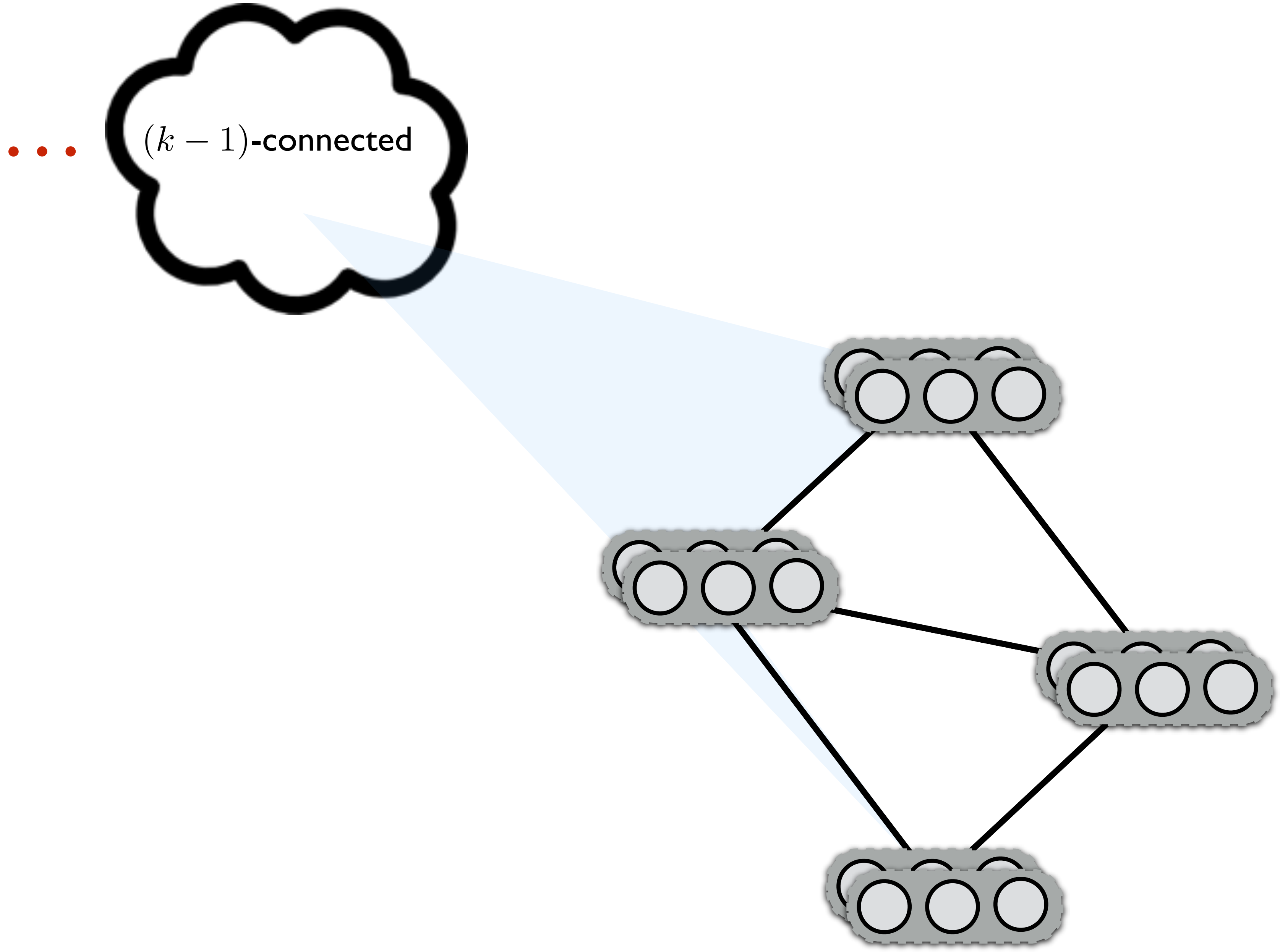

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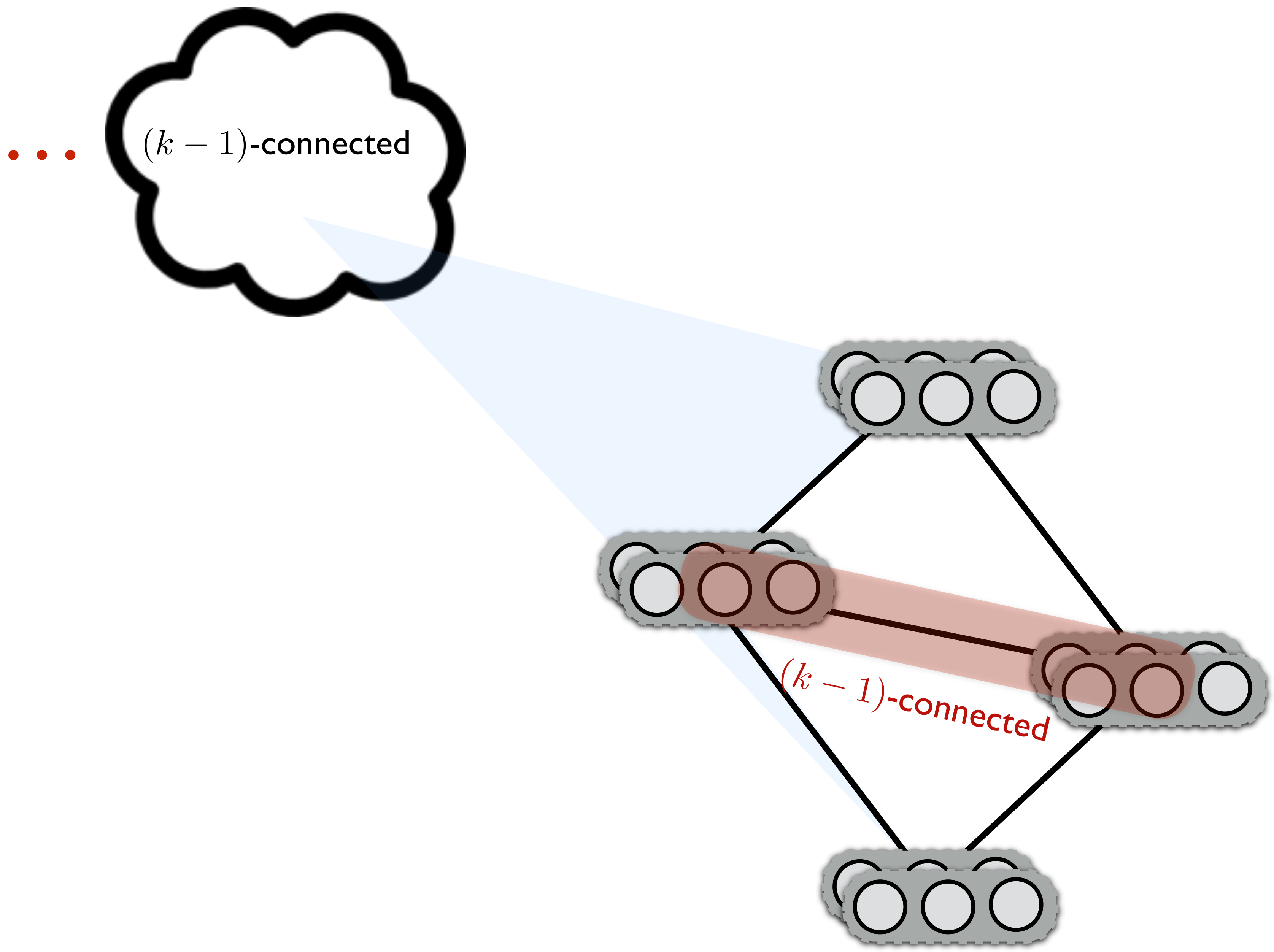

