Method 1:

$T_{\mathbf{a}}(\mathbf{R}^n) = \{(\mathbf{a}, \mathbf{x}) \mid \mathbf{x} \in \mathbf{R}^n\}$

$\phi(\mathbf{ax}) = \mathbf{x} - \mathbf{a}$

canonical basis $\{\phi^{-1}(e_i) \mid i = 1, ..., n\}$

Method 2:

Let $x(t) : \mathbf{R} \to \mathbf{R}^n$, a C^1 curve such that $x(0) = \mathbf{a}$

$x(t) \sim y(t)$ if $x_i'(t) = y_i'(t)$ for $t \in (-\epsilon, \epsilon)$

Let $f([x(t)]) = \mathbf{x}'(0) = (x_1'(0), ..., x_n'(0))$

Let $T_{\mathbf{a}}(\mathbf{R}^n) = \{[x(t)] \mid x \in C^1, x(0) = \mathbf{a}\}$

$[x(t)] + [y(t)] = f^{-1}(x'(0) + y'(0))$

$\alpha[x(t)] = f^{-1}(\alpha x'(0))$

Let $f : \mathbf{R}^n \to \mathbf{R}$ and let $\mathbf{v} \in R^n$ such that $||\mathbf{v}|| = 1$

The directional derivative of f at $\mathbf{a}$ in the direction of $\mathbf{v}$ is

$D_{\mathbf{v}} f(\mathbf{a}) = lim_{h \to 0} \frac{f(\mathbf{a}+h\mathbf{v}) - f(\mathbf{a})}{h}$

$= D[f(\mathbf{a} + t\mathbf{v})]_0 = Df_{\mathbf{a}}\mathbf{v} = Df_{\mathbf{a}} \cdot \mathbf{v} = (\frac{\partial f}{\partial x_1}, ..., \frac{\partial f}{\partial x_n})|_a \cdot \mathbf{v} =$