Topology HW answers (many answers adapted from latexed HW of Colin B. McKinney)

p. 111:

2.) Suppose that $f : X \mapsto Y$ is continuous. If x is a limit point of the subset A of X, is it necessarily true that $f(x)$ is a limit point of $f(A)$?

It is NOT necessarily true that $f(x)$ is a limit point of $f(A)$ if x is a limit point of the subset A of X. The constant function $f : R \rightarrow R$, $f(x) = 0$ is continuous. 5 is a limit point of R, but $f(5) = 0$ is not a limit point of $f(R) = \{0\}$ since $\{0\}$ has no limit points.

9.) Let A_i be a finite collection of subsets of X; let $X = \cup_i A_i$. Let $f : X \mapsto Y$; suppose that $f|A_i$ is continuous for each i.

Since each $f|A_i$ is continuous, if we choose any set C closed in Y, $(f|A_i)^{-1}(C) = f^{-1}(C) \cap A_i$ is closed in A_i. Since A_i in closed in X, $f^{-1}(C) \cap A_i$ is closed in X. Hence $\cup_i (f^{-1}(C) \cap A_i) = f^{-1}(C) \cap (\cup_i A_i) = f^{-1}(C) \cap X = f^{-1}(C)$ is closed since finite unions of closed sets are closed. Hence f is continuous.