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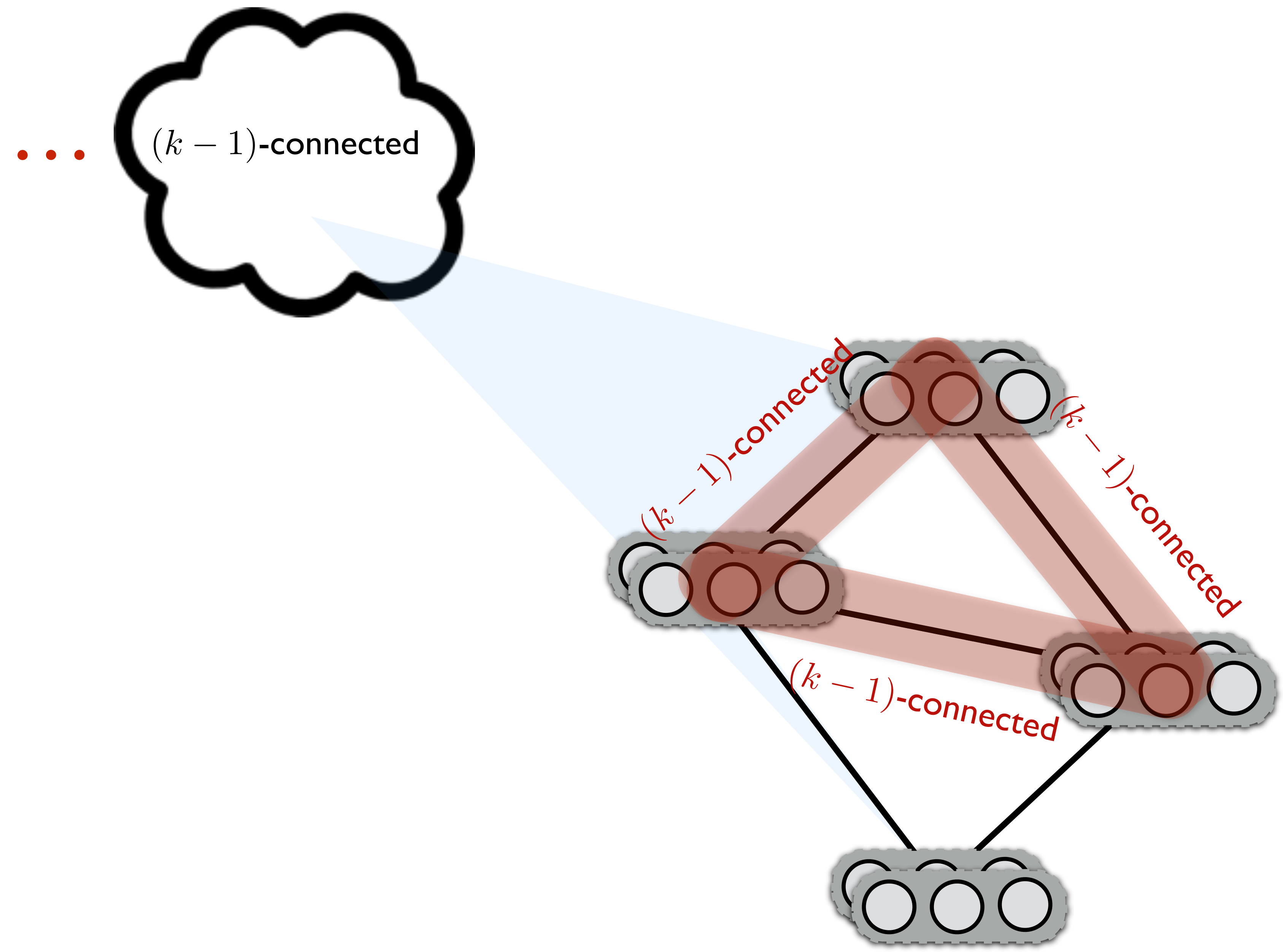

