$bdE = \overline{E} \cap \overline{X - E} = \overline{E} - E^o$
$p \in X$ is a boundary point of E if $p \in bdE$.
p is an isolated point of E if $p \in E$, but $p \notin E'$.
$p \in X$ is a limit point of E if for all U open such that $x \in U$, $U \cap E - \{x\} \neq \emptyset$

A point p is an interior point of E if there exists a basis element B such that $p \in B \subset E$.

$$\begin{aligned} E^o &= \text{the set of all interior points} \\ &= \{x \in E \mid \text{ there exist } B \in \mathcal{B} \text{ s. t. } x \in B \subset E\} \\ &= \text{largest open set contained in } E = \cup_{U^{open} \subset E} U. \end{aligned}$$

Note: $E^o \subset E$.

E is open iff every point of E is an interior point.
E is open iff $E = E^o$.
E is open iff $bdE \subset E^c$

$$\begin{aligned} \overline{E} &= E \cup E' = \text{smallest closed set containing } E = \cap_{F^{closed} \supset E} F \\ &= \{x \in X \mid \text{ for all } U \text{ open such that } x \in U, U \cap E \neq \emptyset\} \end{aligned}$$

E is closed iff E^c is open.
E is closed iff $E = \overline{E}$.
E is closed iff $E' \subset E$.
E is closed iff $bdE \subset E$

Subspace topology: Suppose $Y \subset X$.

E is open in Y if and only if there exists a set U open in X such that $E = U \cap Y$.

E is closed in Y if and only if there exists a set F closed in X such that $E = F \cap Y$.

Questions to consider:

Can a point be both a boundary point and an isolated point?

Can a point be both a boundary point and a limit point?

Can a point be both a boundary point and an interior point?

Can a point be both a limit point and an isolated point?

Can a point be both a limit point and an interior point?

Can a point be both an isolated point and an interior point?

Consider the integers, rationals, $\{\frac{1}{n} \mid n = 1, 2, 3, ...\}$, (0, 1), (0, 1], [0, 1], using standard, subspace, discrete, indiscrete topologies.

Give an example to show that a sequence can have a limit, but when the sequence is considered as a set, the set has no limit points.

Prove that T_1 is a needed part of the hypothesis of thm 17.9 and Hausdorff is a needed part of the hypothesis of thm 17.10.