section 18: 13) Suppose $A \subset X$. Let $f : A \to Y$ be continuous where Y is T_2. If $g_i : \overline{A} \to Y$ is continuous for $i = 1, 2$ and if $g_1|_A = g_2|_A = f$, then $g_1 = g_2$

Pf: Suppose $g_1 \neq g_2$. Then $\exists x \in \overline{A} - A$ such that $g_1(x) \neq g_2(x)$. Y T_2 implies $\exists V_1, V_2$ open in Y such that $g_i(x) \in V_i$ and $V_1 \cap V_2 = \emptyset$

$x \in g_i^{-1}(V_i)$ implies $x \in g_1^{-1}(V_1) \cap g_2^{-1}(V_2)$. g_i continuous implies $g_i^{-1}(V_i)$ open in $\overline{A}$ and hence $g_1^{-1}(V_1) \cap g_2^{-1}(V_2)$ open in $\overline{A}$. Thus there exists U open in X such that $g_1^{-1}(V_1) \cap g_2^{-1}(V_2) = U \cap \overline{A}$. Note $x \in U$.

$x \in \overline{A} - A$ implies $x \in A'$. Thus $\exists a \in U \cap A - \{x\} \subset g_1^{-1}(V_1) \cap g_2^{-1}(V_2)$. But $f(a) = g_i(a) \in g_i(g_i^{-1}(V_i)) \subset V_i$. But $V_1 \cap V_2 = \emptyset$.

NOTE: It may have been simpler to WLOG assume $X = \overline{A}$, but you must state why this WLOG works.