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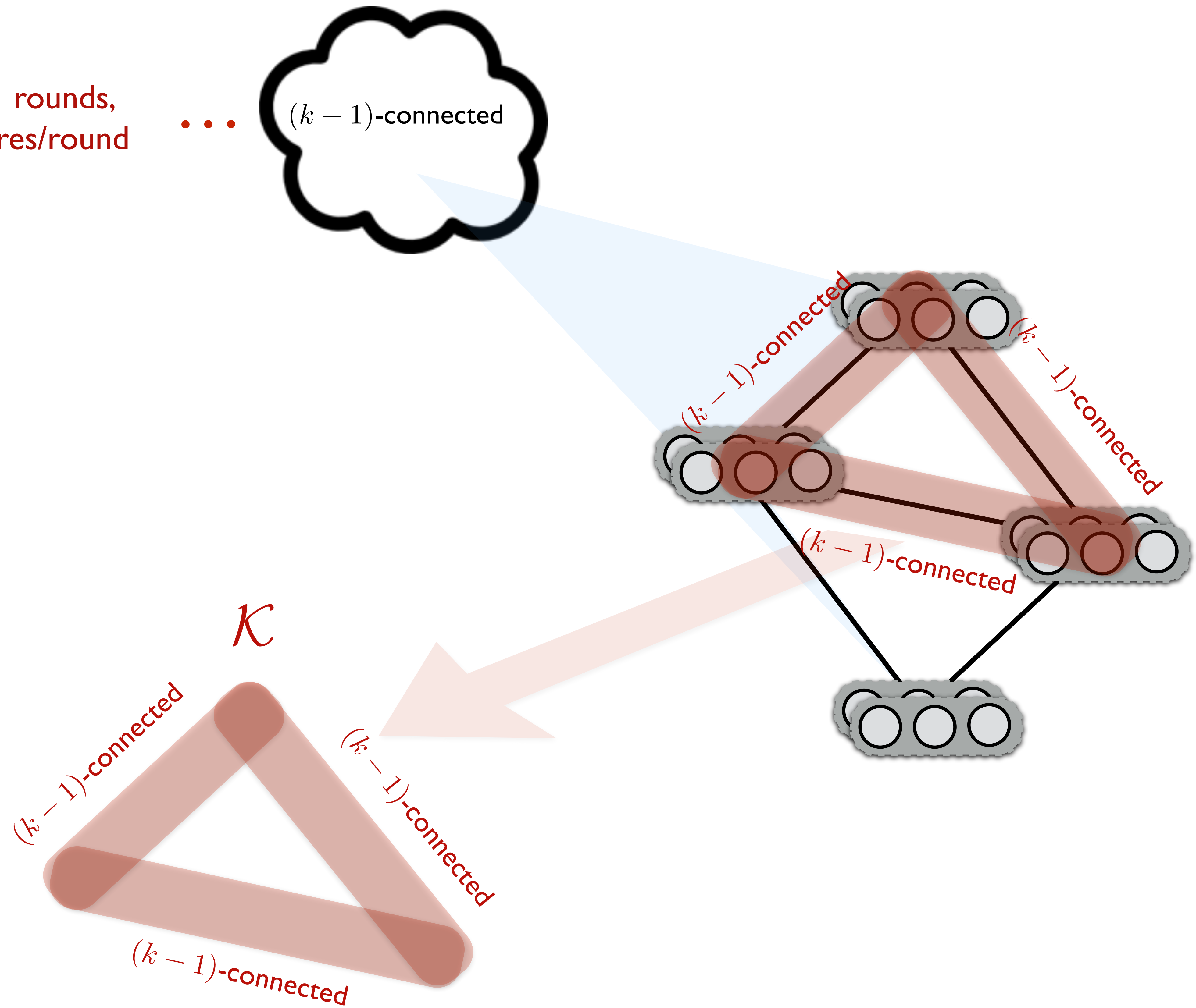

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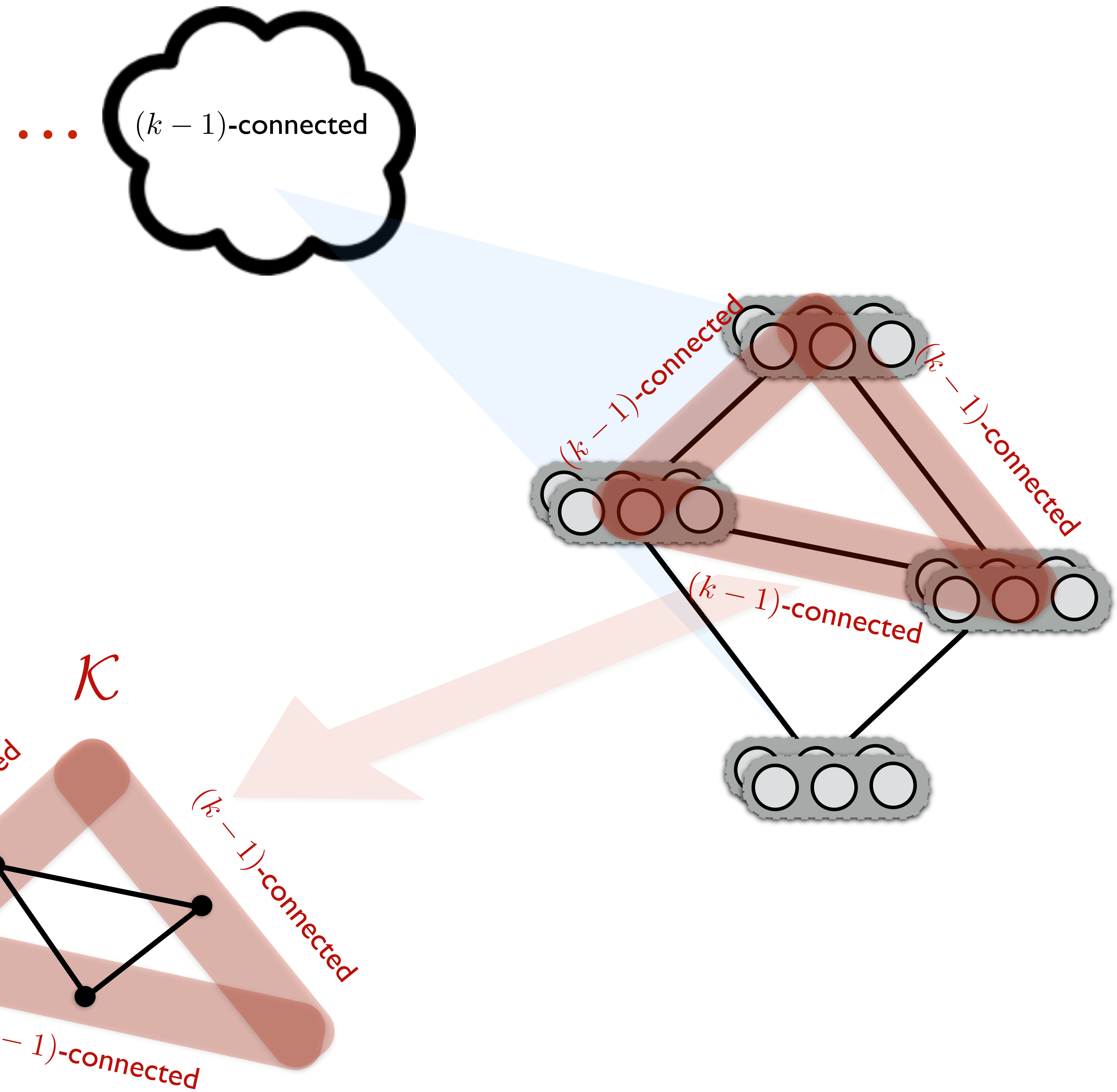

