Suppose $f : N \to M$. Then $d_p f : T_p(N) \to T_p(M)$.

If df_p is $1-1$, for all $p \in N$, then f is called an *immersion*.
I.e., f is an immersion iff f has rank n

If df_p is onto for all $p \in N$, then f is called a *submersion*.
I.e., f is an submersion iff f has rank m

Defn. Suppose $f : M \to N$ is smooth.

$p \in M$ is a *critical point* and $f(p)$ is a *critical value* if rank $df_p < n$.

If $p \in M$ is not a critical point, then it is a *regular point*.

If $q \in N$ is not a *critical value*, then it is a *regular value*.

Note: $q \in N$ is a regular value iff $f^{-1}(q) = \emptyset$ or $\forall p \in f^{-1}(q)$, $df_p = n$.

Defn: K is a *m-submanifold* of N if $\forall q \in K \subset N$, $\exists g^{smooth} : V^{open} \subset N \to \mathbf{R}^{n-m}$, $q \in V$ such that

1.) g is smooth

2.) $K \cap V = g^{-1}(0)$ and

3.) rank $d_p g = n - m$

Defn: Suppose f: $M \to N$ is a $1-1$ immersion, and suppose f: $M \to f(M)$ is a homeomorphism, where $f(M) \subset N$ has the relative topology. Then f is an *embedding*, and $f(M)$ is an embedded submanifold.

Thm. $f : M \to N$ embedding implies $f(M)$ is a submanifold of N.

Thm 2.3.13: Let q be a regular value of f: $M \to N$. Then either $f^{-1}(q) = \emptyset$ or $f^{-1}(q)$ is an $(m-n)$-submanifold of M.