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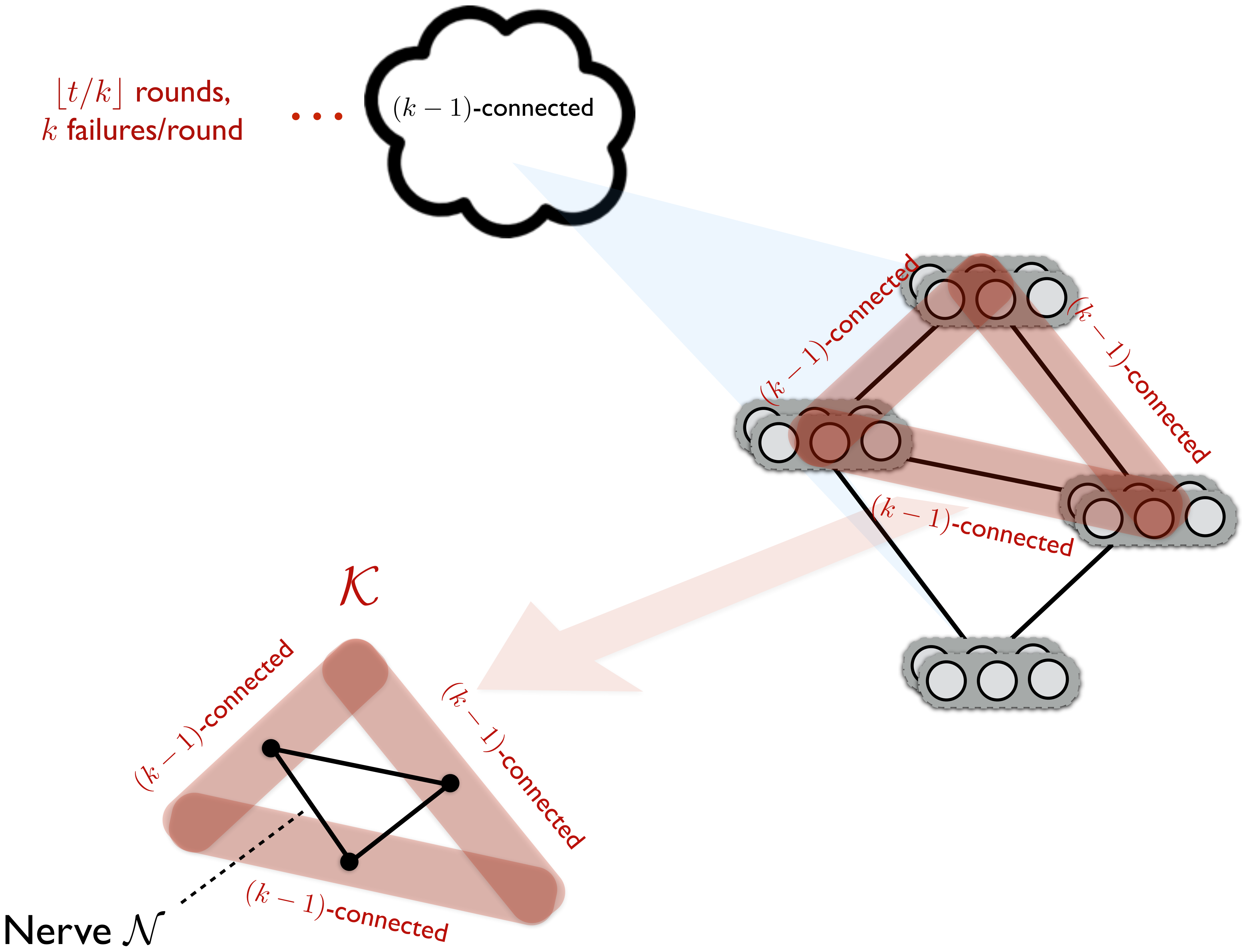

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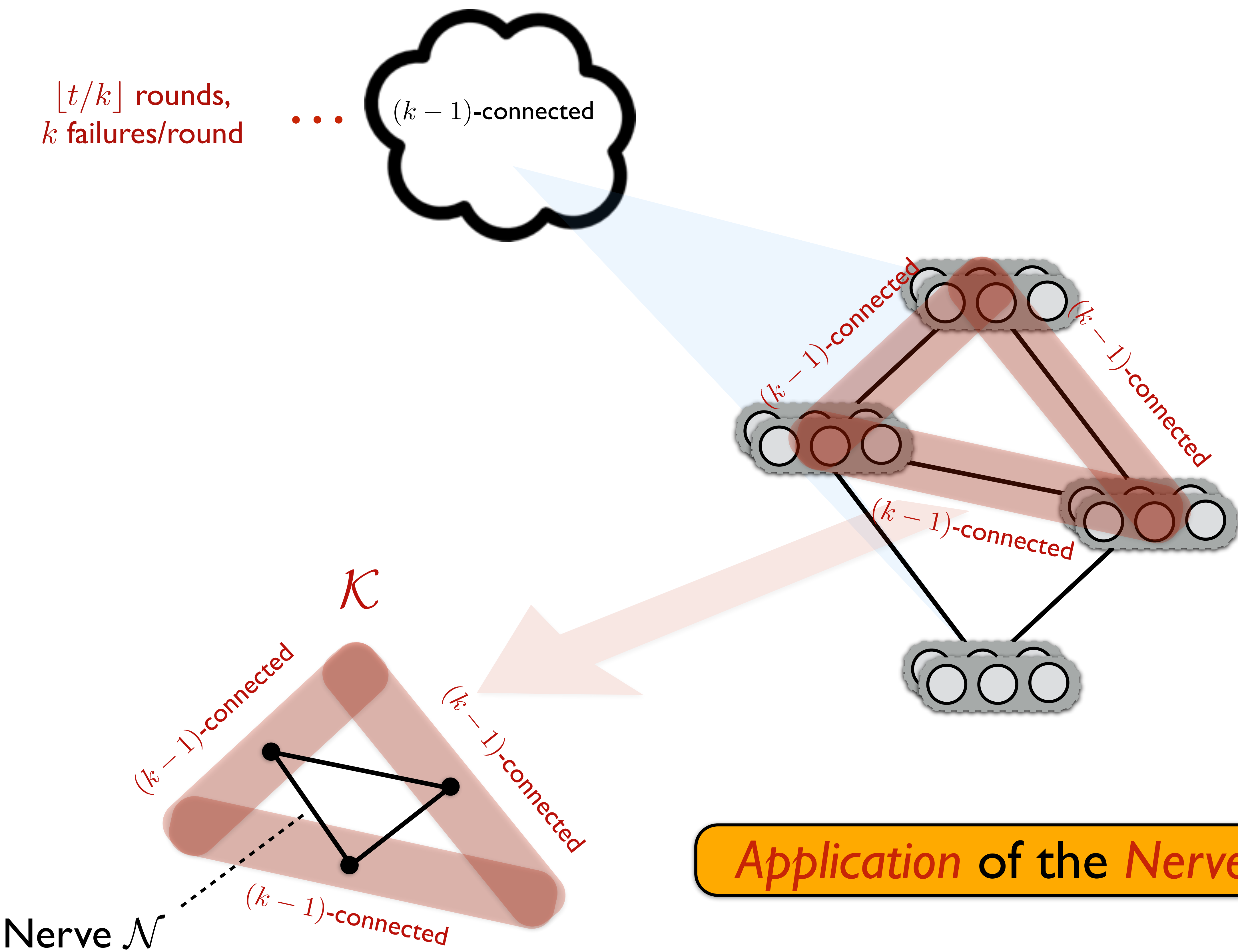

Application of the *Nerve Lemma*

The Equivocation Operator

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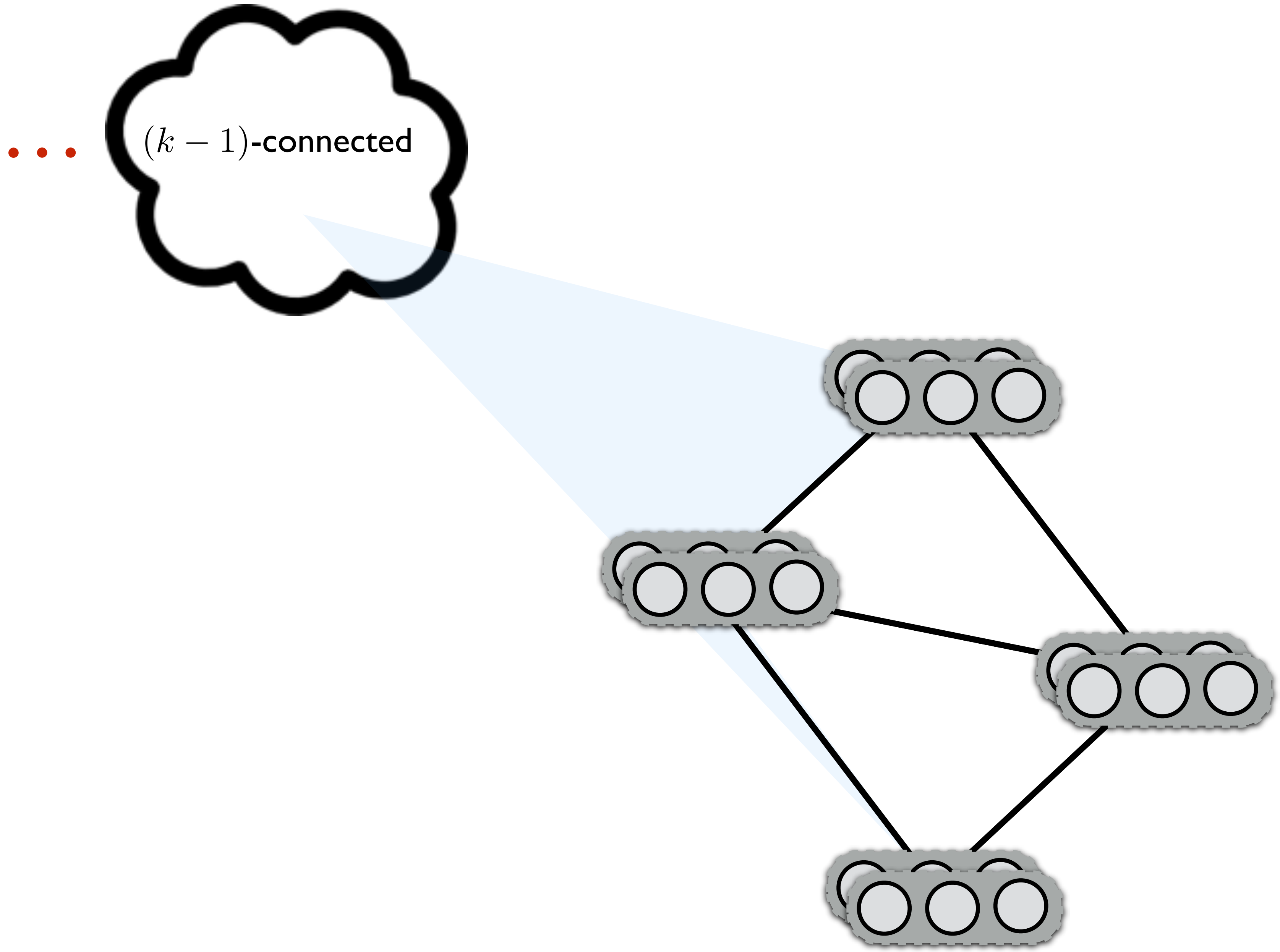

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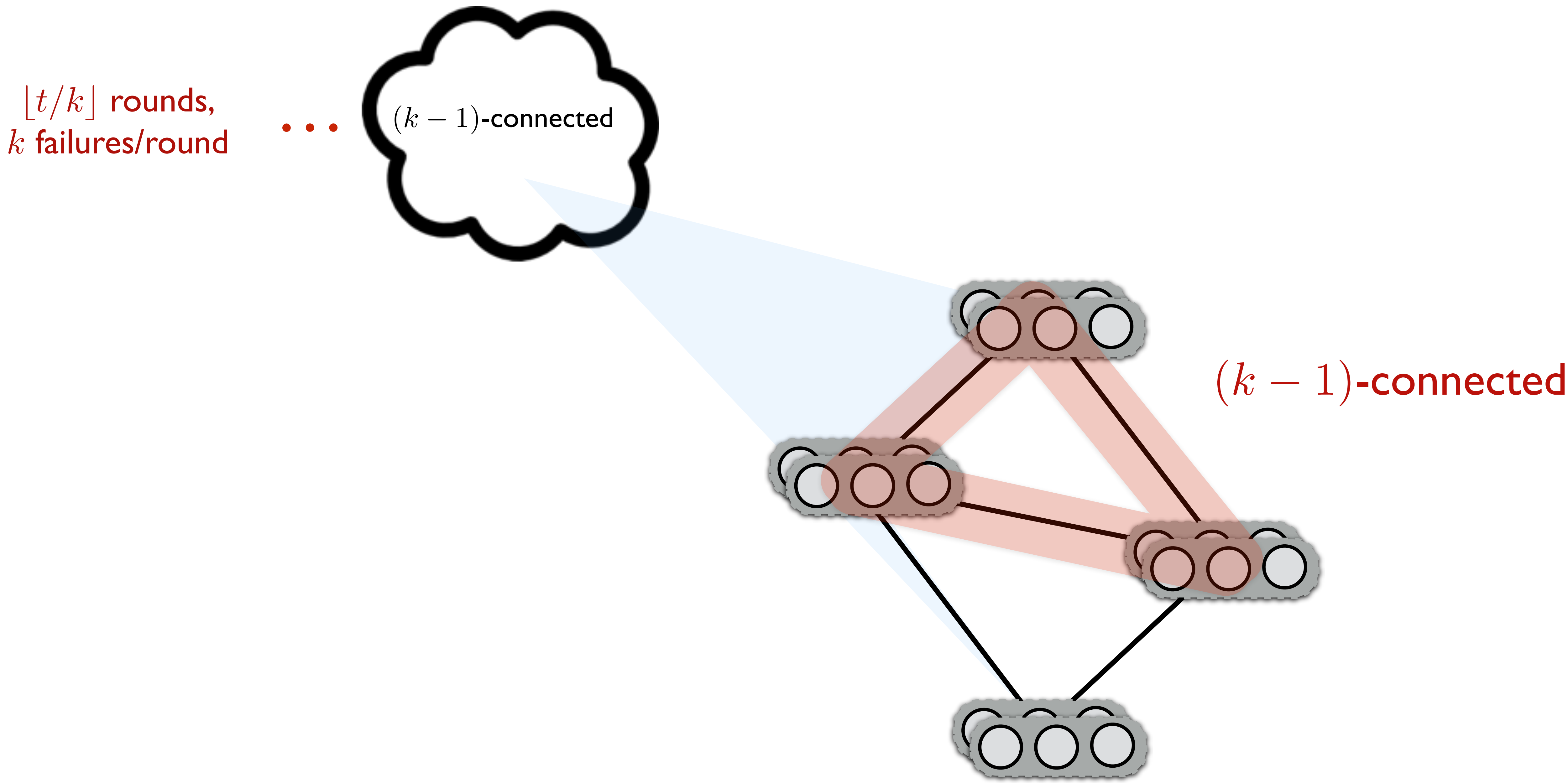

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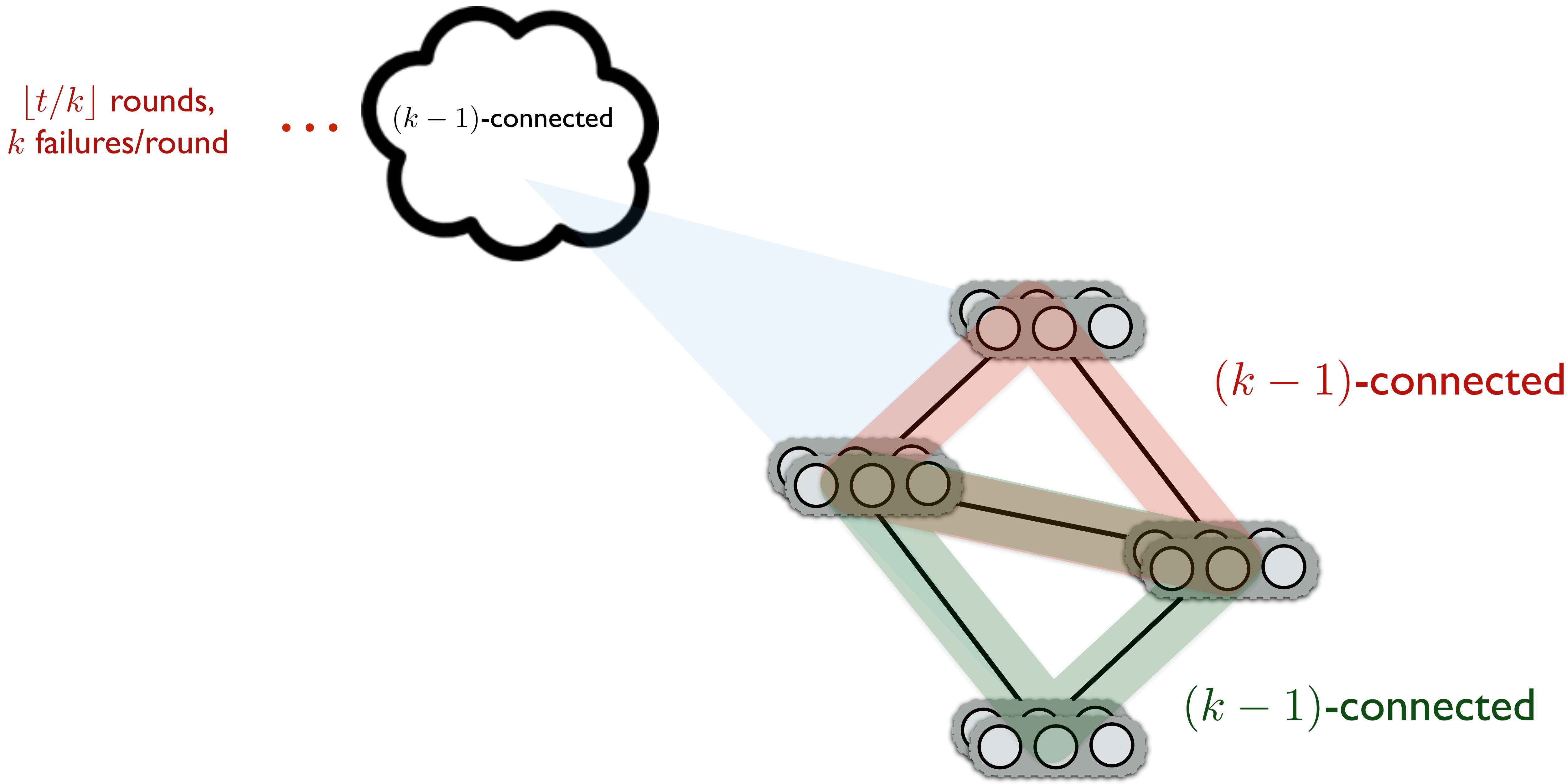

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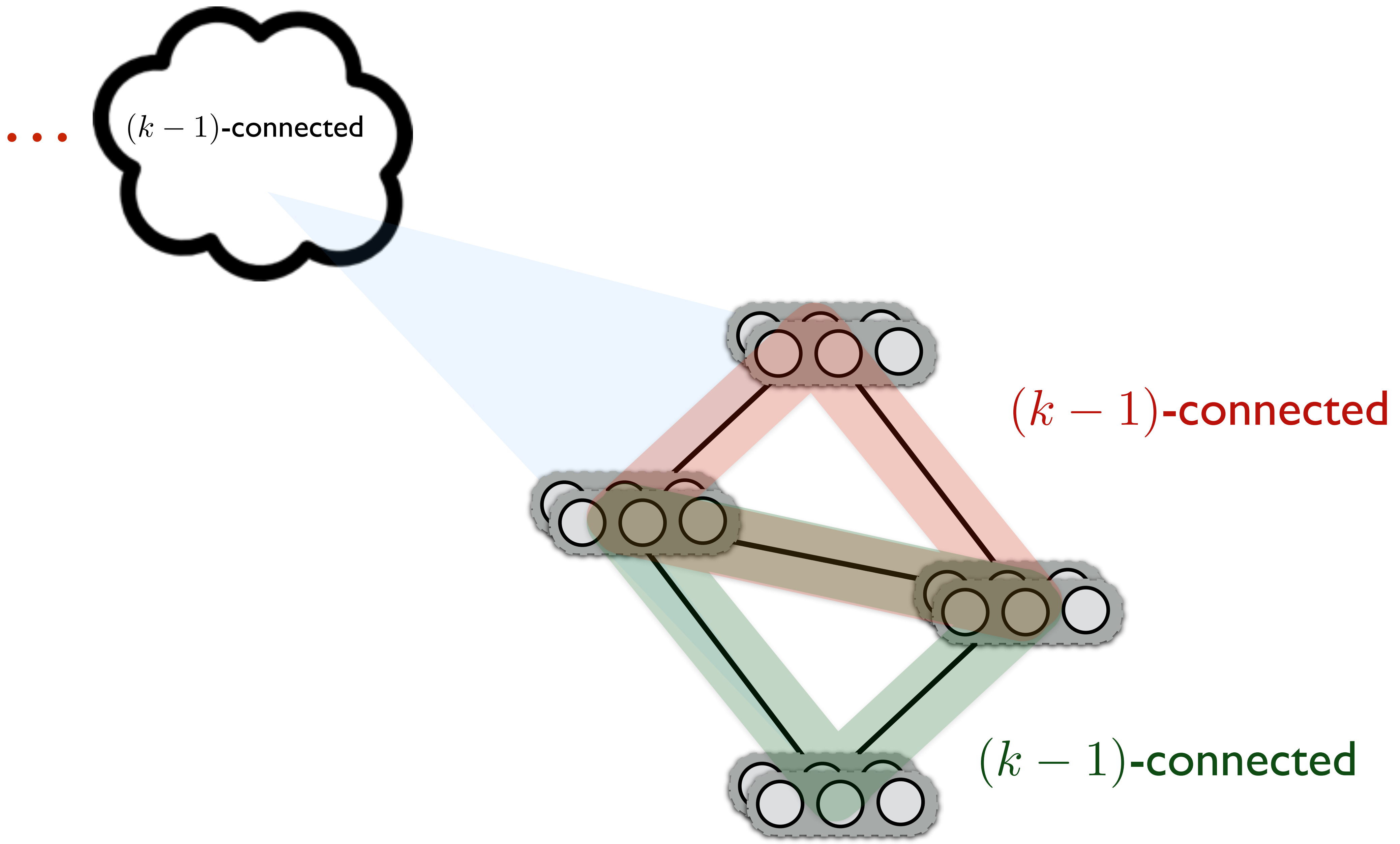
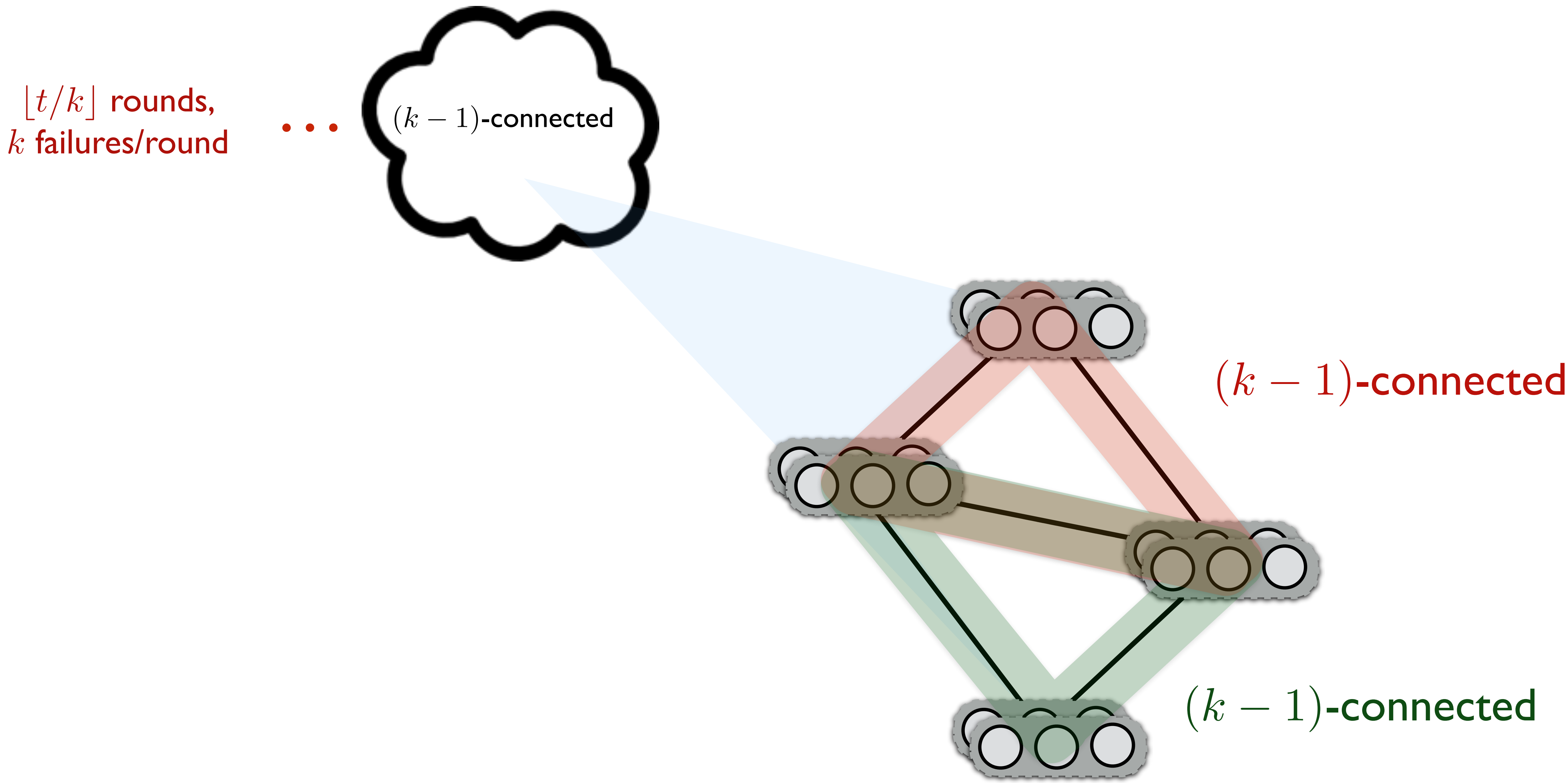


Extend throughout structure

The Equivocation Operator



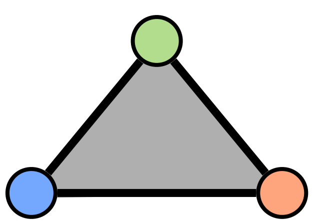
Extend throughout structure

Subsequent applications of the Nerve Lemma

Strategy, Again

We match the bound with an *algorithm*

$\mathcal{K}_0 = \mathcal{I}^*$ $\quad$ $\mathcal{K}_1 = \mathcal{R}_c(\mathcal{K}_0, k)$ $\quad$ $\mathcal{K}_2 = \mathcal{R}_c(\mathcal{K}_1, k)$ $\quad$ $\mathcal{K}_3 = \mathcal{R}_e(\mathcal{K}_2)$



...

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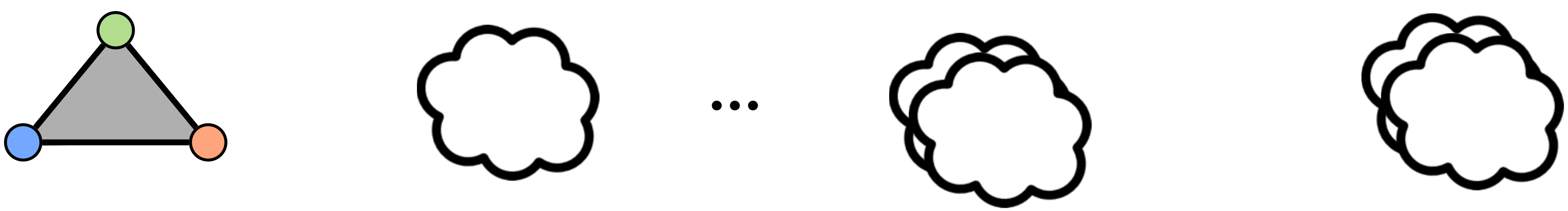

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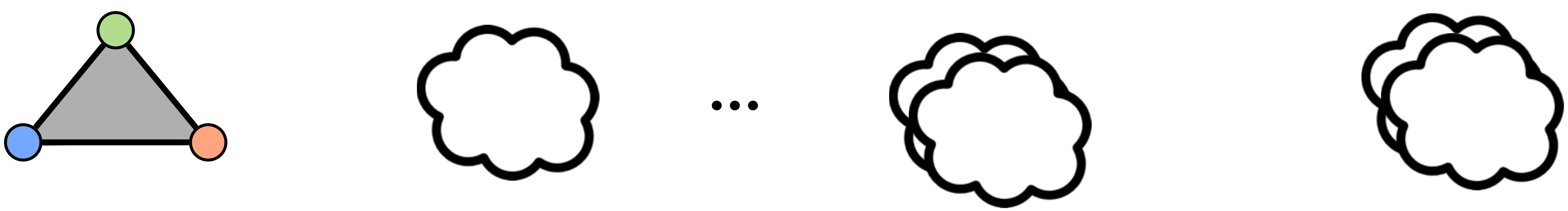

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    - Proofs are not constructive
- Failure Detectors
    - Allow us to detect the crash of peer processes
    - Again, many *impossible* problems now become *possible*

References

Published:

[1] Hammurabi Mendes and Maurice Herlihy. Multidimensional approximate agreement in Byzantine asynchronous systems. In *Proceedings of the 45th annual ACM Symposium on Theory of Computing*, STOC'13, pages 391–400, New York, NY, USA, 2013. ACM.

[2] Hammurabi Mendes, Maurice Herlihy, Nitin Vaidya, and VijayK. Garg. Multidimensional agreement in Byzantine systems. *Distributed Computing*, pages 1–19, 2015.

[3] Hammurabi Mendes, Christine Tasson, and Maurice Herlihy. Distributed computability in Byzantine asynchronous systems. In *Proceedings of the 46th Annual ACM Symposium on Theory of Computing*, STOC '14, pages 704–713, New York, NY, USA, 2014. ACM.

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Thank You!

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